



SU(3) Projections and Hilbert Space

Gudrun Kalmbach HE*

MINT, PF 1533, D-86818 Bad Woerishofen, Germany

*Corresponding Author: Gudrun Kalmbach HE, MINT, PF 1533, D-86818 Bad Woerishofen, Germany.

Received: February 24, 2025

Published: March 06, 2025

© All rights are reserved by **Gudrun Kalmbach HE.**

Abstract

In the QCD theory GellMann 3x3-matrices present 8 gluons. An extended view is that mesons color charge cc pairs arise too and that the gravity particle is found as a strong superposition of three color charges r,g,b, (red-green-blue) found as neutral cc of nucleons. The complex scaling of suitable GellMann matrices allows the use of a complex 4-dimensional Hilbert space. Its projections form an orthomodular closed subspace lattice. Its semantics and structure should be studied in more detail. The wave particle duality is solved using mesons and baryons. Graviton waves get a SU(3) isospin presentation with a model. The big bang model of astronomy is extended.

Keywords: Forces; Hilbert Space; Projective Geometry; QCD; Raypresentation

SU(3) symmetry and octonions

For the symmetry of the strong interaction the 3 Pauli 2x2-matrices are 3x3 extended by inserting a row and column with 0 values.

The multiplication table shows why two matrices are added for getting λ_8 since the three diagonal σ_3 matrices are linearly dependent and generate only 2 gluons. The single matrices are $\lambda_9 = \text{diag}[1,0,-1]$ and $\lambda_{10} = \text{diag}[0,1,-1]$. The matrices with index $j = 1,2,4,5,6,7$ present six gluons with 2 color charges cc, for the indices $j = 3,9,10$ the cc uc(u) are for mesons $u = r,g,b, c(u)$ its conjugate. Since meson decay similar to weak bosons, as replacement for spin/Pauli matrices three isospin coordinates are set to them. These meson decays are well documented. Since stretching uses 345 (dual graviton), 3 as tip of two vectors to 4,5 for a meson (anti-)quark allows for their stretching that the energy input after reaching a threshold splits them into two mesons where the pair adds the missing part.

This shows up in the SU(3) topological product geometry $S^3 \times S^5$ where the first Hopf factor is for the uc(u). S^5 is for a newly defined strong fiber bundle [1] with fiber S^1 . It is for nucleons complex projective 2-dimensional projected space CP^2 with a bounding (real 2-dimensional) Riemannian sphere S^2 . If instead of the λ_9 ma-

trix the extended identity $\text{diag}[1,1,0]$ is used, the scaled and added matrices can have rows $(z_1, z_2, z_3), (-c(z_2), c(z_1), z_4), (-c(z_3), -c(z_4), 0)$. This coordinate presentation is also obtained by using the complex cross product. The complex written spacetime is $z_1 = z + ict$, t time, c speed of light for xyz-space with $z_2 = x + iy$. extended with $z_3 = (m, f)$ for a mass m and frequency f coordinate, extended again with octonions cc force coordinate 0 and octonions coordinate 7 for the electromagnetic interaction EMI. In octonions xyzt-space is 1234. z_3 is 56. Octonions give measuring units for 8 energies, defined by Gleason operators [2] and their spin-like GF base triples, in the Fano figure drawn as lines or circle 123 (spin), 145 (electrical force), 167 (EMI), 246 (heat), 257 (mass), 347 (rotation, angular momentum), 356 (nucleon dynamics, frequency). Added are three strong GF 126 for rgb-gravitons, 345 for its dual and 037 for a cc G-compass (figure 1 right), 0 for the cc input, 7 for the EMI output of atoms (spectral series). G is a rotation matrix of order 6 with first row (1 -1) and second row (1 0).

Hilbert space

The countable infinite Hilbert space is used in quantum mechanics for state presentations of systems. In the MINT wigris library are models for instance of the nucleon states. It is enough to show them in a real 4-, complex 2-dimensional spacetime 1234

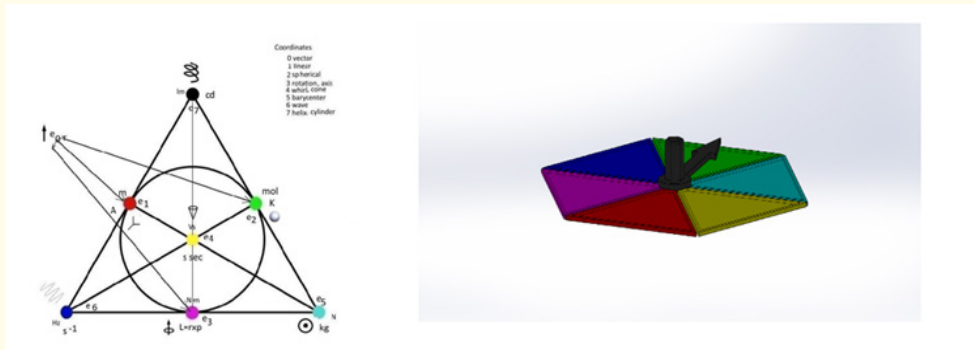


Figure 1: Fano memo at left, at right G-compass.

by adding octonions. The GellMann 3x3-matrix extensions of Pauli matrices show that they are projection matrices. The calculus for Hilbert space projections onto closed subspaces is not commutative. Algebraically the closed subspace lattices satisfy the orthomodula law and are the set theoretical union of blocks as maximal Boolean algebras of pairwise commuting operator projections. An invalid modus ponens or deduction theorem arise for the associated orthomodular logic. Under Wikipedia in 2024 are listed two articles written by quantum philosophers special Paradoxies and Quantum Logic. The second article discusses nondistributivity, but fails for the noncommutative orthomodular logic. The first article

fails to solve paradoxies, check invalid arguments on noncommutative operators. In further research from [3] should also be taken facts concerning the overlapping of Boolean blocks, especially in real 4 dimensions.

In Greechie diagrams for blocks which are drawn as intervals similar as in the Fano figure and contain four atoms there are two requests (figure 2). A Boolean subalgebra cycle with 3 atoms must be extended by a fourth atom to a block. Inside a 4-cycle of blocks must exist 8 different atoms in form of an astroid consisting of four blocks.

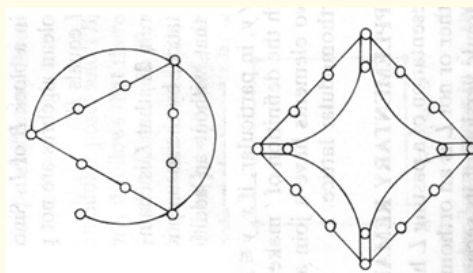


Figure 2: 3-cycle at left, 4-cycle and astroid at right.

Wave particle duality

The inner nucleon dynamics was described in several articles of the author (see the references). In construction barycentrical coordinates (figure 3 left) for nucleons it shows the particle character of fermions, also for leptons with SU(2). The experimentally found wave presentation is derived for the product topology $S^3 \times S^5$ of SU(3) GellMann extended 3x3-matrices.

For dark energy the bounding surface is a pinched torus (figure 3 middle). The pinched point can be at projective infinity and opens for a cylindrical geometry, observed for the electromagnetic interactions EMI exponential psi-wave presentation. The logarithmic surface log potential function inside dark energy is inverted to its exponential function exp and presented as periodic helix line on a cylinder. In case one transversal circle of the pin-

ched torus is at projective infinity its opens as a Minkowski cone with the equation $r^2 = c^2t^2$, c speed of light, r radius, t time. From dark energy with this surface speed $v' > c$ is inverted to universes systems speed v in $v'v = c^2$. Extending diagonal GellMann matrices to id $[1,1,0,0]$ to $[1,-1,0,0]$ $[1,0,-1,0]$ $[1,0,0,-1]$ for three isospin coordinates (replacing spin) allows the CPT discrete and commuting operators of physics as Klein group $Z_2 \times Z_2$. The first λ_3 matrix is for time reversal T. Color charge conjugation for mesons $uc(u)$, $u = r,g,b$ (red-gree-blue) is adding C and the Klein product adds parity P in $(1,-1) \cdot (-1,1) = (-1,-1)$. The cylindrical coordinates for the three

meson cases $uc(u)$ give them and their quarks the Schroedinger psi-wave. The matter wave substitutions are for a wave length λ momentum p in $\lambda p = h$, h the Planck constant, $\omega = 2\pi f$, f frequency for energy $E = hf$. Amplitdes for the waves are given by three cylinder radii in proportions $\frac{1}{2}:1:2$. In case of decays the generated heat phonons have their accoustic interval sounds as for a violins string in proportions for c, c', c'' (figure 3 right). The inflation period after the Planck time for space is related to this, a huge amount of generated mesons decay and the isospin coordinates generate the three xyz-space coordinates of the universe in a short time.

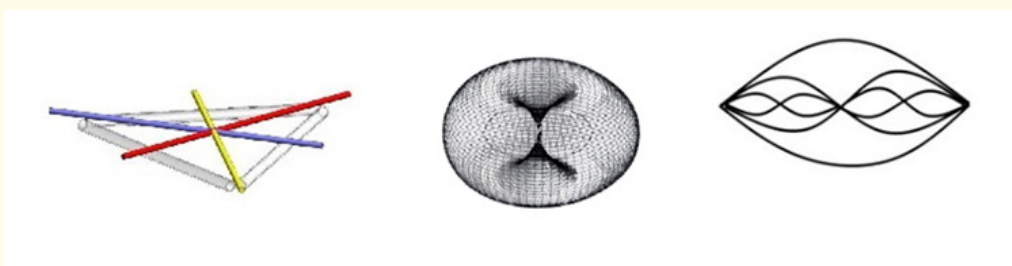


Figure 3: Barycenter of a nucleon quark triangle, pinched torus, heat vibrating interval oscillation.

The time reversal of CPT has the time interval reversed to frequency in $E = hf$ and the differentia d/dt for angle and length in angular speed $\omega = d\phi/dt$ and linear speed $v = ds/dt$. The conjugation is for the cross ratios taken the inverse of a fraction $z, 1/z; (1-z), 1/(1-z), z/(z-1), (z-1)/z$, the Heisenberg uncertainties 51, 46, 23. The parity sets for the rgb -graviton 126 thr rightsrew orintation and for its dual the leftscrew orientation. Also the wave amplitudes are set in their earthworm (figure 4) proportions 2:1:1/2 for spin 2 of the rgb -graviton using a circukar P^1 180 degree rotation, for bosons the usual 360 degree rotation of a circle an for fermions the Moebius strip (spin up/down) 720 degree rotation of its normal on its equatorial circle.

The projection setting of isospin allows also graviton GR waves to be presented in a model. Their GF is entangelt with the 2×2 -Pauli matrix xyz -space coordinates of $SU(2)$. The measuring units of this GF are Minowski metrical rescaled by the GR waves. Two rays in an angle ϕ from one origin in a plane generate for this the Minkowski watch (figure 4). The old metrical units are orthogonal projected from one to the other ray. For the GR wave model it means that it rescales first by emitting phonons/heat the radial r distance which is then $r/(\cos \phi)$ with $\sin \phi = v/c$, v relativistic speed. The product $r \cdot ct$ is unchanged, so time is squeezed. The second dual part is then active, sends phonons/heat to the GR wave and reverses this for

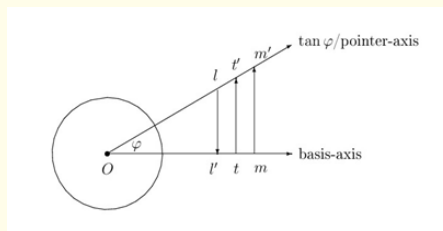


Figure 4: Minkowski watch for rescaling of metrical units.

squeezing radius and stretching time. In the model (figure 5) the isospin 3x3-matrix projections put the frequency helix line for the exponential function on cylindrical coordinates, similar to the EMI waves, but as Schroedinger waves which change their cylinder radius (or diameter) in the proportions 2:1:1/2 in spiralic rotations of $2\pi/3$ for red-blue-green when emitting heat in a right screw. Then

the right screw is for the dual part in a time T of CPT reversal changed to a left screw and the dual process is changing the radial proportions in 1/2:1:2 back by heat/phonons absorbed by the GR wave. The model is repeated when the GR wave travel with its momentum in the SU(2) Pauli/spin generated xyz-coordinates.

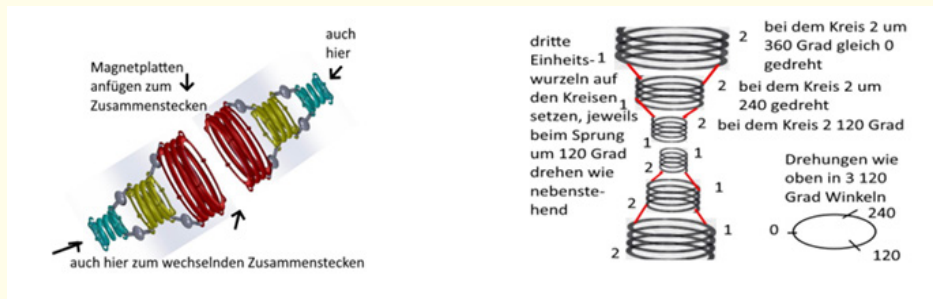


Figure 5: Earthworm as cylindrical GR wave model, exchanging heat for the Minkowski stretching/squeezing observed by LIGO directly

The entanglement of different GF is an important tool for future states of systems. For leptons is set for instance [1] an entanglement between the SU(2) xyz-space 123 GF with the EM 145 GF which explains the gyromagnetic relation and the helicity.

Some more details: Inverting for the universe presentation a dark energy pinched torus with a fixed diameter means that its pinched point at infinity is deleted and an energy winding on its log surface gets on a waves cylinder (for instance of a meson wave) a helix winding for the wave. The Heisenberg uncertainty exchanges quantized with h wave length and momentum. The time interval of the energy winding is inverted by the time energy $E = hf = h\omega/2\pi$ HU use to a angular rotation ω . For an angular momentum inside the dark energy the HU with an angle φ is used for presenting the waves exponential value $\varphi \rightarrow \exp(i\varphi)$ as for the Schroedinger matter wave. This holds for all fermions. The geometrical occurring entanglement is with $S^3 \times S^5$ for the S^3 meson part with the S^5 fiber bundle part of nucleons containing the quark as particle. In experiments the fermion energy can show either its wave or particle character. The particle part is for 3-dimensional quarks with the Kaluza Klein unified EM with GR field a Heegard decomposition in brezels of genus 2 with a 1-dimensional retract as lemniscate in a dark matter location. For leptons it is a Heegard decomposition in tori of genus 1 with a 1-dimensional retract as circle. The localized

energy arises for genus 1 (or 2) by two frequencies hitting orthogonal in a plane in proportion 1:1 (or 1:2).

Big bang extensions

The explosion of a Higgs boson can be taken for a big bang explosion. Changed is that before the big bang energies exist. Dark matter and dark energy are today postulated after a big bang and they exist before it. Added newly is a color charge cc force with a G-compass as model. It consists of the six invariant cross ratios of complex Moebius transformations of a Riemannian sphere S^2 . They are an independent fundamental force, not only a color charge of quarks. Repeater for this use are properties already mentioned in the previous sections. The model is a G-compass where G is a rotational 2x2-matrix of order six and the cc are Heisenberg uncertainties HU paired. The G drives a potential six roll mill constructed as a catastrophe with an elliptic umbilic. It has many cusps for changing potential surfaces. G has three motors POT, SI, WI. Pot is a projective Kaluza Klein unified gravity GR and electromagnetic EM real 5-dimensional field, its points are lines through the origin in a 6-dimensional space. The log surface inside a dark energy location is differentiated to a common POT potential. Log is inverted to its exponential function \exp for wave presentations of energies. This is due to SI where a polar angle φ is mapped to a complex rotation $\exp(i\varphi)$ with the usual complex $x + iy$ cosine, sine projections for

wave presentations. WI is for the Hopf fiber bundle with ray presentations for forces. In a radius inversion for the dark matter energy The 1-dimensional energy carrying circles are projective dual replaced by the xyz-space 3-dimensional radii in the universe. The Hopf fiber bundle is repeated for the S^2 radius inversion to a new S^5 fiber bundle where this sphere is a factor of the SI geometry. The speed inversion is added to dark energy for universes speeds below the speed of light. The 6 roll mill model is listed in octonion coordinates 123456, doubling the WI/EM spacetime 1234 (x,y,z,ict)-coordinates. The octonion coordinate 0 is for cc, the coordinate 7 for the later on generated electromagnetic interaction EMI.



Figure 6: Six roll mill.

On the octonion enumeration the HU 15 POT rolls are driving EM, GR rolls, the HU 23 SI the heat and rotational energy rolls with the same speed and the HU 46 WI the magnetic and kinetic energy rolls with a different speed. Driven is a common quark-gluon plasma. It decays into quark and gluon particles observed in nucleons. They have 2356 space with the S^5 fiber bundle and EMI 1456 space having cylindrical coordinates. In 2356 is observed the neutral cc 126 rgb-graviton superposition of three cc red-green-blue. It is quantized on cylindrical coordinates like photons of EMI as helix lines, but has three amplitudes for the diameter/radius of the cylinder in the proportions of 2:1:1/2 for spin 2 of the rgb-graviton and its dual 345, spin 1 for bosons and spin 1/2 for fermions.

Conclusion

In the articles [1] mathematical solutions are given for the Planck era where physical rules are not valid before the Planck time. As new fundamental force is defined color charges as the six invariant complex cross ratios of a Riemannian sphere. Forces are in different real or complex dimensions of $U^{(n+1)}$ presented as rays through an origin, normed as the unit sphere S^n . The cases are: $n = 1$ for EMI $U(1)$ a unit circle, for the weak interaction WI and leptons the Hopf S^3 with a fiber bundle, the new strong S^5 fiber

bundle space for nucleons and the new S^2 cc unit sphere [2]. WI has also an orientation fixed as clockwise cw rotation of a leptonic negative electrical or neutral charge and opposite mpo for a positive electrical charge, neutral leptons have helicity. Gravity norms force geometries projective. For instance in the unified electromagnetic and gravity Kaluza-Klein model S^5 as 1-point compactification is projective normed by identifying antipods. Lines through an origin in R^6 are presenting the P^5 space. Added in the first section is that the GellMann matrix strong force has to be extended. Included are meson cc, strong GF 126, 345, 037 to the octonions seven Fano GF. Also a 4-dimensional complex Hilbert space is generated. The wave particle duality has mesons for wave and baryons for particle presentations in different experiments. For GR it has the spin 2 particle rgb-graviton neutral color charge of nucleons as inner dynamics and for GR waves the earthworm model.

From physics for the wave-particle duality is quoted: The Wave-Particle Duality theory states that waves can exhibit particle-like properties while particles can exhibit wave-like properties. This definition opposes classical mechanics or Newtonian Physics.

Double slit experiment

In this article it is shown that there are a quantum GF measuring operator and energy exchanges are responsible for this. According to the experiment, the system can change between its two options. Wave frequencies come repeated in helix cylindrical finite parts which can be used for their particle property and particles can show as wave superposition of their parts interference in experiments.

Macroscopic models, running dynamically, are constructed for the new geometries and for states of systems. An infinite dimensional Hilbert space not needed for this. Here is used a complex 4-dimensional Hilbert space. Some models show the unification [3] of forces.

For measuring units of forces the Gleason operators base triples are used which can have three or six real (also complex) values [4]. The three up/down spin lengths [5] are set on circles as a central Moebius strip rotational normal vector in a 720 degree rotation for fermions, for bosons as the usual 360 degree rotation, for the rgb-graviton spin 2 as a P^1 180 degree rotation with antipods in S^1 identified.

An extended big bang view includes before the big bang three energy carrying systems, dark matter, dark energy and the cc force.

Bibliography

1. "For the solution of the Planck era several tools, including dark matter, dark energy". MINT Wigris Library, *Acta Scientific Applied Physics* 4.1 (2024): 36-41.
2. "Color Charge Force and Gravity". *Acta Scientific Applied Physics* 4.2 (2024): 14-16.
3. "Unification of forces with color charges". *Acta Scientific Applied Physics* 4.2 (2024): 17-22
4. Quantum interpretation for measurements in physics". *British Journal of Multidisciplinary and Advanced Studies* 5.6 (2024): 1-5. (for forces the quantum interpretation of measuring units with octonion Gleason operators, spin-like 3-dimensional generated as 123, 145, 167, 246, 257, 347, 356, 126, 345, 037).
5. "Spin 2 rgb-graviton field quantum exist". *Acta Scientific Applied Physics* 4.1 (2024): 32-35.