## The Queen of Rings

Gudrun Kalmbach HE*<br>MINT, PF 1533, D-86818 Bad Woerishofen, Germany<br>*Corresponding Author: Gudrun Kalmbach HE, MINT, PF 1533, D-86818 Bad Woerishofen, Germany.

Received: June 30, 2023
Published: July 17, 2023
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Kalmbach HE.

DOI: 10.31080/ASWH.2023.05.0515


#### Abstract

A tribute to the Noether theorem: the basic forces of physics are discussed by her representation theorem through invariants (field quantums) of the symmetry belonging to a particular force geometry. Her theorem for physics is later than the Erlanger Programm of Felix Klein presented as scientific Habilitation to the Universitaet Erlangen in 1872. To every mathematical geometry belongs a symmetry group and invariants under this group. Noether's transfer to physics includes that field quantums for forces are such invariants of related geometries. They are in general not using the 4 -dimensional spacetime geometry $\mathrm{R}^{4}$ of physics. Observed are invariants in projections. As higher dimensional differently coordinated space the complex 4-dimensional $C^{4}$ extension of $\mathrm{R}^{4}$ is in use. With different force coordinates 8 -dimensional octonians and the strong interactions $\mathrm{SU}(3)$ space are added to $\mathrm{C}^{4}$. The invariants, symmetries and geometries are different for forces and often treatable in $\mathrm{C}^{4}$ only as observable projections.


Keywords: Physics; Internet; Field Quantums

## Quotation from the internet

In Erlangen is a public plaque for Emmy Noether. She developed the mathematical ring theory. In physics her Noether theorem is used for geometries and their symmetry invariants. This guides the standard model of physics: each basic force has its own geometrical symmetry for their field quantums. In numbers, these invariants correspond to the generators of the symmetry.

## List of forces

EMI, Electromagnetic interaction, symmetry $U(1)$, a geometrical circle. As function it is complex written as $\exp (\mathrm{i} \varphi)=$ $\cos \varphi+\mathrm{i} \cdot \sin \varphi$. It has one generator and the field quantum photon. It is demonstrated in its time development as one helix frequency


Figure 1
winding on a unt circle. It is quantized stored in its time generation only in full windings as energy. Observable is only the real cosine 1 projection of the $\exp$ function: $y(x, t)=a \cdot \cos \left(\omega t-2 \pi x / \lambda+\varphi_{0}\right)$, a amplitude of the wave, circular frequency $\omega=2 \pi \mathrm{f}, \mathrm{f}$ frequency, t time, $\varphi_{o}$ phase angle, $\lambda$ wave length, -x space location in case the wave is travelling in this direction, in case it is moving in the opposite direction the $\operatorname{sign}-x$ is changed to $+x$.


Figure 2

Figure 2 at left: two helix lines on a circular cylinder as EMI geometry; in a transversal cut of the cylinder, the circle has as universal cover the helix line, quantized by natural numbers as frequency windings $n=1,2, \ldots$; as real coordinate line the cylinders helix has an additional $U(1)$ dimension to the spacetime coordinates ( $\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t}$ ); the circle $\mathrm{U}(1)$ can be presented as $\mathrm{x}^{2}+\mathrm{y}^{2}=1$ in space. The cylinder above the circle has space axis $z$ and the time development of the helix runs along z. For quantized photons, one full winding in the circle is needed. In mathematics, this is called a complex contour integration about a central pole. The residue theorem allows to count photons. The natural and real numbers use arises. As complex written EMI waves, the real number system is extended to the complex number system for physics.

At right: this figure is from [8], p. 237 (with the permition of the authors); their are transversal (this figure) and longitudinal running waves; in case the station and observer are in relativistic motion $v$ the observed frequency is scaled $f^{\prime}=f \cdot \sqrt{ }\left(1-(v / c)^{2}\right)$, $c$ constant speed of light, v relativistic speed for the transversal case; for the longitudinal case the formulas are not listed here, they are
different for the cases where the observer approaches or remove distance towards the lights direction

WI; EM and Spin, weak interaction, symmetry $\operatorname{SU}(2)$, geometrically spacetime $R^{4}$ provided with the Minkoski metric and generating Pauli matrices, having as subspace geometry for WI the 3-dimensional Hopf (unit) sphere $S^{3}$. The 3 Pauli spin generators of $\operatorname{SU}(2)$ correspond to three WI field quantums $\mathrm{W}^{+}, \mathrm{W}^{-}, \mathrm{Z}^{0}$. They carry mass and are 3-dimensional with the Hopf fiber bundle for their $S^{3}$ geometry. The fiber is a unit circle $S^{1}$ and $S^{3}$ gets the time coordinate deleted in its $S^{2}=h\left(S^{3}\right)$ image as subset of space $R^{3}$. For this projection the three Pauli matrices $\sigma_{j}, j=1,2,3$, are used. The inverse Hopf map $\mathrm{h}^{-1}$ maps latitude circles in $\mathrm{S}^{2}$ to tori in $\mathrm{S}^{3}$. They shrink to a core circle for the $S^{2}$ south poles fiber. The north poles circle is not shown in figure 3 . Opened at a point in projective infinity a central tori-axis coordinate line is shown. Complex spacetime $C^{2}$ coordinates $\mathrm{z}_{1}=\mathrm{z}+\mathrm{it}, \mathrm{z}_{2}=\mathrm{x}+\mathrm{iy}$, ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) space coordinates, t time coodinate, are mapped by the stereographic map st for $S^{2}$ to a complex plane with ( $u, v$ ) coordinates $(u, v)=\operatorname{stoh}\left(z_{2} / z_{1}\right)$ for $\mathrm{z}_{1} \neq 0$, deleting the $S^{2}$ north pole as point at infinity.

For electromagnetism EM, the neutral leptons, a 2-dimensional version together with their helicity, pairing spin with momentum is shown in figure 3. For the electrical charged leptons, the counteclockwise + orientation on latitude circles is for the charge in motion, the clockwise for the -charged ones. Also in this case spin alignes with magnetic momentum (not the momentum) according to the gyromagnetic relation. Weak bosons are intermediate carriers for hitting particles, graphically shown by Feynman diagrams (first line in figure 3); their geometry is not shown and often replaced by a point. They decay again into particles.



Figure 3

Figure 3 first line Feynman diagrams; below: neutral leptons helicity at left, Hopf tori and spin directions at right.

SI strong interaction, $\operatorname{SU}(3)$ geometry, a toroidal toplogical product of a 3 - with a 5 -dimensional unit sphere $\mathrm{S}^{3} \mathrm{x} S^{5}$. A torus product was used by Hopf for his $S^{3}$ fiber bundle with fiber $S^{1}$. For GellMann 3x3-matrix extensions of the Pauli matrices it can be used to project the first three ones with third row and column having entries 0 down to the Hopf Pauli matrices of WI. $S^{5}$ can then be seen as a fiber of this projection. It is possible to extend $S^{3} \mathrm{xS}^{5}$ similarly as the Hopf torus $S^{1} \mathrm{x} S^{1}$ which is spacetime $R^{4}$. This would make an $\mathrm{R}^{10}$ extension. It may be used in string theory, but not here. The part $S^{5}$ is used as space for a nucleon fiber bundle, projecting it down with fiber $S^{1}$ to a complex projective 2-dimensional nucleon space $\mathrm{CP}^{2}$ with a bounding sphere $\mathrm{S}^{2}$. The nucleon dynamics for this is discussed in [1,2,5]. As for EMI this is observed as projections. The bounding spheres come in six polar caps on different radii for the 6 color charges. They can turn their energy vector up/down like spin where in up direction the particluar energy is emitted, in down postition absorbed from the environment. There is a potential (for instance a fold catastrophe having two levels) which opems like a ventil the necessary turn. The polar caps are projective real planes such that the vectors turn can be made on a Moebius strip. Invariants under the SI SU(3) symmetry are 8 gluons. They confine 3 quarks in a nucleon by gluon exchanges between paired quarks.

Figure 4 hedgehog with polar caps at left, G-compass at right.

## Color Charge force, gravity in octonian coordinates

QCD for SI treats color charges as property of (anti-)quarks. A quark has an electrical charge, a mass and a color charge. Nucleons


Figure 4
have always neutral color charge rgb, red-green-blue. This is taken as the field quantum rgb-graviton. It extends spin-like the quark triangle to a tetrahedron as in figure 4. Its dynamical spin generates barycentrical coordinates for the triangle where a Minkowski rescaled (quarks) mass of the nucleon is attached to the barycenter. The finite symmetry is $\mathrm{S}_{4}$ the permutation of four elements. It is factored by its normal CPT Klein group $\mathrm{Z}_{2} \mathrm{xZ}$ to the dynamical triangle symmetry $\mathrm{D}_{3}$. Integrations of forces (with differentials dr radius (or spacial dx, dy, dz), dt time, dA area, dV volume) are performed as a representation of $S_{3}$. The factor classes of $\mathrm{S}_{4}$ contain beside a color charge an octonian coordinate, a $\mathrm{D}_{3}$ symmetry and an energy/forcce in brief with coordinates listed by their indices (red, 1 (space x), $\sigma_{1}$,EM charge); (green, 2 (space y), $\alpha \sigma_{1}$,heat); (magenta,3 (space z), $\alpha^{2}$.rotational energy); (yellow, 4 (time t), $\alpha^{2} \sigma_{1}$, magnetism or neutral leptonic); (turquoise,5,identity id,mass); (blue,6, $\alpha$ (cubic 120 degree rotation),kinetic energy/ frequency). The linearized $U(1)$ EMI coordinate is 7 in octonians. The octonian coordinate 0 is reserved for the color charge force cc. As independent energy from quarks it has the G-compass as model. G is the rotational matrix $\alpha^{2}$ of order 6 with first line ( $1-1$ ) and second line (10). It can turn its needle only in six discrete steps, the sixth' roots of unity (figure 4). Its six fold way is also for six electrical charges, six masses of fermionic series (for instance all three properties of quarks). The G symmetry generates the rgbgraviton earthworm model, described in [14]. The cc geometry is the complex, real 2-dimensional Riemannian sphere $\mathrm{S}^{2}$ together with its stereographic projection onto a complex plane and with the symmetry of Moebius transformations MT. The MT has as six cc invariants the cross ratios $\mathrm{z}, 1 / \mathrm{z}$. ( $1-\mathrm{z}$ ), $1 /(1-\mathrm{z}), \mathrm{z} /(\mathrm{z}-1),(\mathrm{z}-1) / \mathrm{z}$. Beside the complex variable $z$ the three reference points $(0,1, \infty)$ are permuted for the cross ratios. To the triple can be associated a
counterclockwise orientation on a circle like $U(1)$. The clockwise orientation uses $(0,-1, \infty)$ for the tetrahedral $D_{3}$ symmetry. Interprete this as a cc composed rgb-graviton system similar to the nucleon composed three quark system.

## Comparison and Conclusion

The four cases above are using as spherical geometries $\mathrm{S}^{1}$ for EMI, $S^{2}$ for $\mathrm{cc}, \mathrm{S}^{3}$ for WI and in part also for SI. $\mathrm{S}^{5}$ is for SI and in projection for nucleons. The symmetries are similar for the standard models $\mathrm{U}(1) \mathrm{xSU}(2) \mathrm{xSU}(3)$. The different MT symmetry of $\mathrm{S}^{2}$ initiates finite permutational symmetries. For the CPT case (conjugation, parity, time reversal operators of physics) is the Klein group $\mathrm{Z}_{2} \mathrm{xZ}_{2}$, for the tetrahedron $\mathrm{S}_{4}$, for the quark triangle $\mathrm{D}_{3}$, for cc the G matrix of order 6. Beside the 8-dimensional $\operatorname{SU}(3)$ and octonian coordinate description there is the complex 4-dimensional Hilbert space $\mathrm{C}^{4}$ also in countable infinite dimensions for non-commuting projection operators and orthomodular subspace lattice theory [6]. This provides the quantum range with a non-Boolean logic not respected by physics und their paradoxa problems in the internet. Boolean reasoning has to be revised and the quantum measurement process underlies the Copenhagen interpretation.


Figure 5

Figure 5 pinched (Horn) torus for the surface of dark energy at left, at right dark matter radius inversion [11, p.468].

Concerning mathematical inversion with the MT $1 / \mathrm{z}$, dark matter and dark energy postulated by astronomy in the universe has other possible descriptions [15]. For dark matter it is postulated that the inversion of radius as distance measure is at
the Schwarzschild radius Rs of a black hole where for instance quarks length $r$ is inverted to $r^{\prime}$ inside with $r^{\prime} r=(R s)^{2}$. Nucleons quarks are cc 1-dimensional retracted to Lissajous figures. For dark energy, the matter waves $\psi$ could be inverted as functions to $\psi^{-1}=\mathrm{a} \cdot \exp (2 \pi \mathrm{i} / \mathrm{h}(E t-\mathrm{px}))$, exp the exponential function. A meaning for momentum p and energy E is changing inside a bounding pinched torus for dark energy where the matter wave speed $v$ in the universe is inverted to a speed $v^{\prime}$ with $v^{\prime} v=c^{2}$. For instance ( $x, p$ ) of matter waves can be changed to ( $\varphi, \mathrm{L}$ ) angle, angular momentum. The pinched torus is the projective closure of a Minkowski cone. The changed speed uses a scaling $m=m_{0} / \cos \varphi$ of mass $m_{0}$ in dark energy (possibly computed as frequency) by the Minkowski scaling factor $\cos \varphi$ with $\sin \varphi=v / c$ and a computation of the universes Minkowski speed as in optics [11, p.500]. Differentiation is needed for this.

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