



Quark Triangle Coordinates

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Abstract

Octonians are used for a geometrical unification of the standard model with gravity. The method is that color charges are added to the forces of physics as an additional force using for them complex cross ratios.

Keywords: Color Charge Force; Octonians; Standard Model; Quark; Nucleon

Introduction

Geometrical and numerical considerations

Physics uses Pauli matrices for describing spin as eigenrotation of physics systems. For them complex numbers are doubled to quaternions. Spacetime as coordinates for physics 4-vectors $xyzt$, t time xyz space coordinates, are associated by associating the id 2×2 -matrix with t and the three Pauli matrices with x, y, z .

For describing color charges of quarks, another presentation of a number system is used in the MINT-Wigris theory. The quark triangle can be drawn by using third roots of unity. They replace a complex imaginary number i as root of $z^2 + 1$. The new polynomial is $z^3 - 1$ with the roots $1, p_{1,2}$. If the number system for the coordinates is listed as $u = x + yp_1 + zp_2$, the third roots provide an unused numerical presentation for physics quark triangle. Quaternions use $q = t + ix + jy + kz$, i, j, k for quaternion Pauli matrices and id as 1 used for time.

Adding i for 3-vectors as usual. Conjugation $c(u)$ interchanges p_1, p_2 . The distance measure is by $u \cdot c(u) = x^2 + y^2 + z^2$. For complex numbers the generators $1, i$ are signed for presenting the numbers $1, i, -1, -i$ in a complex plane through 4th roots of unity. If the signs are added to the third roots of unities, a hexagon is obtained. The angle between adjacent numbers is 60° not 45° as for complex numbers.

As geometry for quark numbers the Brianchon configuration, interpreted as a G-compass occurs. It has a needle which turns discrete for color charges in multiples of 60° angles.

Take instead of Pauli matrices for the three color charges of quarks a 2×2 -matrix presentation G with first row $(1 \ -1)$ and second row $(1 \ 0)$. Its powers $G^2, G^3 = -id$ can be taken as coefficient matrices of Moebius transformations MT , the symmetry of a complex Riemannian sphere. They are $(z-1)/z, 1/(1-z), z$. For the quark triangle symmetry, the rotation is mpo counterclockwise on the G-compass. If the orientation is reversed to clockwise cw , physics uses the σ_1 Pauli matrix with first row $(0 \ 1)$, second row $(1 \ 0)$. The base vectors for the complex plane are for id as mpo rotation interchanged in the matrix to a cw rotation. If this matrix is added to the mpo rotation α of the quark triangle by 120° , three triangle reflections are generated by $\alpha\sigma_1, \alpha^2\sigma_1, \sigma_1$. The symmetry of the quark triangle is obtained. The new matrices present the MT as $z/(z-1), (1-z), 1/z$. These six MT are the complex cross ratios.

The Noether theorem is used in physics for describing field quantum of forces by invariants of an appropriate geometry. For the weak and strong interaction this is a Hopf sphere S^3 in spacetime R^4 , and a topological product $S^3 \times S^5$ where the powers present dimensions of a unit sphere. The S^5 factor has no use in physics, except that the SI generators are $8 = 3 + 5$ gluons. The WI

generators are 3. The electromagnetism interaction EMI has for photons a rolled circle $U(1)$ as geometry. A problem in physics is how to unify this standard model $U(1) \times SU(2) \times SU(3)$ with a gravity based geometry.

The author suggests that the answer is given above. The Riemannian sphere S^2 fits to the Noether theorem and the spheres S^n , $n = 1,2,3,5$ are a unifying presentation. The MT invariants present color charges as an independent force of physics. It is not directly seen how their use is for gravity.

Einstein's relativities are different and the nonlinear Schwarzschild metric cannot be treated by $U(1) \times SU(2) \times SU(3)$.

In other articles of the author, the former third roots of unity coordinates in form of the G-compass Brianchon and a tetrahedron configuration for nucleons have been added. This needs another extension as doubling up complex numbers to quaternions spin coordinates q for spacetime. Another doubling is described by the Cayley-Dickson theorem where quaternions are (q_1, q_2) doubled to octonians. In octonians the coordinates are enumerated as $0,1,2,\dots,7$. The coordinate 0 is for color charges as an independent basic force of physics. It sets the G-compass six color charges 123456 as MT cross ratios which are perspective projections. This octonian subspace can have 1234 for spacetime coordinates $xyzt$. 56 is for an Einstein projective energy plane with homogeneous coordinates $[m,f,w]$, m mass, f frequency. It contains the line $mc^2 = hf$. Mass is set by a Higgs field or bosons and added through the color charge; frequency is not EMI which is an exponential function for waves.

For it the coordinate 7 stands. The linear real line R of octonians is $U(1)$ Kaluza-Klein rolled up to a circle. Geometrically this means that R occurs for EMI as universal cover of the circle, rolled up in time on a cylinder as helix line. Photons as field quanta are only existing when stored as a full contour winding about the circle. This is a helix winding on the cylinder and quantized by natural numbers in its time expansion as a wave function exponential $\exp(i\omega t + kx)$, $\omega = 2\pi f$, $k = 2\pi/\lambda$, λ wave length on the cylinder. Observable is only the real cosine part of \exp in spacetime. The cylinder is not observed.

The octonians stand for a biological evolution of physics energies. The Brianchon configuration is projective dualized to a Feigenbaum Pascal configuration (Figure 1). The input is 0, bifurcating into 1,5 for EM (electrical force) and mass. 1 bifurcates into 2,4 heat and magnetism as forces.

5 bifurcates into 3,6 rotational and kinetic energy. Then 2,4,3,6 bifurcate into strong SI eight gluons. After that in the evolution of the universe heat chaos occurs for its inflation period after a big bang.

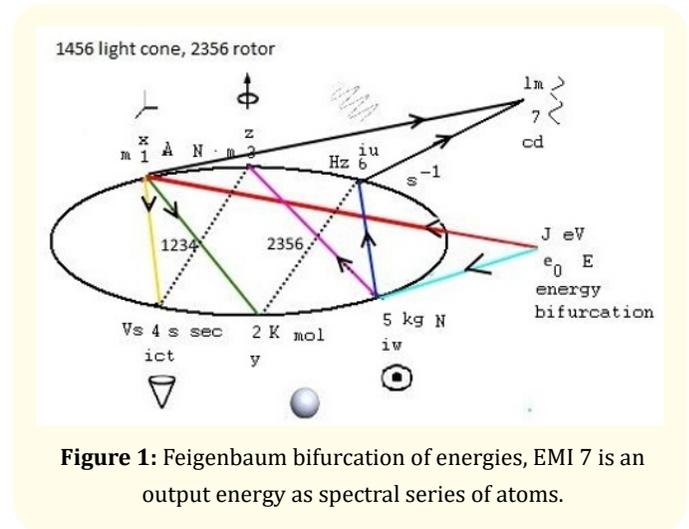


Figure 1: Feigenbaum bifurcation of energies, EMI 7 is an output energy as spectral series of atoms.

It is clear that such an evolution cannot be treated by the standard model or quaternions. It needs octonians. To the energies correspond spin-like base triples as in figure 2. There are 7 cross product like σ_j , not only one quaternion $j = 1,2,3$. In the Fano memo they are drawn as 3 points on an interval or circle. 123 is for the quaternions σ_j . The other ones set measuring Gleason operators for energies. In figure 1 the distribution is 123 for 1 (length or Ampere, eV), 145 for EM 4 (time or TESLA), 167 for EMI 7 (cd candela), 246 for heat 2 (k Kelvin), 257 for mass 5 (kg), 347 for rotational energy, angular momentum 3 (J Joule), 356 for kinetic energy, momentum (Hz Hertz).

To the energies are added configurations. 123 has a base triples for xyz , 2 has a volume, ball with entropy inside, 6 has a harmonic frequency wave-like configuration as well as 7, 4 has a magnetic field quantum cone, generated by a rotating eigenvector; 5 has a Higgs barycenter for mass in a volume or circular disk. For 3 is added an orbit (circle) of a system, rotating in a plane about a normal oriented axis.

The base triples in figure 1 can be 4-dimensional extended, similar to the time extension of space xyz in 1234. The basic subspace lattice is a 4-dimensional real Hilbertspace R^4 . With

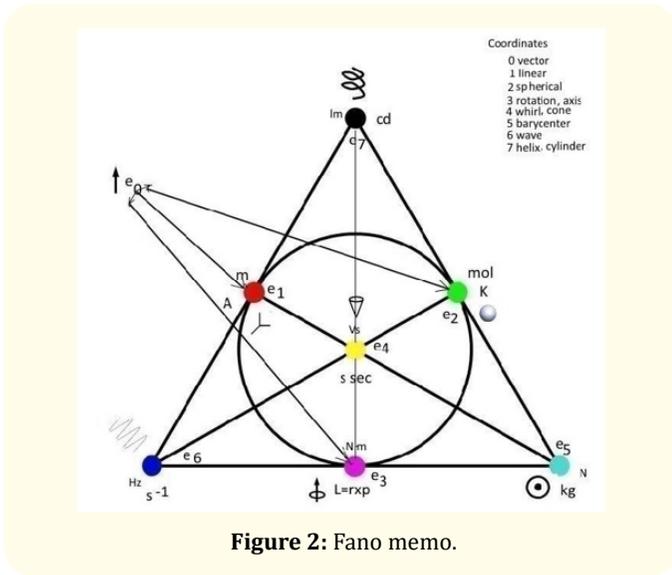


Figure 2: Fano memo.

complex coordinates extended it is C^4 having 8 real dimensions. These extensions use cross products.

As examples, 356 is extended to 2356 and 167 to 1267. 2356 is for a projective complex CP^2 space with a bounding sphere S^2 for nucleons. It arises as a strong SI S^5 fiber bundle with fiber S^1 . Hopf has a weak WI fiber bundle S^3 with fiber S^1 and base projection S^2 .

Extending EMI 167 includes a triple 126 which is not in the Fano memo. It is a strong interaction GellMann SI triple $\lambda_j, j = 1,2,3$. They are σ_j extended 3x3-matrices by adding a third row and column of 0 entries. 126 presents a rgb-graviton as superposition of three color charges r red, green g, blue b. It is observed as neutral color charge of nucleons. The two whirls 126, 167 are in 1 superposition; wave length 1 and the red color charge vector are aligned. As measuring eigenvectors they are oppositely oriented for r,g,b. Equal orientations are for anticolors. For a wave presentation of 126 the cylinder geometry used for EMI is available, but gravity GR is not EMI as energy.

In a geometrical configuration, 126 as projection is drawn like a spin base triple (see Fano) where its whirls center is in the center O of a sphere S^2 as boundary of a nucleon or atomic kernel. The three color charge whirl eigenvectors are oriented on the x,y,(-z) axis of space. Their endpoints carry the barycenters and color charges of three quarks. The quark triangle symmetry was used above for another kind of number system with third roots of unity generators for the color charges. Gravity is cubic, not like Pauli spin or imaginary

2-dimensional. The tetrahedron configuration with vertices Orgb has as symmetry S_4 , the permutations of four elements which is also for color charges as cross ratios. The factorization of S_4 to the triangle D_3 symmetry uses the Klein normal subgroup $Z_2 \times Z_2$. The factor classes contain 4 elements each: a coordinate, a color charge, an energy and a symmetry whose eigenvector sets a measure for the energy.

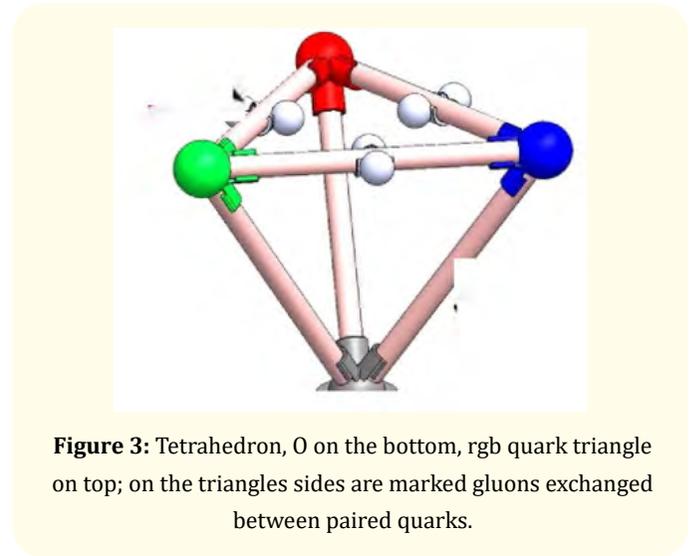


Figure 3: Tetrahedron, O on the bottom, rgb quark triangle on top; on the triangles sides are marked gluons exchanged between paired quarks.

For a long time physics is not reading mathematics. The author developed and published since 1986 her results and a table from 1988 is repeated.

Conclusion

Geometry and octonians give an answer to the unsolved unification problem of physics: color charges of quarks are an independent complex cross ratio force. It uses third roots of unity for xyz coordinates. The numerical conjugation $p_1 p_2 \rightarrow p_2 p_1$ for them is different from complex i or quaternionic ijk conjugation. The last one is cross product bound, the first one cross ratio bound. They cannot be transformed as rectangular measure for the cross product and circular, disk area measure for cross ratios. The unification is through geometry such as unit spheres $S^n, n = 1,3,5$, for EMI, WI, SI using the cross product and S^2 as Riemannian sphere using cross ratios.

In the first line of the table are the spherical SI coordinates, possibly 7- (not 8-)dimensional extended with exponential/polar coordinates. In the second line are the linear Pauli/Euclidean coordinates, in the third line a distribution of color charges to the SI coordinates. The fourth (fifth) line contains the D_3 (SU(2)/Pauli) MTs as cross ratios. Their matrix names are in the sixth line, together with the Einstein matrices. The following line is a numbering for a strong 6-fold integration series (not the Fano figures numbers which are for octonians). The next line contains the Planck numbers. Energy vectors are in the second to last line and the last line contains natural constants and three more operators, C (conjugation for quantum numbers), T (time reversal) and P (space parity) of physics.

r or $re^{i\varphi_1}$	φ	θ	ict	iu	iw
$x \in \mathbb{R}$	$iy \in i\mathbb{R}$	$z \in \mathbb{R}$			
r	g	\bar{g}	\bar{b}	b	\bar{r}
z	$\frac{z}{z-1}$	$\frac{-z-1}{z}$	$(-z-1)$	$\frac{1}{-z-1}$	$\frac{1}{z}$
$\frac{1}{z}$	$-\frac{1}{z}$	$-z$	z		
$id; \sigma_1$	$\alpha\sigma_1; \sigma_2$	$\alpha^2; \sigma_3$	$\alpha^2\sigma_1; id$	$\alpha; \delta$	$\sigma_1; id; \beta$
1	6	4	2	5	3
length λ_P	temp. T_P	dens. ρ_P	time t_P	ener. E_P	mass m_P
EM_{pot}	E_{heat}	E_{rot}	E_{magn}	E_{kin}	E_{pot}
$c, \epsilon_0, \epsilon_0$	k, C	N_A, T	μ_0	h	γ_G, R_S, P

Figure 4: The number line is changed in 2022 to 1 2 3 4 6 5, replacing 1 6 4 2 5 3.

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Figure 5: Models are available in the Emmy Noether Memorial museum.

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