



Gleason Measures, Quasiparticles and Geometries

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In the quantum range the newly introduced nuclear forces and the findings in the (quasi-)particle range allow a new mathematical treatment than 100 years ago. Measurements for quasiparticles are new, geometries are mostly through projective extended octonions and shapes. The figures: Fano memo shows the measures, the hedgehog figure the energy exchange of a nucleon with its environment, the inner dynamics of them is illustrated in the next figures, an energy evolution follows and mass of nucleon is special and general relativistic reset through Einstein's mass-frequency equation.

Keywords: Gleason Measure; Quasiparticle; Projective Geometry**Abbreviations**

EM: Electromagnetism; EMI: Electromagnetic Interaction; GF: Gleason Frame; GR: Gravity; MT: Moebius Transformations; SI: Strong; WI: Weak Interactions

The Fano GF

For quasiparticles like spin the Gleason frame triples GF are introduced for measurements in the Copenhagen interpretation. The Stern-Gerlach experiment for spin shows that a system to be measured has to be prepared. For spin a chosen space direction of the particles is generated. The output of the measurement through an apparatus can be numbers or vectors. For spin prepared in the space direction x for instance the apparatus allows as outcome only the orthogonal z -space direction. The measurement shows that half of the particles turn their spin in the up, the other ones in the down direction of z . The Copenhagen interpretation is that only one of the vector triple can be measured and possibly the weight of the GF as commuting operator, the other two GF vectors remain undetermined. After the experiment the measured system can change its state. In the spin case it is the direction of its spin vector.

Orthogonal triples like spin (s_x, s_y, s_z) for an xyz -frame are taken for GF as a orthogonal base of unit vectors on a unit sphere S in a real, complex or quaternionic inner product space H of dimension $n = 3$ (or higher). They can be turned in any direction on the sphere. The GF carry on its three vectors numbers as scalars, for spin it is the spin length. The weight of the GF is the sum of these three numbers. A projective scaling is allowed for the frame function $f: S \rightarrow \mathbb{R}^*$

where the weights are scaled by a phase angle in form of $f(\lambda u) = f(u) \cdot \lambda$ a (real, complex or quaternionic) number with $|\lambda| = 1$. \mathbb{R}^* are the nonnegative real numbers and the λ scaling is a fiber. For the reals the fiber is $+1, -1$, a 0-dimensional sphere S^0 , for complex numbers a circle $U(1)$, for quaternions a sphere S^3 . The dimensions of the base is chosen for quasiparticles as $n = 3$. For spin H is the real xyz -space. For mass H can be complex which allows six real numbers for the masses of fermionic series. Alternatively, for mass also a real 6-dimensional H can be taken. S^5 is then 5-dimensional, for spin-like GF's S is a 2-dimensional unit sphere S^2 . The generators of an S^2 GF are as 2×2 -matrices the three Pauli spin matrices. In higher dimensions like the octonion inner product space there are seven 3-dimensional subspaces of this kind where the spin triple of space is only one of them (Figure 1). It sets in space the units for length in meters.

In this figure there are two input e_0 and output e_j vectors added to six energy vectors $e_j, j = 1, \dots, 6$.

For $j = 1$, the measure is in meters (spin), $j = 2$ in Kelvin is for temperature, $j = 3$ is measured for rotational energy $E(\text{rot})$ in rad (angular radians) or $\text{kg} \cdot \text{m}^2 / \text{s}^2$, kg for mass, s for time in seconds, $j = 4$ is for time in seconds, $j = 5$ for mass in kg , $j = 6$ for frequency and kinetic energy $E(\text{kin})$ in Hertz Hz or $\text{kg} \cdot \text{m}^2 / \text{s}^2$. The input vector sets in the octonion vector space a vector with an initial point, a direction and the weight as scalar of H attached. The spin-like vector triples for the GF are generating 3-dimensional subspaces of H . The output vector has as measure cd candela for the electromagnetic

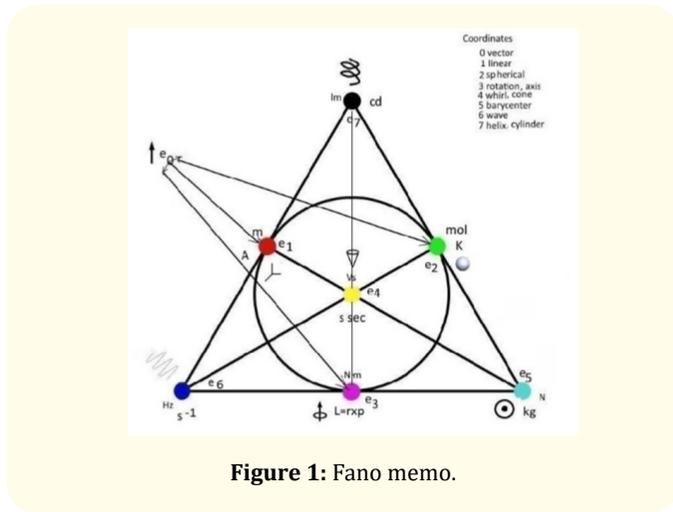


Figure 1: Fano memo.

waves interaction EMI. The electrical EM units can be set as A (Ampere) on e_1 , as T (Tesla) or A/m, Vs, V Volt, on e_4 .

For the associated quasiparticles the indices of the coordinates are written and the GF frame is drawn in figure 1 as a line having three points for the coordinates: 123 is for spin and Euclidean space xyz-coordinates, 145 for conic magnetic field quanta and the cross product defining induction. For 167 is mentioned that its phonons field quanta have in the superposition of EMI waves the triple of a wave length λ , a frequency $E = hf$ (E energy, h Planck constant as energy quantum) and a speed $v = \lambda f = c$. For the EMI symmetry the linear e_7 coordinate is rolled to a Kaluza-Klein circle U(1) and the stereographic map is added to map the circle onto the e_7 line. The geometry of this is a cylinder rolled for the period 2π of the exponential wave function in polar form as complex function $\exp(i\varphi)$. Observed in the universe's space is from the cylindrical coordinates of the wave a real cosine or sine projection. For $j = 2$ there are two new triples, a subspace for heat 246 and 257 for mass and barycentric coordinates. The 245 quasiparticles are phonons for heat and sound. They are volume bubbles with entropy as energy inside. They transport in matter momentum and energy. On the S^2 ball surface of the 3-dimensional volume pressure (in kg/ms^2) is generated and the equation pressure times volume equals scaled temperature holds. 257 is for a Higgs field and its bosons as quanta. 5 is for mass, 7 for a Schwarzschild radius of a system with mass m , 2 sets a barycenter determined by barycentric coordinates where the mass is measured in kg. In the mass-frequency plane 56 holds $(1) mc^2 = hf$ and in the subspace 356 the barycentric coordinates for a nucleon triangle with 3 quarks are constructed through a SI rotor [1-24]. The equation (1) allows Higgs to rescale mass by adding frequencies. The quark mass is only 10 percent of the nucleon's mass having inside high speeds and interaction energies, frequencies. The special relativistic rescaling is for a common group speed of the nucleon's parts and for a matter wave description. The last memo line is 347 for

$E(\text{rot})$ in form of $L = r \times p$, L angular momentum, r radius, $p = mv$ momentum. 7 sets a speed for the system P in motion which has a simple closed contour (circular, ellipse, rosette) orbit in a plane with polar coordinates (r, φ) about a barycenter B in variable speed distance $|BP| = r$. L is parallel to a rotation axis through B orthogonal to the plane. Spherical coordinates for 123 are useful for describing the three vectors 1 spin s, 3 L on 3 measuring an angle towards the positive z-axis for its direction and $J = s+L$ as rotation axis. All three must rotate on cones because of the Heisenberg uncertainty angle-angular momentum. The figures are known for electrons in an atom's shell. Vectorial addition/subtraction is introduced. Quasiparticles for this can be rotons.

Quasiparticles and hedgehog

Quasiparticles for the discrete S^0 sphere +1, -1 are plasmons or excitons which generate Cooper pairs, also the case of an electron bound to a missing electron as hole in a distant matter. They are not a GF. The circle/fiber S^1 for the U(1) photon symmetry is also used not a GF, but a fiber of the Hopf fiber bundle for the weak WI interaction and EM. A rotating electrical or neutral EM point charge on a Hopf S^2 sphere is S^1 blown up to a leaning circle and traces out for the lepton a torus (a replacement of a condenser/capacitance plate, occurring as shape of electrons probability distribution in chemistry models for atoms electrons). The point rotation is on a S^2 latitude circle and S^2 has at its north pole a spin vector as normal. The rotation is cw clockwise for a negative or neutrino charge and positive mpo for a positive or antineutrino charge.

The strong SI interaction has as geometry a trivial fiber bundle $S^3 \times S^5$ where S^3 carries rgb-gravitons as superposition of three color charge whirls red r, green g, blue b. The whirl matrices are the three Pauli matrices blown up to 3x3-matrices by adding a last row and column with coordinates 0. The projection map for the rgb-graviton quasiparticles GF map the strong S^3 fiber to the Hopf sphere S^3 as unit sphere in spacetime. For S^5 the fiber S^1 maps it to an inner complex atomic spacetime CP^2 in form of a complex 2-dimensional space with boundary S^2 , a Riemannian complex sphere. For an atomic kernel AK there are three such atmospheres which decompose in a Heegard splitting along an xy-or xz- or yz-plane in an atmosphere of six hemispheres as polar caps. They are for the energy exchange of the AK with its environment and each carries one of the octonian energy vectors 1 as EM(pot) electrical potential on the +x-axis, Heisenberg joined to the mass potential E (pot) 5 on the (-x)-axis. E (heat) 2 on the +y-axis, E (rot) on the (-y)-axis, E (magn) on the +z-axis and E (kin) on the (-z)-axis.

The particle or quasiparticles for the caps are spin and EM charged leptons for the red cap, the Higgs bosons for the turquoise cap, phonons for the green cap, rotons for the magenta cap, magnetic field quanta, magnons for the yellow cap and helicons for

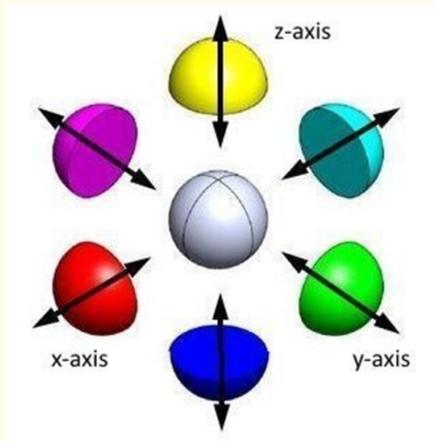


Figure 2: Hedgehog for a deuteron AK with a proton and a neutron.

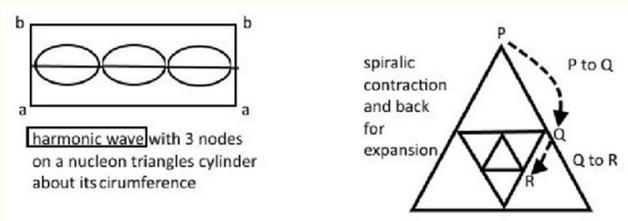


Figure 4: Harmonic wave, mass GF with three weights arising from Compton wavelength scalings.

the circular frequency expansion of EMI waves along a cylinder on its world line in direction of its momentum.

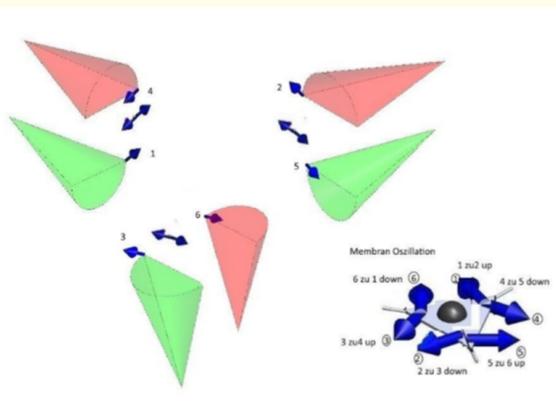


Figure 3: Membrane oscillation of the nucleon triangle and momentum quark locations in the SI rotor.

In the SI rotor the $E(\text{kin})$ vector at a blue quark is rotating in a representation of the quark triangle symmetry D_3 and traces out an oscillation with three nodes as the triangles quark vertices.

The oscillation as circles about the triangles sides spanned by the gluon exchange between paired quarks can be 2-dimensional expanded as a membrane oscillation of the quark triangle in form of three conic whirls for the rgb-graviton. Red or green vectors are at the tip of the cone and blue fixes with its six locations the observable quark vertices of the triangle.

Energy evolution

The energy vector carrying color charges cc are generated by the symmetry of the tetrahedron spanned by the spin-like rgb-graviton which carries at its ends the three cc vectorial energies, The tetrahedron symmetry S_4 of order 24 is factored by the normal CPT Klein group to the triangle symmetry D_3 . Its matrices arise

from the complex Moebius transformation MT invariants as cross ratios. The MT are the S^2 symmetry. The cc caps are located as in the hedgehog figure. The SI rotor generates barycentrical coordinates for the triangle. The difference to the three Pauli matrices is that the last one use the real cross product for the 3-dimensional space and the flat triangle D_3 symmetry has as composition of its members that two reflections composed are a rotation by $+120$ or -120 degree of the triangle.

Solitons are quasiparticles which change densities as mass per volume, measured in kg/m^3 . As a GF it can have radius 1 of a volume as one vector, mass as 5 vector and density as a quotient vector. Real or complex quotients are attributed to real projective normings or the complex norming of the Hopf fiber bundle from complex spacetime Pauli coordinates $z_1 = z+ict, z^2 = x+iy$ to z^2/z_1 for complex coordinates of a tangential plane in the south pole of S^2 . The north pole is at complex infinity $z_1 = 0$ and adds to the Hopf projection the stereographic projection from S^2 to the plane.

For the quasiparticles a change of the normal from input to output in the hedgehog caps or for spin shows that projective planes P^2 are needed. They contain a Moebius strip and the input output change is by a 360 degree rotation in the non-orientable strip. For this identify along the boundary of the polar caps diametrical points $+p$ and $-p$ to obtain P^2 . The planes arise in many orbits for systems and also for equations. The Heisenberg joined vectors 15 have quarks with two poles for their particle quantum and the equation $\lambda p = h$. Setting Coulomb force equal to centrifugal mass related force in 15 allows electrons Schrodinger waves when solved for main quantum numbers. 23 joined $E(\text{heat}), E(\text{rot})$ as L vectors plane generate complex φ polar heat angle on 2 and angular speed $\omega = d\varphi/dt$ for $L = J\omega, J = m r^2$ is an inertial mass momentum and an equation is $\varphi L = h$. The relation between magnetic energy and frequency of a leptonic charge ω rotation in time as inverse frequency requires for the wave presentation of leptons that the wavelength has to fit to the circumference of the rotational latitude circle in S^2 . This generates the Bohr radii of electrons in the AK shells when emitting or absorbing light with linear frequency. 46 joined magnetic/time frequency carries $E = hf$ in a frequency-time plane. 46 joined magnetic/time frequency carries $E = hf$ in a frequency-time

plane. A projective 56 plane has the line $mc^2 = hf$ for measuring kg in $kg \cdot m/s^2$ or inverse seconds and reversely. This extends to mass scalings or relativistic mass for EMI frequencies. As another (r, ict) plane the correlation quadric for Minkowski metric is $r^2 - c^2 t^2 = 0$. If closed a projective infinity by a circle dark sound whirls arise, in space sound expansion is on wave fronts in a Mach cone with phonons as quasiparticles moving in matter. For cylindrically closed 17 coordinates the closure at infinity is similar to a pinched torus. In addition to a Minkowski cone it has the helix energy expansion on the cylinders surface for the EMI frequency. The 12 plane of $z^2 = x+iy$ is for complex and polar coordinates, also in 2×2 -matrix representation, using the identify for the real scalar and the Pauli spin σ_2 matrix for the imaginary part. In 13 n-the roots location divide the $U(1)$ circle for spherical θ -angles as known for several electrons on one shell of an atom. As G. compass for $n = 6$ it sets the six color charges of quarks. The tetrahedron symmetry has then as equivalence classes for a color charge that attached to them is an energy, a symmetry of D_3 and an octonian coordinate 1,...,6. Pauli MT's are also included in the table below (see the next section).

jected support space for GR 1256. Other projections in the 123456 subspace can include 2356 for the nucleon SI rotor and 1456.

The linear subspaces 12 and 34 generate the spacetime vacuum space of physics 4-vectors. The linear 26 and 35 subspaces generate the SI rotor space and the linear 14 and 56 subspaces generate a subspace 1456 for a light cone with radius r on 1, time on 4 and the mass, frequency coordinates for speeds $v = \Delta x/\Delta t$, its mass version on the 5 coordinate and momentum as $p = mv$. Matter wave descriptions are using 14 as (x,t) coordinates and as their scalars $E = hf$ on 6 and p on a combination of m on 5 and v on 14. For EMI waves the substitutions are made that a wave length is replaced by p and an angular frequency $\omega = 2\pi f$ by E. The projectors for 123456 are here complex not real as for the POT field R^5 of Schmutzer. In pairs, the Einstein energy plane 56 is normed to the constant 1 for the vacuum spacetime 1234, the norming of 14 as Minkowski cones space (r,ict) gives the SI rotors space 2356 for nucleons and the norming of 23 gives the space for the affine Minkowski light cone 1456 with relativistic speed v.

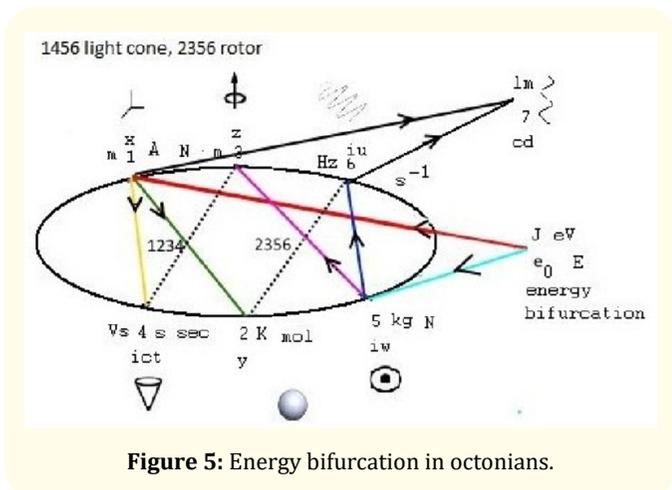


Figure 5: Energy bifurcation in octonians.

In a big bang bifurcation of energies the Pascal situation for three subspaces as points on the Pascal line 1234, 1456, 2356 is shown in figure 5.

Each probability has a support where for instance an energy like mass is 0. For setting mass at barycenters, frequency is set 0. The support N' subspace is the orthogonal complement for a greatest element N of subspaces A with $\mu(A) = 0$. H decomposes vectorial into $H = N + N'$. Since spacetime has 1234 N' coordinates, the vacuum support may have $N = 0567$ as null octonian space. A real 5-dimensional projective R^5 of Schmutzer for a unified potential field of EM and GR with a projector mapping R^5 down to three 4-dimensional spaces has the space 12345 and as one projection 1234. The other projections are an EM and a mass potential field. For mass on 5 the rgb-gravitons add the dimensions 126 as pro-

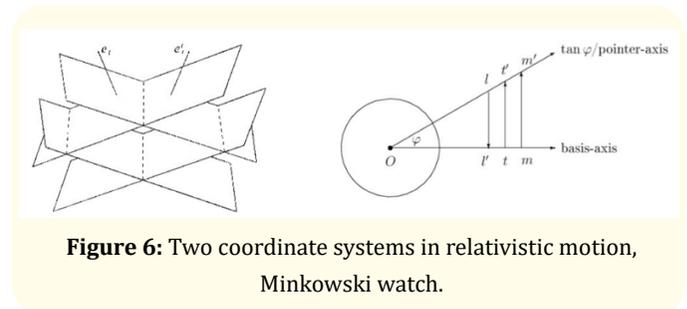


Figure 6: Two coordinate systems in relativistic motion, Minkowski watch.

The turning angle θ between two coordinate systems (x, t) and (x', t') not in GR interaction is given by $\sin \theta = v/c$ and the rescaling of coordinate units is by $\cos \theta$. Mass m for instance on 5 is stretched to $m' = m/\cos \theta$, time to $t' = t/\cos \theta$ on 4 and length contracted on 1 to $l' = l \cdot \cos \theta$. The rescaling of momentum is $p' = m'v = mv/\cos \theta$ and for EMI's frequency it depends in the Doppler effect on a x-space direction for the wave expansion in relation to an observer P of the wave. If P moves with the EMI +x or -x expansion it is longitudinal, if P moves orthogonal to x it is transversal $f' = f \cdot \cos \theta$. The measuring projection between two rays at an angle θ is orthogonal as in figure 6.

Conclusion

It is clear that the old quantum mechanics axioms cannot cover the experimental modern findings. Also Einstein's results are only partly included in the old theories. Measurements for the quasiparticles are by the GF and have the Copenhagen interpretation for quantum measures. An inner dynamics for nucleons is new, the SI rotor. The old fiber bundle method of Hopf for the WI Pauli spin calculus is complemented for this by the new SI geometry with a 5-dimensional fiber bundle for nucleons. The construction uses

rgb-graviton whirls as quasiparticles and neutral color charge of nucleons; they are like spin as color charge superpositions of the first three SI GellMann matrix color charges. Generated is a tetrahedron in the modelling of chemistry, having a base triangle which carries as vertices three nucleon quarks. The finite symmetry S_4 of order 24 can be factored by the normal CPT Klein group for the triangle symmetry which has a representation as the SI rotor. The six equivalence classes of S_4 are listed in the tables rows. Essentially every class has an energy (See figure 5), one of the octonian or Hopf coordinates and symmetries and a color charge.

In the first line of the table are the spherical SI coordinates, possibly 7- (not 8-)dimensional extended with exponential/polar coordinates. In the second line are the linear Pauli/Euclidean coordinates, in the third line a distribution of color charges to the SI coordinates. The fourth (fifth) line contains the D3 (SU(2)/Pauli) MTs as cross ratios. Their matrix names are in the sixth line, together with the Einstein matrices. The following line is a numbering for a strong 6-fold integration series (not the Fano figures numbers which are for octonians). The next line contains the Planck numbers. Energy vectors are in the second to last line and the last line contains natural constants and three more operators, C (conjugation for quantum numbers), T (time reversal) and P (space parity) of physics.

r or $re^{i\varphi_1}$	φ	θ	ict	iu	iw
$x \in \mathbb{R}$	$iy \in i\mathbb{R}$	$z \in \mathbb{R}$			
r	g	\bar{g}	\bar{b}	b	$\bar{\tau}$
z	$\frac{z}{z-1}$	$\frac{-z-1}{z}$	$(-z-1)$	$\frac{1}{-z-1}$	$\frac{1}{z}$
$\frac{1}{z}$	$-\frac{1}{z}$	$-z$	z		
$id; \sigma_1$	$\alpha\sigma_1; \sigma_2$	$\alpha^2; \sigma_3$	$\alpha^2\sigma_1; id$	$\alpha; \delta$	$\sigma_1; id; \beta$
1	6	4	2	5	3
length λ_P	temp. T_P	dens. ρ_P	time t_P	ener. E_P	mass m_P
$E_{M_{pot}}$	E_{heat}	E_{rot}	E_{magn}	E_{kin}	E_{pot}
$c, \epsilon_0, \epsilon_0$	k, C	N_A, T	μ_0	\hbar	γ_G, R_S, P

Figure 5

The octonian system is described by using 8-vectors, doubling up the 4-vectors of physics. This replaces the infinite dimensions of Hilbert space for quantum structures. It allows to describe deuteron and deuteron states.

The construction has many octonian subspaces available which need no additional vector extension. The subspace coordinates are constructed geometrically and not necessarily linear. It is useful to allow fibers S^1 as coordinates with stereographic projections on a linear vectorial coordinate. Used in this article are Euclidean, spherical, cylindrical, barycentric, affine Minkowski and projective coordinates. In addition, real numerical extensions are for complex, quaternionic and octonian coordinates. For real, complex or quaternionic inner product spaces H the Gleason frames in 3 (or more) dimensions generate Gleason measuring operators T which

rescale the inner product in $\langle u, u \rangle$ and this new metrical quadric generates a probability μ with $\mu(N) = \text{tr}(T \text{pr} N)$ on closed subspaces N of H where tr is the trace of the operator and prN is the projection of H onto N.

The new axiomatic treatment adds to or revises quantum mechanics axioms. The old formulations for them cannot be used for modern experimental findings.

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