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Case Study

Conference matrices C_4, C_6, C_8: Expanding Matrices Changing the Element Layout Structure to easy Operation

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Abstract

A conference matrices has the diagonal as "0", the 1st row and 1st column as "1", and the other parts as "-1" and "1". Let's call the former the core (CORE) and the latter the body (A). By changing the positive and negative values of the core and body, they maintain also orthogonal as matrices that are easy to layout. They can accommodate the various levels of experimental design by researchers.

Keywords: Conference Matrices; Optimizing; Analysis; DOE; Taguchi Methods

Introduction

There was the Taguchi design of experiments based on the mixed type orthogonal table and evaluation with SN-ratio [1]. However there were the fatal problems that were to request too much number of trails and poor estimations for optimum condition s by selecting the factor effect charts. The researchers have been expect to improve those kinds of the fatal defects of Taguchi methods [1]. Actually, the optimum conditions with 62% [2] cases studies optimizing by Taguchi methods were under the top of response value data at orthogonal array table L_{18}. So, the 62% optimums were not over the top of trial values of the L_{18} orthogonal Table results. we have been conducted the New WAY:C [a, b] design [3] to reduce the number of experiments with C matrices and to get the better estimation with [a, b] analysis to detect the interaction for layout factors.

The most popular C matrices [4] are C_4, C_6, C_8. We show C_4 matrices to table 1. New WAY:C [a, b] will be applied Belevitch [5] type C matrices.

0	1	1	1
1	0	−1	1
1	1	0	−1
1	−1	1	0

Table 1: $C_4(2^1 3^3)$.

The C matrix satisfies the following three requirements.

1: The diagonal component is ``0''.

2: Each element is either ``-1'' or ``+1.''

3: Each column is orthogonal

When applying the C matrix, the column elements [-1,0,1] are replaced with the ``1st, 2nd, and 3rd'' levels.

The order of allocation levels is often set according to the trends of (small, medium, and large), (short, medium, and long), and (weak, medium, and strong) and etc.

Therefore, when the C matrices is applied to the design matrices, the third level is gathered in each column of the first row.

Comparing the current Hadamard matrices (L_8, L_{27}), which consists of the first row with the first level, and the mixed system orthogonal array (L_{18}, L_{36}), it was evaluated as lacking in convenience.

There were many requests from researchers to make the first level the first row in order to make the C matrices as friendly design matrices. Of course, there was a proposal from the matrices research side to replace [-1, 0, 1] in Table 1 with the "3rd, 2nd, 1st" level. However, researchers at the design site said, ``This proposal can be assumed to be prone to errors.'' We promised to conduct the countermeasure to research C matrices generation to meet this requests.

Therefore, we generated C matrices with the different arrangement of elements, with the core (CORE) and body (A) signs changed to positive or negative.

Generation mechanism for Belevitch type of Conference matrics

The generation mechanism of the C matrix is shown. The Legendre index is shown in table 2. p is a prime number and a is an integer.

Let's take an example of p=5. Since p is 5, squaring 1, 2, 3, and 4 gives 1, 4, 9, and 16. When dividing this by p (5), the remainder is 1 ,4,4,1.

Since both remainders 1 and 4 remain, if we set 1 and 4 of integer to [1], 2 and 3 of integer to [-1], and 0 to [0], and position them at the beginning. [0,1, -1, -1, 1] is the Legendre index of p5.

Legendre symbol $\left(\frac{a}{p}\right)$
for various a (along top) and p (along left side).

p \ a	0	1	2	3	4	5	6	7	8	9	10
3	0	1	−1								
5	0	1	−1	−1	1						
7	0	1	1	−1	1	−1	−1				
11	0	1	−1	1	1	1	−1	−1	−1	1	−1

Only $0 \le a < p$ are shown, since due to the first property below any other a can be reduced modulo p. Quadratic residues are highlighted in yellow, and correspond precisely to the values 0 and 1.

Table 2: Legendre symbol.

Here we show how to generate $C_6(2^1 3^5)$ using the Legendre index of p^5.

The Legendre indices p5 are combined vertically and moved horizontally, one by one, in five columns as shown in table 3. Table 4 shows the obtained 5 rows and 5 columns (BODY) within the square frame. Note that this generated matrices is composed of the cyclic matrices.

0				
1	0			
−1	1	0		
−1	−1	1	0	
1	−1	−1	1	0
0	1	−1	−1	1
1	0	1	−1	−1
−1	1	0	1	−1
−1	−1	1	0	1
1	−1	−1	1	0
	1	−1	−1	1
		1	−1	−1
			1	−1
				1

Table 3: 5 rows with p5 shifted by one.

0	1	−1	−1	1
1	0	1	−1	−1
−1	1	0	1	−1
−1	−1	1	0	1
1	−1	−1	1	0

Table 4: Obtained matrices (BODY).

Add ``0'' to the top left of Table 4 and place ``1'' in the first row and first column, as shown in the completed table 5.

0	1	1	1	1	1
1	0	1	−1	−1	1
1	1	0	1	−1	−1
1	−1	1	0	1	−1
1	−1	−1	1	0	1
1	1	−1	−1	1	0

Table 5: Completed C_6 ($2^1 3^5$) (T).

Table 6 shows the transposed matrices T', the original T', and the product T'T is shown in Table 7. The diagonal components are 5, and other elements are ``0''.

We confirmed the element of Table 5 Orthogonality between each layout factors.

The above describes the C matrices generation method implemented by Belevitch.

0	1	1	1	1	1
1	0	1	−1	−1	1
1	1	0	1	−1	−1
1	−1	1	0	1	−1
1	−1	−1	1	0	1
1	1	−1	−1	1	0

Table 6: Transposed matrix (T').

5	0	0	0	0	0
0	5	0	0	0	0
0	0	5	0	0	0
0	0	0	5	0	0
0	0	0	0	5	0
0	0	0	0	0	5

Table 7: Product T'T: Confirmation Orthogonality.

Definition of Balonin and Jennifer for C matrices

C matrices researchers Balonin and Jennifer [3] defined the Belevitch matrices as shown in Table 8, where BODY is A and the matrices [1,1,1,....] is (e,e^T).

0	e
e^T	A

Table 8: Definition of Belevitch matrices by Balonin and Jennifer.

The definition in Table 8 is the Belevitch-type with 6 rows and 6 columns, where e, e^T, and A are positive (+).However, we confirmed that orthogonality was maintained even when the positive (+) of each term was individually changed to negative (-).

The case of changing "e^T" to "-e^T"

The confirmation of the orthogonality of the corresponding $C_4(2^1 3^3)$ matrices with "e^T" ⇒"-e^T" confirmed here is shown below. The definition of "e^T" ⇒ "-e^T" is shown in table 9, the $C_4(2^1 3^3)$ matrices is shown in Table 10, the transpose T' is shown in table 10, and the product T'T is shown in table 11. In table 11, the elements outside the diagonal are ``0'', so Table 10 of the design matrices is orthogonality defined at table 9.

0	e
$-e^T$	A

Table 9: Definition of "e^T" ⇒ "-e^T".

0	-1	-1	-1
1	0	1	-1
1	-1	0	1
1	1	-1	0

Table 10: Transposed matrices T'.

3	0	0	0
0	3	0	0
0	0	3	0
0	0	0	3

Table 11: Product T'T: Confirmation Ortogonality.

The case of changing "A" to "-A"

We showed the $C_4(2^1 3^3)$ matrices under changing `A'' ⇒ ``-A'' and confirmed orthogonality of it. The definition of ``A'' ⇒ ``-A'' is shown in table 12, the $C_4(2^1 3^3)$ matrices is shown in table 13, the transpose T' is shown in table 14, and the product T'T is shown in Table 16. In Table 15, all the columns except the diagonal are 0, and the columns are orthogonality.

Generating C matrices with different element arrays by reversing the positive/negative of the core and BODY of the C matrix

The researchers will cover all subjects with almost three kind C matrices C_4, C_6, and C_8. For this reason, we limited them in C_4, C_6, and C_8 and generated C matrices with different element arrangements by reversing the positive/negative of the core [e, e^T] and the body [A] of the Belevitch type matrices. We could confirm Orthogonality between columns even if each terms in the core and body were individually reversed. We reversed the positive and negative definitions of [e,eT] and [A] . We generated C_4 matrices are shown in Table 16-1/2, the C_6 matrix is shown in Table 17-1/2, and the C_8 matrices is shown in Table 18-1/2.

0	e
e^T	-A

Table 12: Definition of "A" ⇒ "-A".

0	1	1	1
1	0	1	-1
1	-1	0	1
1	1	-1	0

Table 13: A⇒-A:$C_4(2^1 3^3)$ Matrices.

0	1	1	1
1	0	-1	1
1	1	0	-1
1	-1	1	0

Table 14: Transposed Matrices (T').

3	0	0	0
0	3	0	0
0	0	3	0
0	0	0	3

Table 15: Product T'T: Confirmation Orthogonality.

0	e
e^T	A
[1]	

0	1	1	1
1	0	-1	1
1	1	0	-1
1	-1	1	0

0	-e
e^T	A
[2]	

0	-1	-1	-1
1	0	-1	1
1	1	0	-1
1	-1	1	0

0	e
$-e^T$	A
[3]	

0	1	1	1
-1	0	-1	1
-1	1	0	-1
-1	-1	1	0

0	-e
$-e^T$	A
[4]	

0	-1	-1	-1
-1	0	-1	1
-1	1	0	-1
-1	-1	1	0

Table 16: 1 $C_4(2^1 3^3)$ core positive and negative at A (+).

0	e
e^T	-A
[1]	

0	1	1	1
1	0	1	-1
1	-1	0	1
1	1	-1	0

0	-e
e^T	-A
[2]	

0	-1	-1	-1
1	0	1	-1
1	-1	0	1
1	1	-1	0

0	e
$-e^T$	-A
[3]	

0	1	1	1
-1	0	1	-1
-1	-1	0	1
-1	1	-1	0

0	-e
$-e^T$	-A
[4]	

0	-1	-1	-1
-1	0	1	-1
-1	-1	0	1
-1	1	-1	0

Table 17: 2 $C_4(2^1 3^3)$ core positive and negative at A (-).

0	e
e^T	A

[1]

0	1	1	1	1	1
1	0	1	−1	−1	1
1	1	0	1	−1	−1
1	−1	1	0	1	−1
1	−1	−1	1	0	1
1	1	−1	−1	1	0

0	−e
e^T	A

[2]

0	−1	−1	−1	−1	−1
1	0	1	−1	−1	1
1	1	0	1	−1	−1
1	−1	1	0	1	−1
1	−1	−1	1	0	1
1	1	−1	−1	1	0

0	e
$-e^T$	A

[2]

0	1	1	1	1	1
−1	0	1	−1	−1	1
−1	1	0	1	−1	−1
−1	−1	1	0	1	−1
−1	−1	−1	1	0	1
−1	1	−1	−1	1	0

0	−e
$-e^T$	A

[4]

0	−1	−1	−1	−1	−1
−1	0	1	−1	−1	1
−1	1	0	1	−1	−1
−1	−1	1	0	1	−1
−1	−1	−1	1	0	1
−1	1	−1	−1	1	0

Table 18: $C_6(2^13^5)$ core positive and negative at A(+).

0	e
e^T	−A

[1]

0	1	1	1	1	1
1	0	−1	1	1	−1
1	−1	0	−1	1	1
1	1	−1	0	−1	1
1	1	1	−1	0	−1
1	−1	1	1	−1	0

0	−e
e^T	−A

[2]

0	−1	−1	−1	−1	−1
1	0	−1	1	1	−1
1	−1	0	−1	1	1
1	1	−1	0	−1	1
1	1	1	−1	0	−1
1	−1	1	1	−1	0

0	e
$-e^T$	−A

[3]

0	1	1	1	1	1
−1	0	−1	1	1	−1
−1	−1	0	−1	1	1
−1	1	−1	0	−1	1
−1	1	1	−1	0	−1
−1	−1	1	1	−1	0

0	−e
$-e^T$	−A

[4]

0	−1	−1	−1	−1	−1
−1	0	−1	1	1	−1
−1	−1	0	−1	1	1
−1	1	−1	0	−1	1
−1	1	1	−1	0	−1
−1	−1	1	1	−1	0

Table 18-2: $C_6(2^13^5)$ core positive and negative at A (-).

0	e
e^T	A

[1]

0	1	1	1	1	1	1	1
1	0	−1	−1	1	−1	1	1
1	1	0	−1	−1	1	−1	1
1	1	1	0	−1	−1	1	−1
1	−1	1	1	0	−1	−1	1
1	1	−1	1	1	0	−1	−1
1	−1	1	−1	1	1	0	−1
1	−1	−1	1	−1	1	1	0

0	−e
e^T	A

[2]

0	−1	−1	−1	−1	−1	−1	−1
1	0	−1	−1	1	−1	1	1
1	1	0	−1	−1	1	−1	1
1	1	1	0	−1	−1	1	−1
1	−1	1	1	0	−1	−1	1
1	1	−1	1	1	0	−1	−1
1	−1	1	−1	1	1	0	−1
1	−1	−1	1	−1	1	1	0

0	e
$-e^T$	A

[3]

0	1	1	1	1	1	1	1
−1	0	−1	−1	1	−1	1	1
−1	1	0	−1	−1	1	−1	1
−1	1	1	0	−1	−1	1	−1
−1	−1	1	1	0	−1	−1	1
−1	1	−1	1	1	0	−1	−1
−1	−1	1	−1	1	1	0	−1
−1	−1	−1	1	−1	1	1	0

0	−e
$-e^T$	A

[4]

0	−1	−1	−1	−1	−1	−1	−1
−1	0	−1	−1	1	−1	1	1
−1	1	0	−1	−1	1	−1	1
−1	1	1	0	−1	−1	1	−1
−1	−1	1	1	0	−1	−1	1
−1	1	−1	1	1	0	−1	−1
−1	−1	1	−1	1	1	0	−1
−1	−1	−1	1	−1	1	1	0

Table 19-1: $C_8(2^13^7)$ core positive and negative at A (+).

0	e
e^T	−A

[1]

0	1	1	1	1	1	1	1
1	0	1	1	−1	1	−1	−1
1	−1	0	1	1	−1	1	−1
1	−1	−1	0	1	1	−1	1
1	1	−1	−1	0	1	1	−1
1	−1	1	−1	−1	0	1	1
1	1	−1	1	−1	−1	0	1
1	1	1	−1	1	−1	−1	0

0	−e
e^T	−A

[2]

0	−1	−1	−1	−1	−1	−1	−1
1	0	1	1	−1	1	−1	−1
1	−1	0	1	1	−1	1	−1
1	−1	−1	0	1	1	−1	1
1	1	−1	−1	0	1	1	−1
1	−1	1	−1	−1	0	1	1
1	1	−1	1	−1	−1	0	1
1	1	1	−1	1	−1	−1	0

0	e
$-e^T$	−A

[3]

0	1	1	1	1	1	1	1
−1	0	1	1	−1	1	−1	−1
−1	−1	0	1	1	−1	1	−1
−1	−1	−1	0	1	1	−1	1
−1	1	−1	−1	0	1	1	−1
−1	−1	1	−1	−1	0	1	1
−1	1	−1	1	−1	−1	0	1
−1	1	1	−1	1	−1	−1	0

0	−e
$-e^T$	−A

[4]

0	−1	−1	−1	−1	−1	−1	−1
−1	0	1	1	−1	1	−1	−1
−1	−1	0	1	1	−1	1	−1
−1	−1	−1	0	1	1	−1	1
−1	1	−1	−1	0	1	1	−1
−1	−1	1	−1	−1	0	1	1
−1	1	−1	1	−1	−1	0	1
−1	1	1	−1	1	−1	−1	0

Table 19-2: $C_8(2^13^7)$ core positive and negative at A (-).

The description of the transposed matrix T' and the product T'T is omitted, but orthogonalities have been confirmed.

We expect they will contribute to optimize the researcher subjects with 1/3-1/2 to Taguchi way L_{18}.

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