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Research Article

Analytical Assessment of Favorable Ratio Between Thickness of Knife and Grate of Auger Meat Chopper Aimed to Improve Minced Meat Quality and Reduce Cutting Pair Wear Rate

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Abstract

A mathematical model has been developed for the bending process of the outlet grinding grate as a circular perforated plate under the load of the meat product parabolically decreasing along the radius. The deflection of the knife blade under this parabolic load of the food medium has been determined analytically. Based on the analysis of the features of the physical model representing the process of contact interaction between the elements of the cutting unit of meat raw materials, the conditions of fastening are formulated. The mentioned conditions exclude the concentration of mutual friction forces of the knife and grate and ensure a decrease in the rate of their wear and high quality of the meat product. Based on the principle of deformations compatibility, an analytical dependence of the required thickness of the knife blade on the thickness of the outlet grinding grate was established with the objective to reduce the wear rate by ensuring the equidistance of elastic lines of their bending under the raw meat load parabolically decreasing along the radius. The equidistance of the elastic deflection lines of the knife blades and the annular grinding grate excludes the gap between the contact surfaces of the knife and the grate. In this case, the penetration of the fibers of the crushed meat into the contact joint of the cutting pair is prevented. Consequently, the absence of joint wedging eliminates the material crumpling and the useful meat juice squeezing. Thus, reducing the loss of the meat liquid phase in the prepared minced meat ensures the high quality of the output product.

Keywords: Analytical; Auger Meat; Minced Meat; Wear Rate

Introduction

Analysis of the mathematical model of raw meat auger grinders, extruders and tops, as well as the results of experimental studies of the dependence of the energy-power parameters of the knife-grate pair on the central clamping nut torque have shown that the mentioned unit is the most dynamically and thermally stressed [1-3]. This is confirmed by the experience of industrial operation of the tops since frequent re-sharpening of knives and change of cutting sets of the tops are a common vulnerability of grinding and cutting equipment. According to [2], the full resource of one cross knife is deployed during two months of ordinary operation.

As follows from [2,4], in the process of operation, the cutting pair wears out and the quality of the meat product significantly decreases, while the power consumed by the auger meat chopper increases by 8-20%. Experimental studies indicate that the temperature of raw meat rises and reaches ten degrees at the knife-grate junction, and this is a significant factor in increasing the wear rate of the grate and knives [3] and reducing the quality of the meat product. In addition, in the case of a discrepancy between the thickness of the knife and the chopping grate, there contact surfaces adhere not tightly, raw meat fibers get into this gap, the fibers are crushed and grinded, and useful components contained in the meat liquid fraction are squeezed and lost as well. An in-depth analysis of physical and mechanical processes occurring during the operation of auger grinders' cutting unit has shown that the pressure of the raw meat exerted on the grate and knife reaches significant values, specifically, 0.5-1.2 MPa and more.

In the traditional conditions of fixing the grate and the knife with a clamping nut along the outer annular contour, and this pressure makes the grate to bend with a bulge outward of the top body in the direction of the product outlet, and the knife in this case rests with its peripheral sections on the outer annular part of the grate turning into a two-post simple beam. As a result, when rotating the knife creates a significant concentration of normal stresses on the peripheral annular surface of the grate at the junction with it, which leads to the concentration of friction forces in this zone and to accelerated wear of both the knife and the grate. The foregoing requires a fundamental change in the fastening conditions (not along the outer annular contour of the central mounting hole of the grate, but along the inner one), and a correct mathematical description of the process of interaction between the knife and the grate aimed to optimize the design and technological parameters of the cutting unit for raw meat.

The designated problem can be solved for the uniform [5] distribution of the pressure of the extruded meat raw material, as well as for the linearly decreasing [6] load along the radius of the output grinding grate. Considering the more complex nature of the meat product load distribution along the radius of the output grinding grate, a new task has been posed.

The purpose of the work is to provide a mathematical description of the dependence of the required thickness of the knife blade on the thickness of the output grinding grate under the load parabolically decreasing along the radius and do it based on the condition of ensuring the equidistance of the elastic deflection lines of the grate and the knife, which prevents the penetration and crushing of meat fibers in the knife-grate joint, eliminates fibers squeezing out and the useful liquid loss and, thereby, guarantees an increase in the quality of the minced meat produced.

Research Objects and Methods

The object of the research is the process of interaction between the knife and the grate in the cutting pair of a raw meat auger grinder. The subject of the research is the conditions of fastening as well as the ratio of the knife and grate thickness, which ensures the equidistance of the elastic lines of their deflection to exclude the concentration of stresses in the contact zone, which leads to a decrease in the wear rate under the raw meat load parabolically changing along the grate's radius and provides an increase of the minced meat quality.

The chosen research method is a systematic analysis of possible conditions for fixing the grate, as well as mathematical modeling through the theory of elasticity and differential calculus, the processes of the grate and knife bending aimed to optimize the thickness of the knife blade based on the equation of compatibility of deformations under the pressure parabolically changing along the radius of the grinding grate with regards to the influence on deformation processes of physical, mechanical and rheological characteristics of raw meat.

Results and Discussion

Justification of perspective conditions for fixing the knife-grate cutting pair in raw meat auger grinders

As the analysis of the characteristics and parameters of physical processes carried out in auger grinders shows, the pressure of the extruded meat raw material exerted on the grate and knife reaches $1.2 \cdot 10^5$ Pa and more [7,8]. Therefore, in accordance with the traditional conditions of fixing, under this meat product pressure, the grate bends with a bulge outward of the top body, and the knife blades rest on the peripheral annular platforms of the grate, and in this case represent a two-post simple beam. Thus, this meat pressure creates a significant concentration of stresses at the knife-grate junction on the peripheral annular surface, which leads to accelerated wear of both the knife and the grate in the case of these pressures' relative rotation. Schematically, this process is shown in figure 1.

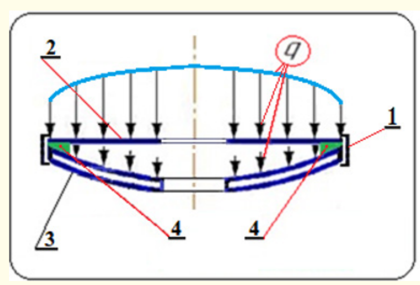

Figure 1: Scheme of traditional conditions for fixing the annular meat-grinding grate in the form of rigid fixation along its outer contour; 1 is outer annular rigid grate termination; 2 is knife blade; 3 is outlet chopping grate; 4 is zones of stress concentration and increased knife blades wear rate.

Thus, there is the need to synthesize and substantiate a promising scheme for fixing the cutting pair.

From the viewpoint of the elasticity theory, the knife grate of the auger grinder is a thin circular perforated annular plate with a central mounting hole fixed in the body of the auger meat grinder. In the most general case, more than two dozen options for fastening the knife grate by means of a clamping nut have been considered [5,8,9]. As a result of the system analysis, it is shown that the main options are the ones presented in figure 2. The diagrams of option A in this figure reflect fixing the grate by means of a rigid annular termination (1 is external termination of the ring, 2 is an internal one, 3 is a double-sided one). Diagrams B in figure 2 relate to pivot ring supports (4 is external, 5 is internal, 6 is double-sided supports). Schemes C illustrate a mixed fastening of the annular grate (7 is outer annular hinge support with an internal rigid ring termination, 8 is external rigid termination with an internal annular articulated support).

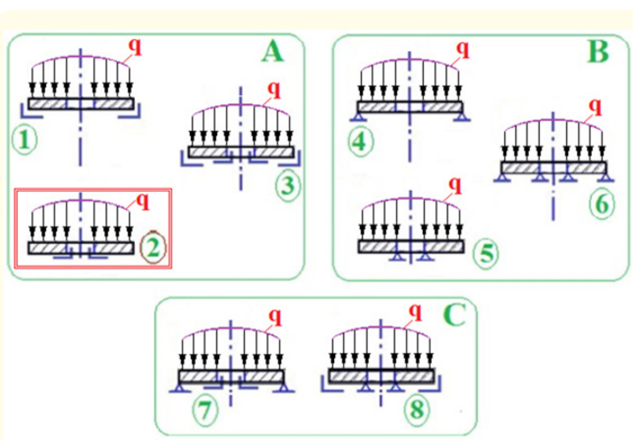

Figure 2: Main possible schemes for fixing grates in auger meat choppers; 1 is traditional grate fixing scheme; 2 is perspective fixing diagram.

Analyzing the main schemes, it should be concluded that out of the proposed options for fixing the grinder grate, it is necessary to pay attention to the case being the most promising from a practical perspective and presented by option 2 of scheme A in Figure 2. In a reasonable solution, the ring plate is fastened along the inner border of the ring mounting holes by means of a rigid seal. This choice is based on the results of the analysis covering the process of joint deformation of the grate and the knife during the meat chopper functioning. Indeed, in this case, the plate deformation is caused by the bulge inside the top body, as well as the knife; therefore, it is possible to provide the same displacement values of the peripheral annular sections of the grate and knife blades. Thus, the equidistance of elastic deflection lines and uniform forces of interaction between the knife and the grate in the zone of their joint are ensured, stress concentration is excluded, and, as a result, the wear rate of the knife and the grate is reduced, as well, the quality of minced meat does not decrease due to possible crumpling of raw meat. Moreover, frictional loss is reduced and the temperature load on meat raw materials and the cutting head at the knife-grate joint is reduced.

In this case, in the problem under consideration, the external force of raw meat is a load *q* parabolically decreasing along the radius of the grate and uniformly distributed along the circumferences of the area of the perforated plate (paraboloid of revolution).

The physical model of the grate and knife blade deflection for the perspective version of the cutting unit attachment by means of rigid termination along the inner annular mounting hole is shown in figure 3.

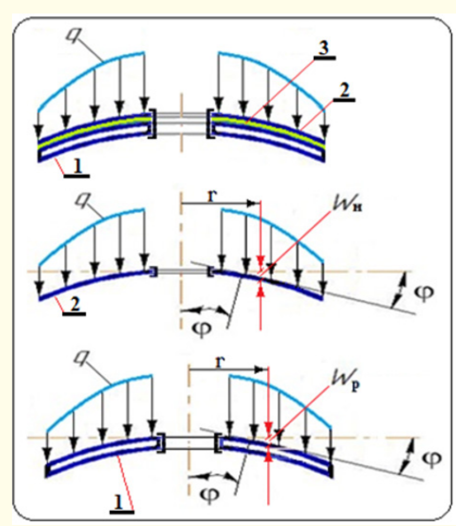

Figure 3: Perspective diagram of joint loading by pressure caused by raw meat, cutting knife-grate pair and individual knife and grater deflection; 1 is outlet grille; 2 is knife blade; 3 is zone of uniform contact stresses; φ is angle of rotation of grate and knife cross-section.

An obvious difference in the nature and features of the meat force at the knife-grate junction and the specific of their deflection is visible when comparing Figures 1 and 3, which is clearly illustrated in figure 4.

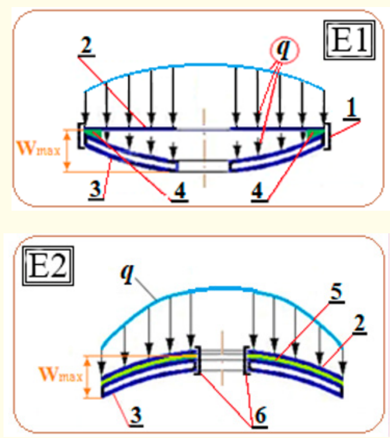

Figure 4: Comparative representation of deflection and features of interaction between elements at the junction of cutting knife-grate pair under raw meat pressure in traditional (E1) and prospective (E2) fixing schemes; E1 is traditional grid embedding scheme along the outer annular contour; E2 is perspective scheme for embedding grate along the contour of the inner hole of the grate; 1 is outer annular rigid grate termination; 2 is knife blade; 3 is outlet chopping grate; 4 is zones of stress concentration and increased wear rate of knife blades; 5 is zone of uniform contact stresses; 6 is internal annular rigid grate termination.

Thus, a mathematical model of the interaction of a knife and a grate of auger grinders under the pressure of raw meat is to be developed for a promising scheme for fixing a cutting pair with a rigid embedment along the inner annular surface of the mounting hole, which eliminates the concentration of internal stresses in the joint and maintain the quality of minced meat.

Approximation of load on knife and grate by parabolic dependence on the radius of exit grinding grate under raw meat pressure

Under the conditions of applying a parabolic load to the cutting pair, to reduce the wear rate of the knife and grating, it is necessary to ensure the equality of their deflections, which will minimize contact stresses and exclude the concentration of friction forces.

As shown in [3], in order to guarantee the equality of the deflection deformations of the output grinding grate and the knife blade, it is necessary to ensure the equidistance of their curved median surfaces. In [5], a mathematical model of the deflection of a perforated grate with a mechanical load uniformly distributed over its surface and a thermal bending moment uniformly distributed along the peripheral outer, free of bonds, annular boundary of the grate and linearly distributed over its thickness was developed. However, as is shown in [6], the load distributed over the annular surface of the grinding grate is not uniform. In the mentioned study, the load applied to the cutting unit of the auger grinder is approximated by a linearly decreasing load along the radius of the outlet grate.

The problem of determining the optimal dependence of the thickness of the knife blade on the thickness of the grate under the conditions of a parabolic load is reduced at the first stage to determining the corresponding law of distributing the pressure of the extruded material along the radius of the grinding grate.

The pressure value of the extruded material in the center of the grate P_c (in the absence of sluicing) was estimated in [7]

$$P_c = \frac{4P_{sp}}{d_h - \frac{2 f \nu_m \delta_g}{1-\nu_m(1-f)}}. \qquad \text{----------- (1)}$$

Where:

P_c is pressure in the central part of the annular grinding grate, Pa.

P_{sp} is specific cutting force of the material crushed in the extruder, N/m;

d_h is hole diameter of the grinding outlet grate, m.

ν_m is Poisson's ratio of the extruded material.

f is coefficient of material sliding friction on the working surfaces.

δ_g is the thickness of the outlet grinding grate, m.

The pressure P_{sl} on the outer boundary of the grinding grate (on its periphery) is set for slugging food material through the annular gap ξ between the surface of the ridges of the counter-rotation beads made on the inner surface of the extruder body and the outer surface of the ridges of the screw auger. The diagram of the forces and parameters of the annular gap are shown in figure 5. The lock pressure P_{sl} under the raw meat pressure is determined from the equilibrium equation of the projections of the forces acting on the extruded material pushed through the annular gap with height ξ and elementary length dl

$P_{sl}\,\xi\,dl = P_{sp}\,dl.$ --------------(2)

Thus, based on equation (2), the value of the meat pressure at the outer peripheral boundary of the annular grate will be of the following value

$P_{sl} = P_{sp}/\xi.$ -------------(3)

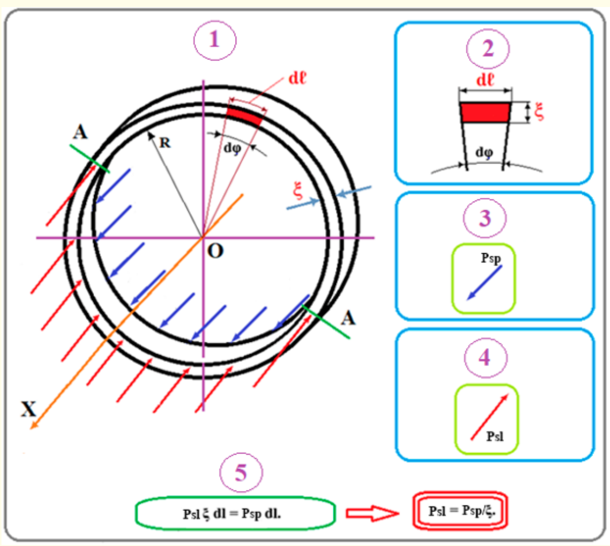

Figure 5: Scheme of action forces and parameters of annular gap, which determine raw meat sluice pressure; 1 is ring cylinder of material in sluice zone (the loads are conventionally not shown above section A-A); 2 is element of the annular cylinder; 3 is specific cutting force of material Psp in sluice gap, N/m; 4 is pressure Psl on selected element of material (ξ * dl) in airlock gap, Pa; 5 is equilibrium equation of material in projection on axis OX.

As shown in [6], it follows from the analysis of the relations (1) and (3) that the only case of forming load distributed uniformly under the influence of the raw meat pressure over the area of the annular grinding grate is possible under the condition of equal pressures on the central and outer peripheral surfaces; thus, it is possible to obtain the relation corresponding to the following condition

$\xi = (d_h/4) - \nu_m f \delta_g/2[1- \nu_m(1- f)].$

Under conditions of the technology of manufacturing the elements of the extruder or the top and the accuracy of foundry, the size of the gap ξ significantly (from one and a half to two times) exceeds the obtained value. Therefore, in the general case and in accordance with the expression (3), the raw meat pressure on the outer peripheral part of the annular grate is one and a half to two or more times less than the central region ($P_{sl} < P_c$) [6,8].

Having determined the edge values of the ring pressures P_c and P_{sl} on the outlet grinding grate, we can write down the corresponding parabolic approximating equation for the distribution of this load under the raw meat pressure along the grating radius *r* (in the form of an equation for a paraboloid of revolution). The axonometric diagram of the impact on the perforated outlet grinding grate with the raw meat load parabolically distributed along the radius is shown in figure 6.

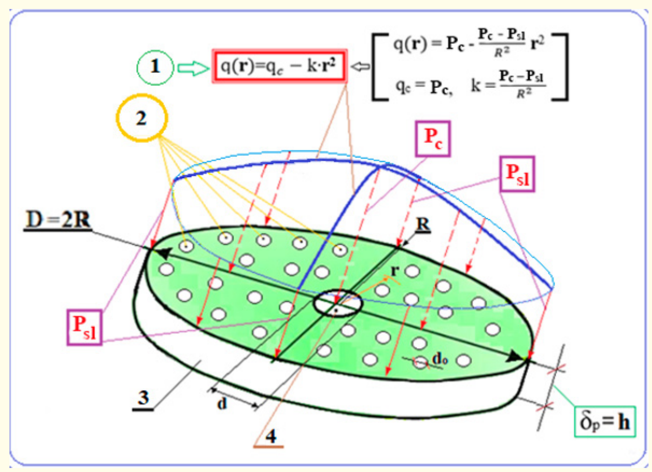

Figure 6: Axonometric diagram of loading annular grinding grate of extruder by raw meat pressure load parabolically distributed along the radius: 1 is equation of load in the form of a paraboloid of revolution on the grate of the top; 2 is holes with a diameter dh, perforating the grate; 3 is outlet chopping grate; 4 is landing (installation) hole with diameter d; R is radius of the grate; h is the grate's thickness; r is current value of the grate radius.

The equation of the parabolically distributed load of meat raw materials along the radius of the output grinding grate is generally written in the following form

$q(r) = a_1 r^2 + a_2 r + a_3.$

An analytical expression for determining the unknown coefficients a_1, a_2, a_3 of this external load q (r) can be found from the boundary conditions

$q(0) = P_c$; $q(R) = P_{sl}$; $q(-R) = P_{sl}$.

Elementary calculations lead to the following
$a_3 = P_c$; $a_2 = 0$; $a_1 = -\frac{Pc - Psl}{R^2}$.

Thus, the parabolic load equation takes the following form
$q(r) = P_c - \frac{Pc - Psl}{R^2} r^2$.

To shorten the entries, we introduce new designations
$q(r) = q_c - k \cdot r^2$, ----------(4)

where

$q_c = P_c$,

$k = \frac{Pc - Psl}{R^2}$.

After finding the analytical dependence of the magnitude of the external load under the raw meat pressure on the radius in the form of the equation of the second degree (4), it seems possible to solve the problem of determining the deflection of the annular perforated grate under its influence.

Assumptions made during mathematical modeling of the bending deformation process of the outlet grinding grate under raw meat pressure

The outlet grinding grate of the extruder is an annular perforated plate with a diameter D = 2R and a thickness δ_p. Such a plate will be calculated in accordance with the theory based on three hypotheses proposed by Kirchhoff for a thin plate [9-11], since its geometric and deformation parameters satisfy the required conditions

$\frac{1}{5} \geq \frac{\delta p}{D} \geq \frac{1}{80}$, as well as: $w \leq \frac{\delta p}{4}$.

Where

W is the magnitude of the plate deflection, m.

is the maximum value of the plate deflection? m.

R = D/2 is the maximum value of the annular plate radius, m.

r the current value of the annular plate radius, m.

It should be noted that a round perforated plate is considered annular provided that

$[(R - b)/R] << 1$,

where: b = d/2 is the central hole radius of the plate, m.

Roughly, the real designs of extruders are characterized by the following proportions: R = 0.03 m, b = 0.004 m, (m is the scale factor), therefore $[(R - b)/R] = 0.867 \approx 1$. Since the initial condition is not met, then in the case under consideration, a round perforated plate can be considered a perforated solid, which greatly simplifies the solution of the problem of determining its deflection. In this case, the influence of perforating holes with a diameter d_h on the deflections of the plate under the raw meat pressure is regarded when calculating the value of its cylindrical stiffness.

Construction of the equation for curved median surface of a solid circular perforated plate under the load parabolically distributed along the radius under conditions of rigid embedding in inner annular part of central hole

The problem posed will be solved by the method of direct integration of the well-known inhomogeneous differential equation of the third order (5) for the plate deflection, which has the following

$$\frac{d^3w}{dr^3} + \frac{1}{r}\frac{d^2w}{dr^2} - \frac{1}{r^2}\frac{dw}{dr} = \frac{Q}{D}, \quad \text{-------------------(5)}$$

where Q is shearing force per unit length of a cylindrical section of radius r, N/m

D_p is cylindrical stiffness of a solid plate (H m), determined by the relation (6)

$$D_p = \frac{E(\delta_p)^3}{12(1-\nu^2)}. \quad \text{-----------------------(6)}$$

When quantitatively evaluating the obtained mathematical models of the plate bending under the influence of the raw meat pressure, we will use the refined value of its cylindrical stiffness (7) regarding the effect of perforated holes on the value of expression (6)

$$D_p = \frac{E(\delta_p)^3}{12(1-\nu^2)}\left(\frac{a - n_r d_h - b}{a}\right), \quad \text{-----------------------(7)}$$

where n_r is the number of holes in the cross-section of the perforated circular annular plate (grate).

d_h is the diameter of the perforating holes of the plate, m.

E is modulus of longitudinal elasticity of the plate material, Pa.

ν is Poisson's ratio of the plate material (grate);

δ_p is plate thickness (δ_p h), m.

For a plate with an external load $q(r)$ distributed over the area, we can obtain the value of $Q(r)$ from the equilibrium equation

$$2\pi r = \int_0^r q(r) 2\pi r dr. \quad \text{-----------------------(7)}$$

Thus, considering the axisymmetry of the external load $q(r)$ and relations (5) and (6) or (7), the differential equation of the curved middle surface of a solid circular plate is most conveniently written for integration in the following form:

$$r \cdot \frac{d}{dr}\left[\frac{1}{r} \cdot \frac{d}{dr}\left(r \cdot \frac{dw}{dr}\right)\right] = \frac{1}{Dp} \cdot \int_0^r q(r) \cdot r \cdot dr. \quad \text{---------------- (8)}$$

The analytical expression for the external load from the pressure of raw meat $q(r)$ on the plate can be represented as follows

$$q(r) = P_c - \frac{\mathrm{Pc} - Psl}{R^2} r^2.$$

Introducing new designations for the convenience of notation, we obtain the following

$$q(r) = q_c - k \cdot r^2,$$

where $q_c = \mathrm{P}_c$, -------------- (9)

$$k = \frac{\mathrm{Pc} - \mathrm{Ps}l}{R^2}.$$

Considering relation (9), equation (8) takes the following form

$$\mathrm{r} \cdot \frac{d}{dr}\left[\frac{1}{r} \cdot \frac{d}{dr}\left(r \cdot \frac{dw}{dr}\right)\right] = \frac{1}{Dp} \cdot \int_0^r (\mathrm{q}_c - \mathrm{k} \cdot r^2) \cdot r \cdot dr.$$

After integrating the right-hand side, we get

$$\mathrm{r} \cdot \frac{d}{dr}\left[\frac{1}{r} \cdot \frac{d}{dr}\left(r \cdot \frac{dw}{dr}\right)\right] = \frac{1}{Dp}\left(\mathrm{q}_c \frac{\mathrm{r}^2}{2} - k\frac{\mathrm{r}^4}{4}\right). \quad \text{-----------------(10)}$$

Having divided both sides of relation (10) by r, and integrating the resulting equation for the first time, we write

$$\frac{1}{r} \cdot \frac{d}{dr}\left(r \cdot \frac{dw}{dr}\right) + \mathrm{C}_1 = \frac{1}{Dp}\left(\mathrm{q}_c \frac{\mathrm{r}^2}{4} - k\frac{\mathrm{r}^4}{16}\right). \quad \text{---------------- (11)}$$

Having multiplied both sides of relation (11) by r, and integrating it for the second time, we obtain

$$r \cdot \frac{dw}{dr} + \mathrm{C}_1\frac{\mathrm{r}^2}{2} + \mathrm{C}_2 = \frac{1}{Dp}\left(\mathrm{q}_c \frac{\mathrm{r}^4}{16} - k\frac{\mathrm{r}^6}{96}\right). \quad \text{------------------(12)}$$

Having divided both sides by r and integrating relation (12) for the third time, we write the expression for W in the following form

$$\mathrm{W} + \mathrm{C}_1\frac{\mathrm{r}^2}{4} + \mathrm{C}_2 \ln \mathrm{r} + \mathrm{C}_3 = \frac{1}{Dp}\left(\mathrm{q}_c \frac{\mathrm{r}^4}{64} - k\frac{\mathrm{r}^6}{576}\right). \quad \text{------------------(13)}$$

From the physical conditions of the boundedness of the deflections W, it is obvious that $C_2 = 0$. Then the equation (13) takes the following form:

$$\mathrm{W} = \mathrm{q}_c \frac{\mathrm{r}^4}{64Dp} - k\frac{\mathrm{r}^6}{576Dp} - \mathrm{C}_1\frac{\mathrm{r}^2}{4} - \mathrm{C}_3. \quad \text{------------- (14)}$$

It follows from the boundary condition for the zero deflection of the plate at its center that $C_3 = 0$. Then relation (14) will be simplified

$$\mathrm{W} = \mathrm{q}_c \frac{\mathrm{r}^4}{64D} - k\frac{\mathrm{r}^6}{576Dp} - \mathrm{C}_1\frac{\mathrm{r}^2}{4}. \quad \text{------------ (15)}$$

The indefinite coefficient C_1 in equation (15) is determined from the boundary condition of equality to zero of an internal bending moment along the external circumferential (tangential) section of the plate: $M_r(R) = 0$.

Considering the dependence of M_r on r, in accordance with [10-13], we write

$$M_r(r) = \frac{d^2w}{dr^3} + \frac{\nu}{r}\frac{dw}{dr}, \quad \text{-----------(16)}$$

where ν is Poisson's ratio of the plate material.

With and , it follows from relation (16)

$$\frac{d^2w}{dr^2} + \frac{\nu}{r}\frac{dw}{dr} = 0. \quad \text{------------ (17)}$$

Differentiating relation (15) twice, we obtain

$$\frac{dw}{dr} = \mathrm{q}_c \frac{\mathrm{r}^3}{16Dp} - k\frac{\mathrm{r}^5}{96Dp} - \mathrm{C}_1\frac{\mathrm{r}}{2}; \quad \text{--------------(18)}$$

$$\frac{d^2w}{dr^2} = 3\mathrm{q}_c \frac{\mathrm{r}^2}{16Dp} - 5k\frac{\mathrm{r}^4}{96Dp} - \frac{\mathrm{C}_1}{2}. \quad \text{--------------(19)}$$

Substituting expressions (18) and (19) into equation (17), we write

$$3\mathrm{q}_c \frac{\mathrm{r}^2}{16Dp} - 5k\frac{\mathrm{r}^4}{96Dp} - \frac{\mathrm{C}_1}{2} + \frac{\nu}{r}\left(\mathrm{q}_c \frac{\mathrm{r}^3}{16Dp} - k\frac{\mathrm{r}^5}{96Dp} - \mathrm{C}_1\frac{\mathrm{r}}{2}\right) = 0. \quad \text{---------------(20)}$$

Solving the resulting equation (20) with respect to C_1, with $r = R$, we obtain

$$\mathrm{C}_1 = q_c \frac{R^2}{8Dp}\frac{(3+\nu)}{(1+\nu)} - k\frac{R^4}{96Dp}\frac{(5+\nu)}{(1+\nu)}. \quad \text{-------------(21)}$$

Substituting the obtained expression (21) into equation (15), we obtain the equation of the curved middle surface of a solid circular perforated plate under the raw meat load parabolically distributed along the radius amid its central part rigid embedding

$$Wp(r) = q_c \frac{r^4}{64Dp} - k\frac{r^6}{576Dp} - q_c \frac{R^2}{32Dp}\frac{(3+v)}{(1+v)}r^2 + k\frac{R^4}{192Dp}\frac{(5+v)}{(1+v)}r^2$$ ---------------(22)

In this case, the maximum value of the deflection of the perforated grate according to relation (22) reaches with and gets the value

$$Wp.max = Wp(R) = \frac{R^4}{Dp}\{\frac{q_c}{64}[1 - \frac{2(3+v)}{(1+v)}] - k\frac{R^2}{576}[1 - \frac{3(5+v)}{(1+v)}]\}.$$ ------------(23)

Having deduced similar terms, we simplify the resulting expression (23) to the form

$$Wp.max = Wp(R) = -\frac{R^4}{64Dp}\left[q_c\frac{(5+v)}{(1+v)} - k\frac{2R^2}{9}\frac{(7+v)}{(1+v)}\right].$$ -------------(24)

After substituting relation (6) into equation (24), as well as the values for q_c and k from explication to relation (4), we obtain

$$Wp.max = -\frac{R^4(1-v)}{48E\,(\delta_g)^3}[P_c(31-7v) + 2\,Psl(7+v)].$$ ----(25)

The next step in solving the problem is to determine the deflection of the knife blade under the raw meat load parabolically distributed along its length.

Solution of differential equation for bending knife blade under raw meat load parabolically distributed along its length amid cantilever rigid embedding in central part of grate

The problem of minimizing the contact stresses of the elements of the knife-grate pair, thus, reducing the rate of their wear, as well as the quality of minced meat makes it necessary to ensure equality of the deflections of the knife blades and the perforated grate. Thus, the urgent task of this section of the article is to correctly write and solve the differential equation for the knife blade bending as a cantilever is rigidly sealed with one short side of a rectangular plate, which is under the pressure of raw meat loading parabolically distributed along its length. The design diagram of the formulated problem is shown in figure 7.

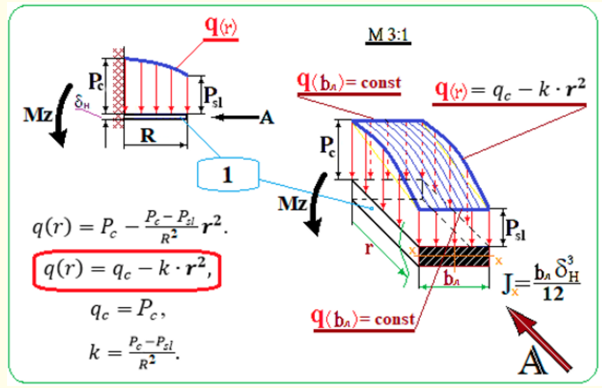

Figure 7: Scheme of loading a segmented knife with a load parabolically distributed along its length r; 1 is knife blade; Mz is moment in termination; Jx is axial moment of rectangular section inertia.

Figure 8 (Scheme 1) shows an equivalent flat design pattern of knife loading regarding the constancy of the distributed load of raw meat along the width of the blade.

Due to the laboriousness of the analytical writing of the resolving equation of the knife bending under a parabolic raw meat pressure given in a generalized form (4), it is advisable to seek the solution of the problem of knife blade deflection by the superposition method [14,15]. Figure 8 (Scheme 2) explains the principle of superposition of purely parabolic (II) and uniform (I) loads acting on the knife.

The geometric coordinates of the centers of pressure of the resultant equivalent forces T and N and the analytical characteristics of these force factors are illustrated in figure 9.

Thus, the problem of determining the deflection deformation W = Wk of the knife blade under the raw meat pressure will be solved by the superposition method considering it to be loaded with a constant load $Q(r) = P_{sl}b_k$ = const, from the pressure P_{sl} and a purely parabolic distributed load Q_{kv} (r), decreasing from the value (Pc - Psl) b_k at r = 0 to zero at r = R as shown in schemes (I) and (II) in figure 9.

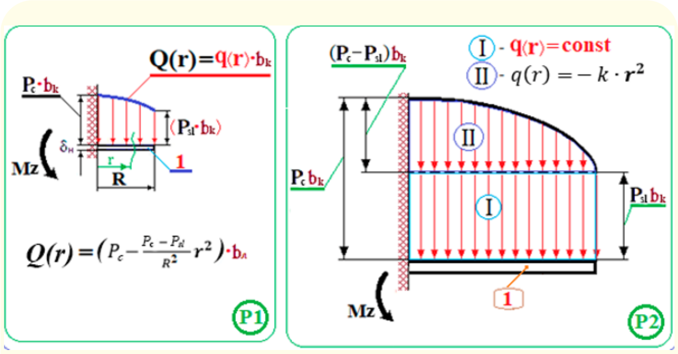

Figure 8: Plane equivalent diagram of loading segmented knife with raw meat load parabolically distributed along its length r; P1 is equivalent flat diagram of the total knife loading; P2 is scheme of superposition of constant (I) and parabolic (II) loads; 1 is knife blade.

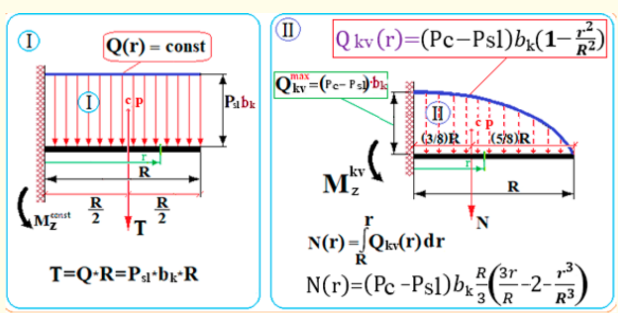

Figure 9: Schemes of superposition loading of a segmented knife with constant load, and load strictly parabolically distributed along its length r.

For a knife blade of constant thickness and width, we write down the resolving equation of bending in the well-known form [13,14]

$$\frac{d^2w}{dr^2} = -\frac{m}{E_k J}. \qquad \text{-------------(26)}$$

M is the value of the bending moment in the section of the blade with the coordinate *r*, N m;

$J = J_x$ is axial moment of inertia of a rectangular section of a knife blade, m^4.

$$J = \frac{b_k}{12}(\delta_k)^3, \qquad \text{-------------(27)}$$

b_k, δ_k are width and thickness of the knife blade, m.

E_k is modulus of longitudinal elasticity of the knife blade material, Pa.

Solution of differential equation for knife blade bending under load uniformly distributed along its length

In the case of a uniformly distributed raw meat load along the length of the knife blade, the value of the bending moment in the section of the blade with the coordinate *r* is determined in accordance with Figure 9, scheme (I) by the ratio

$$M = P_{sl} b_k (R-r)^2/2. \qquad \text{-----------(28)}$$

In this case, the solution to the constitutive differential equation (26) of the knife blade bending under the raw meat pressure can be written in the following algebraic form [14]

$$W_{k1} = -\iint \frac{M}{E_k J} dr\, dr + Cr + B, \qquad \text{-----------(29)}$$

Here *C* and *B* are integration constants determined from the boundary conditions:

$$W = 0; \frac{dw}{dr} = 0, \text{ with } r = 0. \qquad \text{-----------(30)}$$

As a result of integrating equation (29), under boundary conditions (30), we have obtained the deflection value of the knife blade from the constant component of the total load of raw meat depending on the coordinate *r* of the considered section

$$W_{k1} = -\frac{b_k P_{sl} R^4}{24 E_k J}\left[\left(1-\frac{r}{R}\right)^4 + \frac{4r}{R} - 1\right]. \qquad \text{-----------(31)}$$

$$W_{k1} = -\frac{b_k P_{sl} R^4}{24 E_k J}\left[\left(1-\frac{4r}{R}+\frac{6r^2}{R^2}-\frac{4r^3}{R^3}+\frac{r^4}{R^4}\right) + \frac{4r}{R} - 1\right].$$

$$W_{k1} = -\frac{b_k P_{sl\text{ш}} R^2}{24 E_k}\frac{r^2}{J}\left(6 - \frac{4r}{R} + \frac{r^2}{R^2}\right).$$

In this case, the maximum value of the deflection is determined by relation (31) at r = R

$$W_{k1max} = -\frac{b_k P_{sl} R^4}{8 E_k J}. \qquad \text{-----------(32)}$$

Solution of differential equation for knife blade bending under raw meat pressure distributed along its length according to parabola law

In the case of a parabolic raw meat load along the knife blade length, the value of the bending moment in the blade section with the coordinate *r* is determined in accordance with Figure 9, scheme (II) by the ratio

$$M = N(r)\frac{3}{8}R\left(1-\frac{r}{R}\right). \qquad \text{----------(33)}$$

The value N(r) is obtained by integrating the value of $Q_{kv}(r)$ along the radius r [16]

$$N(r) = \int_R^r Q_{kv}(r)dr. \qquad \text{----------(34)}$$

It is easy to show that the algebraic value of Q(r) is determined by the relation

$$Qkv(r) = (Pc - Psl)b_k\left(1-\frac{r^2}{R^2}\right). \qquad \text{----------(35)}$$

After substituting (35) into (34) and integrating, we obtain the expression

$$N(r) = (Pc - Psl)b_k\frac{R}{3}\left(\frac{3r}{R}-2-\frac{r^3}{R^3}\right). \qquad \text{----------(36)}$$

Considering the obtained value (36) of the resultant force N(r), relation (33) for the bending moment in the section of the knife blade with the coordinate *r* takes the following form

$$M = (Pc - Psl)b_{bl}\frac{R^2}{8}\left(\frac{5r}{R}-3\frac{r^2}{R^2}-\frac{r^3}{R^3}+\frac{r^4}{R^4}-2\right). \qquad \text{-----------(37)}$$

In the section of the rigid knife embedment (r = 0), in accordance with (37), we obtain

$$M_{em} = M(0) = -(Pc - Psl)b_k\frac{R^2}{4}.$$

As a result of integrating equation (29), under boundary conditions (30), and the value of the bending moment in the form (37), we obtain the value of the knife blade deflection from the purely parabolic component of the total raw meat load depending on the coordinate *r* of the section under consideration, as is illustrated in Figure 9, scheme (II). For this, we introduce the notation

$$V = (Pc - Psl)b_k\frac{R^2}{8}. \qquad \text{----------(38)}$$

Then we write equation (37) in the following form

$$M = V\left(\frac{5r}{R}-3\frac{r^2}{R^2}-\frac{r^3}{R^3}+\frac{r^4}{R^4}-2\right). \qquad \text{----------(39)}$$

At r = 0, the deflection and the rotation angle of the knife section are equal to zero

$$W_{k2} = \frac{V}{EkJ}\iint\left(\frac{5r}{R}-3\frac{r^2}{R^2}-\frac{r^3}{R^3}+\frac{r^4}{R^4}-2\right)dr\,dr + Cr + B. \qquad \text{---------(40)}$$

$$\frac{dW_{k2}}{dr} = \frac{V}{EkJ}\int\left(\frac{5r}{R}-3\frac{r^2}{R^2}-\frac{r^3}{R^3}+\frac{r^4}{R^4}-2\right)dr + C = 0. \qquad \text{---------(41)}$$

From boundary condition (41) we obtain

$$\frac{V}{EkJ}\left(\frac{5r^2}{2R}-\frac{r^3}{R^2}-\frac{r^4}{4R^3}+\frac{r^5}{5R^4}-2r\right)+C = 0,$$

Thus: C = 0

From boundary condition (40) it follows

$$W_{k2} = \frac{V}{EkJ}\int\left(\frac{5r^2}{2R}-\frac{r^3}{R^2}-\frac{r^4}{4R^3}+\frac{r^5}{5R^4}-2r\right)dr + B = 0.$$

After integration, we get

$$W_{k2} = \frac{V}{EkJ}\left(\frac{5r^3}{6R}-\frac{r^4}{4R^2}-\frac{r^5}{20R^3}+\frac{r^6}{30R^4}-r^2\right)+B = 0,$$

Thus: B = 0.

Thus, we can finally write

$$W_{k2} = \frac{V}{EkJ}\left(\frac{5r^3}{6R}-\frac{r^4}{4R^2}-\frac{r^5}{20R^3}+\frac{r^6}{30R^4}-r^2\right). \qquad \text{---------(42)}$$

Considering (38), relation (42) takes the following form

$$W_{k2} = \frac{(Pc - Psl)b_kR^2}{8EkJ}r^2\left(\frac{5r}{6R}-\frac{r^2}{4R^2}-\frac{r^3}{20R^3}+\frac{r^4}{30R^4}-1\right). \qquad \text{---------(43)}$$

The superposition of deformations from constant (W_{k1}) and purely parabolic raw meat loads (W_{k2}) gives the total maximum deflection of the knife blade.

In this case, the maximum deflection value for r = R will be written

$$W_{k.2max} = -\frac{13(Pc - Psl)b_kR^4}{240EkJ}. \qquad \text{-----------(44)}$$

The superposition of deformations from constant (W_{k1}) and purely parabolic loads (W_{k2}) gives the total knife blade deflection as a function of the radius r

$$W_k = -\frac{b_kR^2r^2}{8EkJ}\left\{\frac{P_{sl}}{3}\left(6-\frac{4r}{R}+\frac{r^2}{R^2}\right)+(Pc - Psl)\left(\frac{5r}{6R}-\frac{r^2}{4R^2}-\frac{r^3}{20R^3}+\frac{r^4}{30R^4}-1\right)\right\}$$

For maximum values of deformation from constant (W_{k1}) and purely parabolic (W_{k2}) loads of meat raw materials, at r = R, we obtain the total maximum knife blade deflection

$$Wk.max = W_{k1.max} + W_{k.2max} = -\left[\frac{b_kP_{sl}R^4}{8EkJ}+\frac{13(Pc - Psl)b_kR^4}{240EkJ}\right].$$

Deducing similar terms, we get

$$Wk.max = -\frac{b_k R^4}{240EkJ}[17P_{sl} + 13Pc]. \quad \text{---------(45)}$$

Considering relation (27) for the moment of inertia of the knife cross-section, we finally write

$$Wk.max = -\frac{R^4}{20Ek(\delta_k)^3}[17P_{sl} + 13Pc]. \quad \text{---------(46)}$$

Formation of criterion ratio for optimizing knife blade thickness with parabolic nature of raw meat pressure depending on the thickness of outlet grinding grate as well as geometric and elastic characteristics of top elements

The condition for the optimal ratio of the knife and grate thickness can be formulated as the equation of compatibility of their deformations under the raw meat pressure with the addition of the value to the knife blade movement, which provides a contact stress equal to the crushing stress of the raw meat, thereby eliminating the occurrence of stress concentrators in a knife-grate pair, and guarantees the lowest wear rate of the cutting elements and the non-penetration of meat fibers into the area of the junction of the knife blades planes and the grate.

In this case, the value is determined by Hooke's law

$$W_к = \frac{\delta_k\, \sigma_{str}}{Ek},$$

where

σ_{str} is tensile strength (crushing) of the minced material.

The criterion equation for the ratio of the knife and grate thickness will be written in this case as follows

$$Wk.max = Wp.max + \frac{\delta_k\, \sigma_{str}}{Ek}$$

Let us estimate the order of the added component of the elastic base (grate) effect on the crushing deformation of the minced meat. For this, we find the ratio of the quantities and with

$$W_к/Wp.max = \frac{\delta_k\, \sigma_{str}}{Ek} \Big/ \frac{R^4}{20Ek(\delta_k)^3}[17P_{sl} + 13Pc] \text{ or}$$

$$W_к/Wp.max = \frac{2(\delta_k)^4\, \sigma_{пр}}{3P_c R^4}.$$

We will take roughly the following values of the parameters of this relation [17]

$\delta_k = 6$ mm; $R = 30\ mm$; $\sigma_{str} = 0{,}25$ MPa; $P_c = 1{,}2$ MPa.

Then we get

$W_к/Wp.max = 0{,}0036$.

Thus, considering the smallness of the added component (less than 0.4%), we can write the criterion equation for the ratio of the knife and grate thickness as the equality of the right-hand sides of equations (25) and (46),

W k. max = W p. max.

Or

$$-\frac{R^4}{20Ek(\delta_k)^3}[17P_{sl} + 13Pc] = -\frac{R^4(1-\nu)}{48E\,(\delta_p)^3}[P_c(31-7\nu) + 2\,Psl(7+\nu)].$$

After a series of algebraic transformations, assuming E = E_k, we get

$$(\delta_k)^3 = \frac{12\,(\delta_p)^3[17P_{sl}+13Pc]}{5(1-\nu)[P_c(31-7\nu)+2\,Psl(7+\nu)]}. \quad \text{----------(47)}$$

Considering that ν = 0.25-0.3, relation (47) can be simplified to the following form

$$(\delta_k)^3 = \frac{12\,(\delta_g)^3[13Pc+17P_{sl}]}{5(0{,}75\div 0{,}7)[P_c(31-4{,}9\div 5.25)+2\,Psl(7{,}25\div 7{,}3)]}. \quad \text{----------(48)}$$

Averaging the range of variation ν, and regarding the real ratio for the meat product $P_{sl} = 0.75P_c$, from relation (48) we obtain

$$(\delta_k)^3 = 2.5(\delta_g)^3.$$

$\delta_k = 1.36\ \delta_g$.

Thus, in order to eliminate the concentration of contact stresses in the knife-grate joint, and to exclude the grinding period of the knife, as well as to exclude the penetration and crushing of meat fibers in the knife-grate joint gap, the knife blade thickness should be 1.36 times the thickness of the grate. In this case, there is no excessive squeezing out of the liquid meat fraction from the fibers, and an increase in the quality of the minced meat is ensured. As shown in [18], in this case, the process of knife wear and an increase in the temperature of raw meat, as a quality factor, under steady-state conditions is two times slower than with the traditional scheme of fixing an annular grinding grate. In this case, the period of knife re-sharpening can be increased from 90 hours to 180 hours [17], and the service life of the knives to the limiting state is significantly increased.

Conclusion

- The work approximates the law of change in the food material pressure along the radius of the outlet grinding grate by a parabolic function.

- The bending process of the outlet grinding grate of the top as a circular perforated plate under the raw meat pressure parabolically decreasing along the radius for a perspective fixing scheme and the corresponding boundary conditions have been mathematically modelled.
- The deflection of the knife blade and grate has been analytically determined under the variable raw meat load parabolically decreasing along the radius.
- The condition for excluding the concentration of internal forces during the contact interaction of the knife blade and grate, which ensures a decrease in the wear rate of the contacting elements and an increase in temperature in the cutting zone.
- An analytical dependence of the thickness of the knife blade on the thickness of the outlet grinder grate has been established depending on their physical and mechanical characteristics and the geometric parameters of the grinder elements, as well as on the physicomechanical and rheological characteristics of raw meat.
- The established analytical dependence of the thickness of the knife blade on the thickness of the output grinding grate ensures an increase in the quality of minced meat by preventing the gap in the knife-grate joint, penetration of meat fibers into the gap, eliminating excessive crushing of raw meat and squeezing out the liquid fraction.
- It is shown that due to the parabolic nature of the changing load from the raw meat pressure, the regrinding period for a knife-grate pair increases from 90 hours to 180 hours, and the operating time to the limiting state of the knife doubles.

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