



Optimal Energy Regulation for Robotic Hexoskeleton Manipulation

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Abstract

In this paper we present a basic model of understanding the optimal energy regulation for the power supply of a servomotor, which could be used in an upper limb prosthesis. The results obtained indicate that the use of this tool improves performance as long as the parameters are adjusted for each particular case.

KEYWORDS: REGULATOR; CONTROLLABILITY; STATE SPACE

INTRODUCTION

According to figures from the WHO (World Health Organization) in its Fact Sheet No. 352 of September 2013 on Disability and Health [1], they assure that more than 15% of the world's population suffers from some form of disability and that disability rates are increasing due to the aging of the population and the increase in chronic diseases. among other causes, since there is 6% of the population worldwide with disability problems, and 25% of people affected by their care. Today's civilization is aging. Many diseases that until a century ago were fatal have ceased to be so due to advances in medical therapies. A consequence of this is that more patients need some type of motor assistance due to sequelae of some pathology or due to the same muscle weakness suffered by long-lived individuals [2].

For its part, Engineering has evolved in such a way that its participation in the biological sciences has generated relatively recent disciplines. Within the biomedical area, it can be considered that the area of rehabilitation engineering is the one that produces

the most impact. The contribution of Biomedical Engineering to this problem is the design of useful devices to automate these therapies and provide the patient with the necessary autonomy for their best development. Today's robotic technology provides suitable solutions to assist in many of the rehabilitation activities [3].

In this paper we present a model of energy optimization of a servomotor intended to generate the articulation movements of a robotic arm designed for the motor rehabilitation of a patient.

Kinematics specifically of a mechanical arm is interested in the analytical description of the robot's spatial displacement as a function of time, in particular of the relationships between the position of the variables of articulation, position and orientation of the robot arm [4].

Direct kinematics tries to determine the position coordinates from the knowledge of the articulation variables, that is, from the angles they form. The Cartesian coordinates depend on the articular coordinates.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \vec{f}(\theta_1, \theta_2, \theta_3) \quad \text{----- (1)}$$

inverse kinematics seeks to determine the angles that each joint must form in order to bring the arm to a known desired position (or coordinates).

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \vec{f}^{-1}(x, y, z) \quad \text{----- (2)}$$

It is possible to try to choose a method that allows us to control the manipulator. This control tends to allow the end effector to be located in a certain position, and with a given orientation, in order to perform some task.

To reach any point P in the accessible space of the manipulator, it means placing the kinematic torques, which in this type of robot are all rotational, in a given angular position.

If you want to move to another point P1 in space, what you have to do is calculate what will be the angle that all the joints should rotate. Since the solution to the problem is not unique, some kind of strategy must be used to make the moves, both in their sequence and in their value. It is unlikely that these movements will be made all together, but it is appropriate that they take place one at a time. The acceleration and speed of movements must also be controlled, so as not to exceed certain maximum values.

In systems that require a lot of precision, direct current motors are used, and those that are also of low power.

The objective of this work is to find the desired energy optimization so that the motor drives the movements of the manipulator arm as efficiently as possible.

TERIALS AND METHODS

Servo motor model

Consider the DC motor in Figure 1 that drives a load through a rigid shaft. If the field current is kept constant at an I_f value or the field flow comes from a permanent magnet, this machine can be controlled only by the voltage $v_a(t)$ applied to the armature [5]. In this case, the electric torque equation can be written as

$$T_e(t) = K_t i_a(t) \quad \text{----- (3)}$$

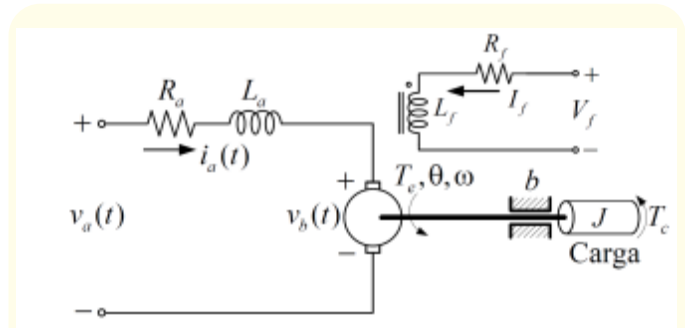


Figure 1: Servo Motor Model.

Where, $K_t = KI_f$ it is a constant. When the motor drives the load, a counter-electromotive force develops in the armature circuit opposite to the applied voltage $v_a(t)$. This tension is linearly proportional to the angular velocity developed in the axis, that is:

$$v_b(t) = K_b \frac{d\theta(t)}{dt} \quad \text{----- (4)}$$

Applying Kirchhoff's law of stresses to the mesh of the armature circuit yields the dynamics of the armature circuit,

$$v_a(t) = u(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + K_b \frac{d\theta(t)}{dt} \quad \text{----- (5)}$$

Let be J the moment of total inertia of the load, the shaft and the rotor of the motor; θ the angular displacement of the load; b the viscous friction coefficient and T_c the torque produced by the load.

The torque that the engine must develop necessary to overcome inertia, friction, and the reaction torque of the load is given by:

$$T_e(t) = J \frac{d^2 \theta(t)}{dt^2} + b \frac{d\theta}{dt} + T_c \quad \text{----- (6)}$$

$$K_t i_a(t) = J \frac{d^2 \theta(t)}{dt^2} + b \frac{d\theta}{dt} + T_c \quad \text{----- (7)}$$

For this first model T_c , the disturbance caused by the load on the motor shaft is not considered.

We chose as state variables of this system the following

$$\begin{aligned} x_1 &= \theta(t) \\ x_2 &= i_a(t) \end{aligned} \quad \text{----- (8)}$$

The representation in state variables is:

$$\begin{bmatrix} \dot{\omega}(t) \\ \dot{i}_a(t) \end{bmatrix} = \begin{bmatrix} -\frac{b}{J} & \frac{K_t}{J} \\ -\frac{K_e}{L_a} & -\frac{R_a}{L_a} \end{bmatrix} \begin{bmatrix} \omega(t) \\ i_a(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_a} \end{bmatrix} u(t) \quad \text{----- (9)}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \omega(t) \\ i_a(t) \end{bmatrix}$$

Be a control system

$$\dot{x} = Ax + Bu \quad \text{----- (10)}$$

$$y = Cx + Du$$

The control signal is selected as

$$u = -Kx = [K_1 \ K_2 \ \dots \ K_n] [x_1, x_2, \dots, x_n]^T \quad \text{----- (11)}$$

This closed-loop system has no inputs. Their goal is to keep the exit to zero. Because disturbances can occur, the output will deviate from zero. This output will return to the zero reference input due to the feedback scheme. A system of this nature is known as a regulatory system.

When replacing

$$\dot{x} = (A - BK)x \quad \text{----- (12)}$$

The eigenvalues of the matrix $A - BK$ are called the poles of the regulator.

Cost function

The dynamics of the output signal depend on the eigenvalues of the matrix $(A - BK)$. If all eigenvalues (closed-loop poles) of this matrix have a negative real part or are in the left half-plane of the plane s , then for any initial state $x(0)$ other than zero, the output of the system will tend to the desired value when $t \rightarrow \infty$. Therefore, if the pair $[A, B]$ is controllable, we can find a profit matrix K in such a way as to place the eigenvalues of $(A - BK)$ in the desired positions. There is a constraint on $u(t)$, which can limit the speed with which the output tends to the desired value.

One of the systematic and widely used methods to determine K values is through the use of optimal control techniques, through the minimization of quadratic performance indices by means of the following scalar function.

$$J = \int_0^\infty [x^T Q x + u^T R u] dt \quad \text{----- (13)}$$

This is a quadratic function commonly referred to as the cost function, and it is directly related to the energy involved in the control process. In the quadratic function the matrix Q is a positive definite (or semi-defined) real hermitian or symmetric matrix, the matrix R is a positive definite hermitian or real symmetric matrix. x is the state vector and u is the input vector.

If the equation of state $\dot{x} = Ax + Bu$ is controllable then the matrix of state feedback gains is given by the following equation

$$K = R^{-1} B^T P \quad \text{----- (14)}$$

Where P is a symmetric, positive defined matrix that satisfies the following equation:

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \quad \text{----- (15)}$$

This equation is called Riccati's algebraic equation, and it must be solved numerically. This equation can have one or more solutions but only one solution is symmetric and positively defined, resulting in an optimal profit matrix K . An advantage of using the quadratic optimal control scheme is that the designed system is asymptotically stable, except in the case where the system is not controllable.

$$u(t) = -Kx(t)$$

Where K is a matrix of dimension $r \times n$, where r is the dimension of the vector $u(t)$.

The matrices Q and R are generally chosen as being diagonal matrices as follows:

$$Q = \begin{bmatrix} q_{11} & 0 & \dots & 0 \\ 0 & q_{22} & \dots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \dots & q_{nn} \end{bmatrix}, \quad R = \begin{bmatrix} r_{11} & 0 & \dots & 0 \\ 0 & r_{22} & \dots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \dots & r_{rr} \end{bmatrix} \quad \text{----- (16)}$$

Where, the q_i elements of the Q matrix are associated with the energy of the states and the r_i elements of the R matrix are associated with the energy of the control action.

RESULTS

The parameters of the powerplant are as follows:

R_a = Resistencia de armadura = 1Ω
 L_a = Inductancia de la armadura = 5 mH
 K_t = Constante del par motor = 1 N.m/A
 K_e = Constante de la f.c.e.m = 3 V.s/rad
 b = Coeficiente de rozamiento de los cojinetes = $0,1\text{ N.m.s/rad}$
 J = Momento de inercia del motor y la carga = $0,2\text{ N.m./rad/s}^2$

Variing[Q]:

$$1) \mathbf{Q} = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}; R = 1$$

$$\mathbf{K} = \begin{bmatrix} 1.2672 & 0.6228 \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} 0.0791 & 0.0063 \\ 0.0063 & 0.0031 \end{bmatrix}$$

$$2) \mathbf{Q} = \begin{bmatrix} 100 & 0 \\ 0 & 1 \end{bmatrix}; R = 1$$

$$\mathbf{K} = \begin{bmatrix} 7.2330 & 1.3699 \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} 0.4286 & 0.0362 \\ 0.0362 & 0.0068 \end{bmatrix}$$

Ranging [R]:

$$1) \mathbf{Q} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; R = 0.1$$

$$\mathbf{K} = \begin{bmatrix} 1.0997 & 2.3985 \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} 0.0219 & 0.0005 \\ 0.0005 & 0.0012 \end{bmatrix}$$

$$2) \mathbf{Q} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; R = 0.25$$

$$\mathbf{K} = \begin{bmatrix} 0.4658 & 1.2876 \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} 0.0247 & 0.0006 \\ 0.0006 & 0.0016 \end{bmatrix}$$

Figures 2 and 3 show the adjustments made to the original system (blue), first modifying Q (states) and then R (control), since it does not reach the appropriate values for the natural response of the system to the unit step.

For the system model expressed in the state space, the LQR technique calculates the control matrix $[\mathbf{K}(t)]$, such that performing a control with state feedback expressed by the equation $[\mathbf{u}(t)] = [-\mathbf{K}(t)][\mathbf{x}]$ minimizes the cost function J in the time interval considered (optimization interval). If the system is controllable, there is a single optimal control matrix $[\mathbf{K}]$ such that the closed-loop system with control $[\mathbf{u}(t)] = [-\mathbf{K}(t)][\mathbf{x}]$ is asymptotically stable.

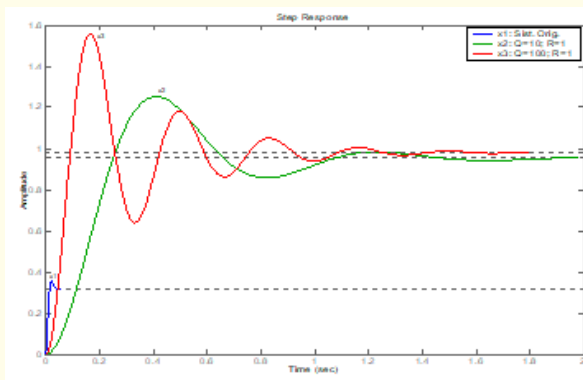


Figure 2: Response to the step. Q varies. Constant R.

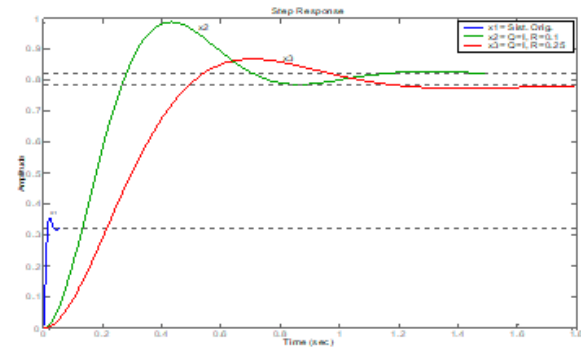


Figure 3: Response to the step. Q constant. R varies.

For a system to be controllable, it is required that the matrix of dimension $n \times n$ $\mathbf{C} = \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \dots & \mathbf{A}^{n-1}\mathbf{B} \end{bmatrix}$ has range = n , or that it contains n linearly independent vectors, or that the $\det(\mathbf{C}) \neq 0$ [6].

For our case study:

$$\mathbf{A} = \begin{bmatrix} 5 & 50 \\ -600 & -200 \end{bmatrix}; \mathbf{B} = \begin{bmatrix} 0 \\ 200 \end{bmatrix}; \mathbf{C} = [\mathbf{B} \quad \mathbf{AB}] = \begin{bmatrix} 0 & 10000 \\ 200 & -40000 \end{bmatrix}$$

$$\text{rank}(\mathbf{C}) = 2; \det(\mathbf{C}) \neq 0$$

The system is controllable.

CONCLUSIONS

There are no simple analytical methods that allow the designer to define the values that make up these matrices depending on the system, the control to be carried out and the efforts of the control variables. Arrays $[\mathbf{Q}]$ and $[\mathbf{R}]$ are then proposed, the controller is calculated, and the behavior of the closed-loop system is checked.

Higher values of the components of the matrix $[\mathbf{Q}]$ imply a more demanding control over the variables to be regulated.

Higher values of the components of the matrix $[\mathbf{R}]$ imply an increase in the influence of control actions on the plant.

Having made the corresponding adjustments, both in the variables and in the plant, a better gain of the system can be achieved through a stable control signal, optimizing the system for a lower energy cost function.

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