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Research Article

A Regulation Model of the Drive Servo System of Prosthetic Joint: Pole–Placement Method

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Abstract

Engineering has evolved, its involvement in the sciences that affect people's lives has given rise to relatively new disciplines. Rehabilitative engineering is the most impactful biomedical field. Assistive devices and technologies such as wheelchairs, prosthetics, mobility aids, hearing aids, visual aids, specialty software, motor assistance equipment, Hearing and communication skills are used most frequently. Thanks to these developments, people with impaired functioning are better able to live independently and participate in society. This article seeks to find the locations of the desired servomotor poles that drive the myoelectrical prostheses of the upper limbs, from the point of view of automatic control. The modeling and programming of these systems is critical before their structural and functional design. Control Engineering allows us to work in the space of states, where it is possible to find new information that can be used to enrich the findings of the investigation. Work points in the complex plan, relevant gains and surface graphs will be new parametric indicators to be considered. The results achieved show that the use of the pole location system as a control tool must be taken in mind for the analysis of the functioning of the drive system of the prosthesis, $P_{desired}$= [-8 ; -4 + 4i ; -4 - 4i], integrate more controllability of the system, as long as it adapts to the needs of each patient.

Keywords: Controllability; Work Points; State Variables

Introduction

The contribution of biomedical engineering to the problem of disability is the design of useful devices to automate these therapies and give patients the autonomy necessary for better performance [1]. An estimated 1.3 billion people, or 1 in 6 people worldwide, suffer from a major disability. Some people with disabilities die up to 20 years earlier than people without disabilities. People with disabilities are twice as likely to develop conditions such as depression, asthma, diabetes, stroke, obesity or oral health problems [2]. Access to rehabilitation and habilitation can lessen the consequences of illness or injury, improve health and quality of life, and decrease the use of health services. While global data on the need for recovery and adaptation, the type and quality of planned measures, and estimates of unmet needs do not exist, data at the national level reveal large gaps in the provision of and access to such services. As engineers, we apply the laws of mathematics and physics to solve problems, such as those mentioned above. Through Modern Control Theory, any problematic situation can be treated by means of equations of state. Some mathematical rules, such as the principle of superposition, present in recursive algorithms are excluded in this new approach. The consequent simplification of developments will be reflected in new findings, giving value to new concepts such as the Controllability of dynamical systems [3].

According to statistical studies by the WHO (World Health Organization) in its fact sheet No. 352 in September 2013 on Disability and Health, more than 15% of the world's population suffers from some type of disability and the rates of these are growing due to the aging of the population and the increase in chronic diseases. Many diseases that were fatal a century ago are no longer fatal due to advances in medicinal therapies. A consequence of this is that more patients need some kind of help because of some disease or effect of some weak muscle that older subjects suffer from [4].

Recently, the principles of Controllability were used to characterize and validate a novel robotic system and its corresponding stochastic self-assembly programmed by fluids, carrying out a statistical analysis to show that the system is governed by the dynamics of reaction diffusion and to validate the applicability of the kinematic model to standard physics [5]. Many recently published papers investigate state-space models of the mobility of a robotic arm using EMG signals [6]. Other works consulted emphasize the analysis of the controllability of the sliding mode [7]. It is essential to arrive at a useful application model, once the prosthesis for people with disabilities has been completed, to evaluate previous cases that have to do with an efficient model of energy management and to present an adequate regulator of a servo motor designed to generate movements of the joint of a robotic arm in the rehabilitation of a patient.

The aim of this work is to find the desired working points of the motor to drive the movements of the manipulator arm as efficiently as possible.

Materials and Methods

Servo motor model

The direct current motor in Figure 1 is considered to drive a load through a rigid shaft. If the field current is kept constant at a value If or the field flux comes from a permanent magnet [8], this machine can be controlled only by the voltage va(t) applied to the armature [9].

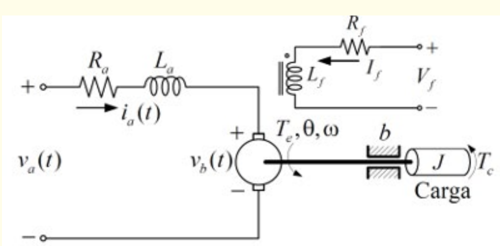

Figure 1: Electrical schematic of the servomotor.

As described in a previous paper [10], the electric torque equation can be written as:

$$T_e(t) = K_t i_a(t) \quad (1)$$

Where, $K_t = KI_f$ it's a constant. When the motor drives the load, a counter-electromotive force develops in the armature circuit opposite to the applied voltage va(t). This voltage is linearly proportional to the angular velocity developed on the axis, i.e.:

$$v_b(t) = K_b \frac{d\,\theta(t)}{dt} \quad (2)$$

By applying Kirchhoff's law of stresses to the mesh of the armature circuit, we have:

$$v_a(t) = u(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + v_b(t) \text{------------}(3)$$

Let be J the total moment of inertia of the load, shaft, and rotor of the motor; angular displacement of the load; b the viscous friction coefficient and the torque produced by the load.

The torque that the motor must develop necessary to overcome inertia, friction, and the reaction torque of the load is given by:

$$T_e(t) = J\frac{d^2\,\theta(t)}{dt^2} + b\frac{d\,\theta}{dt} + T_c \text{-------------}(4)$$

$$K_t i_a(t) = J\frac{d^2\,\theta(t)}{dt^2} + b\frac{d\,\theta}{dt} + T_c \text{----------}(5)$$

For this first model, it is not considered T_e, which would represent the disturbance caused by the load on the motor shaft.

We choose as state variables of this system a,

$$x_1 = \theta(t)$$
$$x_2 = \dot{\theta}(t) = \omega(t) \quad (6)$$
$$x_3 = i_a(t)$$

The representation in state variables is a:

$$\begin{bmatrix}\dot{\theta}\\ \dot{\omega}\\ \dot{i}_a\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & \frac{-b}{J} & \frac{K_t}{J}\\ 0 & \frac{-K_b}{L_a} & \frac{-R_a}{L_a}\end{bmatrix}\begin{bmatrix}\theta\\ \omega\\ i_a\end{bmatrix}+\begin{bmatrix}0\\ 0\\ \frac{1}{L_a}\end{bmatrix}u(t) \quad \text{----------------(7)}$$

$$y=\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}\theta\\ \omega\\ i_a\end{bmatrix}$$

Location of poles

The location of the roots of the desired characteristic polynomial will depend on the performance criteria of the closed-loop system, including peak time, establishment time, overshoot, bandwidth, etc. This region is bounded as follows:

- On the right by a vertical line separated by a value equal to the imaginary axis. As the distance from this line to the jω axis increases, the response of the system is faster, since the damping coefficient increases proportionally;
- By two straight lines starting from the origin at an angle θ. As this angle grows, the overshoot of the transient response increases proportionately;
- If we place all the eigenvalues in the same point or small region, as shown by point "a", the response will be slow and the control action can be high. This leads the system to saturate or act on the limiters of the control action, so that the system behaves in a non-linear manner.
- It is therefore appropriate to place the eigenvalues within the region **C**. The larger the radius "r" the faster the response, but the higher the actuation signal u(t) can be high. In addition, the bandwidth of the closed-loop system can be large and with this the system ends up amplifying external disturbances and noises, which are usually found at high frequencies.

In summary, determining the position of the desired eigenvalues is a trade-off between the speed of the transient response (or the speed at which the error goes to zero) and the sensitivity to disturbances and measurement noises. This tells us that before the practical implementation of the system, the dynamic range of each internal variable of the system must be verified by simulation in a closed loop, to avoid problems of saturation of the plant and/or actuators.

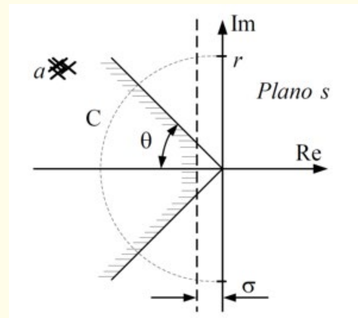

Figure 2: Desired region where to locate the poles in closed loop.

The design technique begins with the determination of the desired closed-loop poles from the transient response and/or frequency response specifications, as well as the steady-state requirements. By selecting an appropriate gain matrix **K** for state feedback, it is possible to have the system have the poles in a closed loop at the desired positions, as long as the original system is fully controllable.

Let be a control system [11],

$$\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{B}u \qquad (8)$$
$$y=\mathbf{C}\mathbf{x}+Du$$

The control signal is selected as

$$u=-\mathbf{K}\mathbf{x}=\begin{bmatrix}K_1 & K_2 & \dots & K_n\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ \vdots\\ x_n\end{bmatrix} \quad \text{----------(9)}$$

Al sustituir

$$\dot{\mathbf{x}}=(\mathbf{A}-\mathbf{B}\mathbf{K})\mathbf{x} \quad \text{------(10)}$$

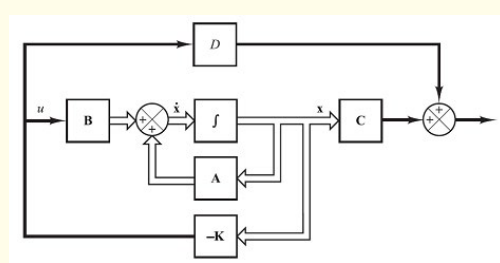

Figure 3: Functional diagram of the servo-system.

If the system defined by equation (10) is completely controllable, specifying the desired eigenvalues for the matrix $\mathbf{K}$, the profit matrix can be determined by the pole location technique.

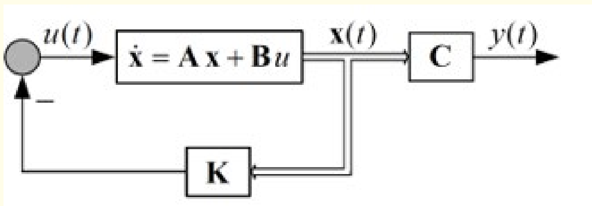

Figure 4: Control diagram of the servo-system.

Akermann's formula

Ackermann's formula is based on the similarity transformation that transforms a given state model into its controllable canonical form $(\mathbf{AB}) \rightarrow (\mathbf{A}_c\mathbf{B}_c)$, through a new state vector $\mathbf{x} = \mathbf{Tz}$, Second, you get the profits K_i, resulting in the law of control $u = -\mathbf{K}_c\mathbf{z}$. To get the gains for the original equation of state, thirdly the gain matrix is transformed back through the matrix $\mathbf{T}$, that is $\mathbf{K} = \mathbf{K}_c\mathbf{T}^{-1}$.

These three steps are grouped together in Ackermann's formula, given by:

$$\mathbf{K} = [0\,0\,0 \ldots 1][\mathbf{B}\;\mathbf{AB}\;\mathbf{A}^2\mathbf{B}\;\ldots\;\mathbf{A}^{n-1}\mathbf{B}]^{n-1}\alpha_c(\mathbf{A}) \text{ -------(11)}$$

Where $\alpha_c(\mathbf{A})$ is a polynomial of matrices formed with the coefficients of the desired characteristic equation $\alpha_c(\mathbf{A}) = \mathbf{A}^n + \alpha_1\mathbf{A}^{n-1} + \alpha_2\mathbf{A}^{n-2} + \ldots + \alpha_n\mathbf{I}$.

Results

The data for the simulation of the servo motor, whose model we developed in the previous section, were obtained from a real motor (Maxon®'s RE 40-40 mm model) [12] frequently used to drive myoelectric anthropomorphic prostheses [13].

The following values were taken from the manufacturer's catalogue:

R_a = Armor Strength = 1,16Ω.

L_a = Armature inductance = 0,329 mH.

K_t = Torque Constant = 60,3 mNm/A.

K_b = Constant Velocity = 158 rpm/V.

b = Coef. Bearing friction = 3,04 rpm/mNm.

J = Motor moment of inertia and load = 138 gcm^2.

The Transfer Function corresponding to the system proposed in (7) is,

$$G(s) = \frac{K_t}{s^3 JL_a + s^2(JR_a + bL_a) + s(bR_a + K_tK_b)}$$

$$G(s) = \frac{1,328}{s^3 + 3,548\,s^2 + 209,9\,s}$$

From the characteristic polynomial (denominator) we find the original eigenvalues (poles) (Table 1).

Designation	Eigenvalues	Gain K
Original Poles	[0; -1,7739+14,3797i; -1,7739-14,3797i]	[0 0 0]
Desired Poles 1	[-8; -4+4i; -4–4i]	[192,75 -85,98 4,1]
Desired Poles 2	[-1; -1+14i; -1-14i]	[148,33 -8,2 -0,18]
Desired Poles 3	[-3; -3+14i; -3-14i]	[463,05 9,76 1,79]
Desired Poles 4	[-2; -2+10i; -2-10i]	[156,61 -73,8 0,8]
Desired Poles 5	[-4; -2+20i; -2-20i]	[1,22 0,16 0,0015]

Table 1: Poles - gains.

In the same way, it is possible to find the other Transfer Functions from each of the other desired poles.

For a system to be controllable, the nxnr dimension matrix is required to

$$\mathbf{C} = [\mathbf{B}\;\mathbf{AB} \ldots \mathbf{A}^{n-1}\mathbf{B}]_{xnr} \text{ -------(12)}$$

Has range = n, i.e., containing n linearly independent vectors, or that the det $(\mathbf{C}) \neq 0$ [14].

Teniendo en cuenta lo establecido por la región deseada para ubicar los polos en lazo cerrado (Figure 2), trasladamos los puntos de trabajo normales a otras posiciones moviéndonos a lo largo y a lo ancho del plano s de Laplace.

The Controllability matrix of the original system is,

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 1,3281 \\ 0 & 1,3281 & -4,7120 \\ 3,0395 & -10,7168 & -600,0421 \end{bmatrix}$$

El determinante, $\det(\mathbf{C}) = -5,3615 \neq 0$.

The other determinants also resulted in $\neq 0$ values, since all systems must be controllable in order to use the Pole Location Method.

Discussion

In the present work, the Matlab® "acker" statement [15] of the Ackermann formula was used to find the components of the profit matrix **K**.

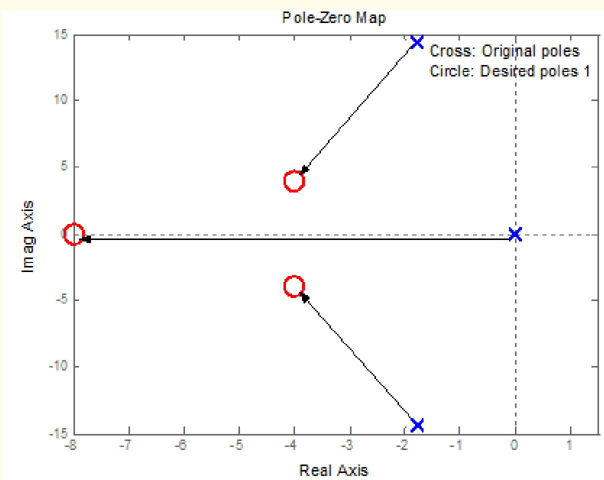

Figure 5: System poles.

Figure 5 depicts the location of the Desired Poles 1 (Table 1) and the influence of the K matrix to move the original working points of the system to the one set by the corresponding gain.

The dynamics of the output signal depend on the eigenvalues of the array (A–BK). If all eigenvalues (closed-loop poles) in this matrix have a negative real part or are in the left half-plane of the s-plane, then for any non-zero initial state x(0), the output of the system will tend to the desired value when $t \rightarrow \infty$.

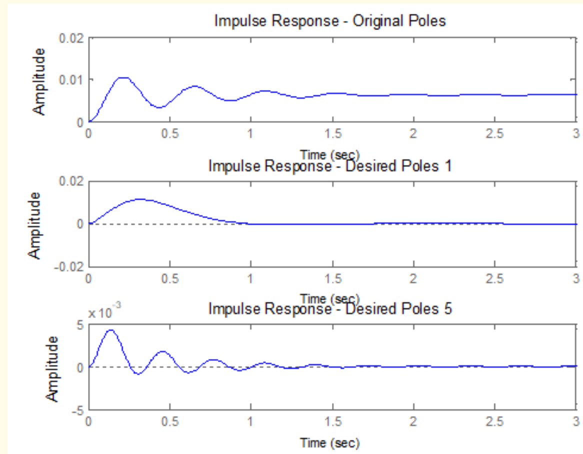

Figure 6: Response from the different points of work.

Figure 6 shows the different responses of the system according to the different positions of the working points (eigenvalues 1 and 4 of Table 1), indicating for each particular case how quickly the system loses its oscillatory effect. This same analysis can be performed with any of the chosen locations and verify their response.

The engineer must have a notion of the dynamics of the system to be evaluated in order to record the location of the different desired poles.

From Figure 7, the control action acquires a significant value to bring the states to the desired final values, stabilizing once the motor has acquired the desired position.

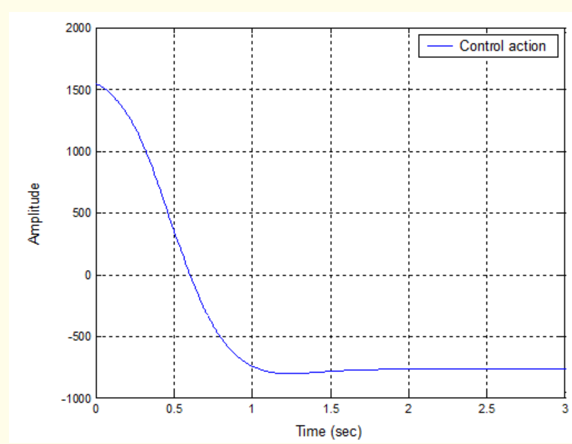

Figure 7: Control action.

Conclusion

It is important to note that the K matrix is not unique to a given system, but depends on the desired positions of the closed-loop poles (which determine the speed and damping of the response). It should be noted that the selection of the desired closed-loop poles, or the desired characteristic equation, is a compromise between the speed of response and the sensitivity to disturbances and noise in the measurement. That is, if the speed of response is increased, the adverse effects of disturbances and noise on the measurement are usually increased. Therefore, when determining the K-state feedback gain matrix for a given system, it is convenient to examine the dynamics of the output signal depend on the eigenvalues of the array and response of the system for several different gain matrices (based on some different desired characteristic equations) and to choose the one that gives the best overall system performance.

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