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Review Article

Stability Control Applied to the Cardiovascular System. Analysis of its Internal Dynamics

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Abstract

We present an alternative analysis of the stability of the cardiac system using Control Engineering tools. We will seek to determine the Lyapunov Function, which is a parameter that indicates the degree of stability of a dynamic system. The model presented in this work is designed in the state space. Elements of Applied Mathematics such as equations of state, transfer function and the dynamic response related to physiological systems will be used to find indicators and factors that from Biomedical Engineering will contribute to the understanding of cardiovascular functionality. The controlling action that is sought will determine in the system a convergence to an equilibrium point before the start of each heartbeat. Usually the variables that intervene in the pulsations of the heart are not keeping in mind, but if they are not regulated, the work required of the heart is excessive. The energy invested in activating the cardiac wall in both hypertensive patients and those with normal blood pressure is related to the control action required to avoid instabilities. The results achieved by the use of this new application have been to find a shorter path from one state to another, ensuring greater stability and lower energy expenditure.

Keywords: Stability; Servomotor; Modeling

Introduction

Mathematical modeling is currently applied in physiology and medicine to support the activities of the scientist and clinical worker. A model is, by definition, an approximation of a system in terms of its representation [1]. The heart, the main organ in the circulatory system, is made up of specialized muscle fibers that supply the driving force to propel blood through the body, like a pump. The elementary functions of the pump are to carry blood throughout the body and that this flow arrives continuously. The vascular system is a widely studied physiological system. Its hemodynamic characteristics, such as total peripheral resistance, total arterial compliance and characteristic impedance of the proximal aorta, allow us to understand this system [2]. The mechanical pumping function of the heart is always preceded and regulated by an electrical activity originated in an exchange of electrolytes, mainly Sodium (Na+), Potassium (K+) and Calcium (Ca++), across the cell membrane in both directions. The contraction of each cell is associated with an action potential, PA. A BP is a very rapid change in cell membrane polarity from a negative normal resting value to a positive value and finally an almost equally rapid change towards negative values again. Heart muscle is mostly composed of cells called cardiac myocytes, whose main function is to produce mechanical contraction of muscle. Although mathematical modeling and parameter estimation can help understand system performance, strong interactions between vascular system features make deeper understanding difficult.

A well-known model for operating with vascular features is the Windkessel model.

The Windkessel model developed by Otto Frank, a German physiologist, defines the heart and systemic arterial system as a closed hydraulic circuit comprising a water pump connected to a chamber. The water pumped into the chamber compresses the air inside and pushes the water out of the chamber, causing the water to return to the pump. This allows us to simulate the elasticity and extensibility of the main artery, as the ventricle of the heart pumps blood to it. This effect is known as arterial compliance, represented by capacitor C of the analogous Windkessel electric model. The term compliance is the parameter that specifies the elastic nature of blood vessels. It is defined as the incremental change in volume that would result from an incremental change in pressure.

The resistance that water encounters as it leaves the Windkessel model and flows back to the pump simulates the resistance to flow that blood encounters as it flows through the arterial tree from the major arteries, minor arteries, arterioles, and capillaries, due to the decrease in vessel diameter. This resistance to flow Rp, is known as peripheral resistance [3,4].

We consider the four-element Windkessel model [5]. This model is shown schematically in figure 1 and consists of a parallel connection of resistor and capacitor.

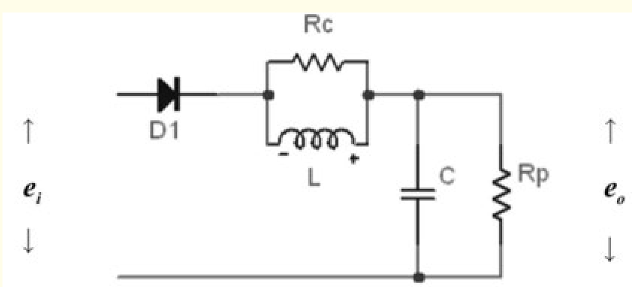

Figure 1: Windkessel model of 4 elements.

The resistor Rp represents the total peripheral resistance and the capacitor C represents the compliance of the vessels. Another resistive element between the pump and the air chamber simulates the resistance to blood flow due to the aortic or pulmonary valve, indicated by the acronym Rc in Figure 1. L is an inertial element in parallel with the characteristic impedance. With this arrangement, the model can account for the inertia of the entire arterial system at low frequencies and, at medium and high frequencies, allows the characteristic impedance to come into play [6]. Other authors have modeled the cardiovascular system based on the Windkessel model of two, three or four elements, to estimate the parameters [7], simulate the waveform of the pressure signal [8], or to study some specific characteristics [9].

The great arteries fulfill 2 different but interrelated functions: a) they constitute low-resistance blood distribution ducts to deliver an adequate blood supply to the peripheral organs, called conduction function and related to the static component of blood pressure (mean arterial pressure), and b) they dampen the pressure oscillations caused by the intermittent nature of ventricular ejection, called damping function related to the pulsatile component (pulse pressure) [10].

Our objective is to describe the dynamics of the cardiovascular system and perform an analysis of the associated stability by searching for input-output relationships in state space, assuming that the heart is a stable biological system with feedback [11,12], determining its influence on the controllability of dynamic systems.

In this article we use equations of state and control engineering criteria to simultaneously calculate multiple relationships that characterize part of cardiovascular dynamics.

Materials and Methods

Data were provided by Dr. Ricardo L. Armentano and his group at Favaloro University with clinical information from hypertensive and normal patients.

Tables 1 and 2 list the data corresponding to two categories of patients, with normal blood pressure and hypertension (hypertension). Columns 2 and 3 show Systolic Pressures (PS) and Diastolic Pressures (PD), respectively. Column 4, Mean Pressure (PM). Column 5 shows the Pulse Wave Velocity (PWV). Column 6 lists the Compliance (Cm) calculated for each record. The values in column 7 indicate L, a parameter that is calculated by subtracting the diastolic pressure from the systolic pressure. In column 8, Polo, gives the idea of system stability. We used in particular the registration of a population of eleven normotensive patients and that of fourteen hypertensive patients, of which records have been taken directly (systolic and diastolic pressures) and indirectly (pulse wave velocity and compliance).

Normotensive	SP	DP	avP	PWV	avC	L	Pole
Pacient	mmHg	mmHg	mmHg	m/s	e-4cm/ mmHg	mmHg	(Control)
1	93	57	69	10.39	4.03	36	-1.1550
2	127	80	96	12.25	4.15	47	-1.1626
3	104	66	79	9.05	5.03	38	-1.2049
4	120	89	99	10.78	3.43	31	-1.1063
5	91	75	80	10.94	3.14	16	-1.0731
6	97	70	79	11.11	3.57	27	-1.1197
7	118	66	83	9.28	5.36	52	-1.2165
8	85	51	62	7.80	6.40	34	-1.2443
9	96	61	73	8.83	4.08	35	-1.1563
10	117	77	90	9.80	3.62	40	-1.1241
11	119	64	82	10.69	4.00	55	-1.1530

Table 1: Normotensive patients.

Hypertensive	SP	DP	avP	PWV	avC	L	Pole
Pacient	mmHg	mmHg	mmHg	m/s	e-4 cm/ mmHg	mmHg	(Control)
1	146	96	113	15.21	2.00	50	0,6849 ± 0,7757i
2	106	84	91	14.58	2.21	22	-0.8139
3	116	65	82	10.07	3.49	51	-1.1122
4	157	89	112	11.15	3.76	68	-1.1357
5	166	98	121	14.25	2.68	68	-0.9948
6	164	92	116	16.26	2.22	72	-0.8216
7	127	82	97	11.44	3.76	45	-1.1357
8	155	92	113	17.16	2.48	63	-0.9417
9	155	70	98	10.83	3.70	85	-1.1309
10	139	100	113	11.28	4.04	39	-1.1557
11	134	84	101	10.27	4.82	50	-1.1965
12	114	75	88	11.31	3.12	39	-1.0704
13	117	78	91	14.07	2.31	39	-0.8753
14	125	83	97	14.78	1.68	42	0,6849 ± 0,3551i

Table 2: Hypertensive patients.

We use the Branwell and Hill equation [13] to find compliance,

$$Cm = \frac{1334.Dm}{2.r.VOP^2}$$

Where factors such as pulse wave velocity, blood viscosity and artery diameter are involved.

The pulse wave velocity was calculated graphically using the following formula,

$$VOP = \frac{D}{\Delta x.T}$$

Where parameters such as the distance separating both transducers, the distance on the x-axis between the two lower peaks of the carotid and femoral pressures and in addition, the sampling interval one beat one second participate. For the measurement, two tonometers should be placed with the patient in dorsal decubitus that record the passage of the pulse wave at the level of the common femoral artery and the homolateral primitive carotid artery. Once the distance between them is obtained, they separate the two wavefeet (carotid and femoral) and report the PWV by application of the formula specified above.

As the functioning of the cardiac system involves many variables such as pressure, blood density, compliance, resistance of the arterial wall, among others, we will model the system in the state space.

We use the concept of Transfer Function, which expresses the output-input ratio in Laplace space of the analogous electrical circuit of the Windkessel model.

Starting from the transfer function of the system considering normal pulsations of low frequency Rc = 0. For the equations Rp=R and Cm=C.

$$G(s) = \frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}} = \frac{E_o(s)}{E_i(s)} \text{ ----- (1)}$$

Where $RC = \tau$, which is the time constant of the fall in diastolic isovolumetric pressure = 0.73 sec. (relaxation phase before the heart restarts its work cycle).

$\frac{E_o(s)}{E_i(s)}$ is the ratio of systemic pressure to blood pressure (Laplace). $\frac{1}{LC}$ Relates inertia blood flow to compliance.

Every system can be described through equations of state, which contain all the information of the internal dynamics of the same, and allow to easily include the initial conditions. Equations of state are 1st order differential equations, simple to solve. In a system there may be internal dynamics (states) that instead of tending to zero or a limited value, increase their energy in the sense of making the system unstable. Considering the heart as a system with automatic control, we can analyze it from the point of view raised above.

The state of a system is the smallest set of variables (called state variables), so knowledge of these variables at t = t 0, together with knowledge of the input for t $_{t0}$, completely determine the behavior of the system for any $\geq$ t.

Operating in the fall,

$$s^2.E_0(s) + s.E_0(s).\frac{1}{RC} + E_0(s).\frac{1}{LC} = E_i(s).\frac{1}{LC} \text{ -------------(2)}$$

To define the state variables x $_1$ and x 2, we anti transform the systema from equation (2),

$$\ddot{e}_o + \frac{1}{RC}.\dot{e}_o + \frac{1}{LC}.e_o = \frac{1}{LC}.e_i \text{ -------- (3)}$$

$$x_1 = e_o \text{ ------------- (4)}$$

$$x_2 = \dot{e}_o \text{ ------------- (5)}$$

x_1 represents the systemic pressure, and x_2 the derivative of it.

The input variable u together with the output variable and define them by,

$$u = e_i \text{ ---------------- (6)}$$

$$y = e_o = x_1 \text{ ---------- (7)}$$

Matrixly, the equations of state of a control system can be written,

$$\mathbf{x} = \mathbf{A.x} + \mathbf{B.u} \text{ ® Ecuación de Estado ------ (8)}$$

$$\mathbf{y} = \mathbf{C.x} + \mathbf{D.u} \text{ ® Ecuación de Salida ------ (9)}$$

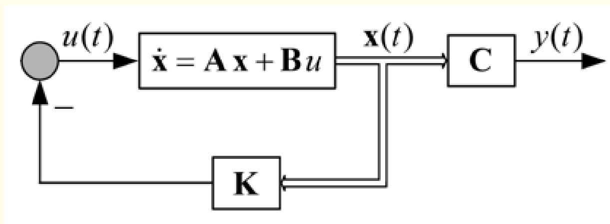

Figure 2: System block diagram.

The following equation of states and output of our system is then posed.

In matrix form,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-1}{LC} & \frac{-1}{RC} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{LC} \end{bmatrix} . u \quad \text{--------------(10)}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad ; D=0 \quad \text{-------------- (11)}$$

Method of location of poles

One of the widely used systematic methods for determining the values of the profit matrix K is by locating desired poles [14].

The control signal is selected as,

$$u = -\mathbf{Kx} = \begin{bmatrix} K_1 & K_2 & \dots & K_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad \text{------------- (12)}$$

When substituting in (8),

$$\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{BK})\mathbf{x} \quad \text{----------- (13)}$$

Classical procedures are based on the system transfer function, while the pole location method is based on the system state model.

It is also assumed that all system state variables can be measured and are available for feedback. In this way, it is possible to locate a set of poles of the system in closed loop, in desired locations, by feedback the states in order to meet certain specifications of transily dynamic response and permanent regime.

The pole location method is summarized in two steps:

- Specify the location of the desired roots of the characteristic equation of the closed-loop system;
- The calculation of earnings to be able to locate these roots in the places determined in the previous point.

There are two (2) ways to determine the components of the K matrix:

Direct substitution method:

The characteristic equation of the closed-loop system in figure 3 is given by:

$$\det[s\mathbf{I} - \mathbf{A} + \mathbf{BK}] = 0 \quad \text{----------- (14)}$$

When this determinant develops it results in a polynomial of order n in s containing the gains of the matrix K. Suppose now that the desired locations of the poles are given by the roots then the desired characteristic equation is given by: $-\lambda_1,\ -\lambda_2,\ \dots,-\lambda_n$

$$\alpha_c = (s+\lambda_1)(s+\lambda_2)\dots(s+\lambda_n) \quad \text{-------- (15)}$$

The project is completed by equalizing the coefficients of equal power in s of the equations of the determinant and the desired characteristic polynomial.

Ackermann's formula

Ackermann's formula is based on the similarity transformation that transforms a given state model into its controllable canonical form, $(\mathbf{AB}) \rightarrow (\mathbf{A}_c\mathbf{B}_c)$ through a new state vector, $\mathbf{x} = \mathbf{Tz}$ secondly the gains K_i are obtained, which results in the law of control. To obtain the gains for the original equation of state, thirdly the gain matrix is transformed back through the matrix, i.e.

$$u = -\mathbf{K}_c\mathbf{z} \ \ \mathbf{T} \ \ \mathbf{K} = \mathbf{K}_c\mathbf{T}^{-1}.$$

These three steps are grouped in Ackermann's formula, given by:

$$\mathbf{K} = [0\ 0\ 0\ \dots\ 1][\mathbf{B}\ \ \mathbf{AB}\ \ \mathbf{A}^2\mathbf{B}\ \dots\ \mathbf{A}^{n-1}\mathbf{B}]^{n-1}\alpha_c(\mathbf{A}) \quad \text{----------(16)}$$

Where is a polynomial of matrices formed with the coefficients of the desired characteristic equation $\alpha_c(\mathbf{A})$,

$$\alpha_c(\mathbf{A}) = \mathbf{A}^n + \alpha_1\mathbf{A}^{n-1} + \alpha_2\mathbf{A}^{n-2} + \dots + \alpha_n\mathbf{I} \quad \text{---------------(17)}$$

Stability criterion

The equation,

$$\mathbf{A^T P + PA = -Q} \text{ ------------ } (18)$$

It is known as the Lyapunov equation. The stability criterion is based on finding P from this equation, previously choosing Q. Q = I is generally used.

Let $x_e = 0$ be the steady state of the invariant linear system described by the equation.

$$\dot{\mathbf{x}}(t) = \mathbf{Ax}(t) \text{ ---------- } (19)$$

The state x_e is asymptotic and globally stable if and only if all eigenvalues of A have a negative real part.

The quadratic form called the Lyapunov function, it must meet Sylvester's criterion and be definite positive [15].

$$\mathbf{V(x) = x^T Px} \text{ ------------- } (20)$$

Results

Then with the data obtained from Tables 1 and 2, the eigenvalues of the systems are determined to find the natural working location in the Laplace space s and the desired gain necessary to move them to desired locations.

Patient 8 with lower systolic pressure from Table 1 and patient 5 with higher systolic pressure from Table 2 were taken.

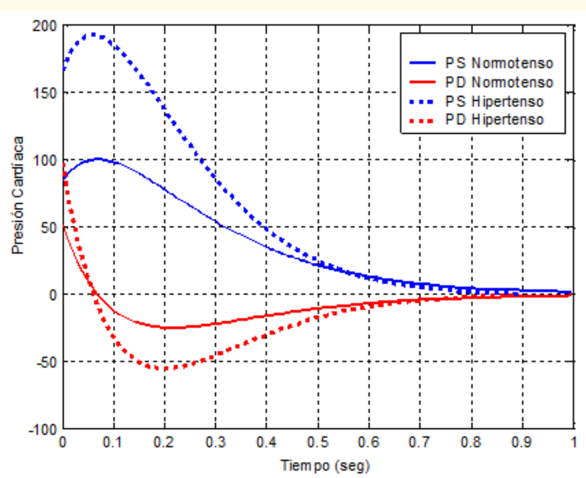

Figure 3: Impulse response – Patient $_{N8}$ – Patient$_{H5}$.

Normotensive patient (Table)

C= 6,40

L= 34

Natural self-values

Poles= [-0.5871 -0.7858]

Hypertensive patient (Table)

C= 2,68

L= 68

Natural self-values

Poles= [-0.0040 -1.3658]

The poles thus obtained from both systems indicate to me in the Laplace s plane the location of the natural working points.

In this work the Ackermann formula was used.

From l method of location of poles is obtained.

Normotension:

K_{N8}= [76,551 21,7302]

Desired poles = [-0.42 -0.85]

Hypertensive:

K_{H5}= [114,0886 63,1338]

Desired poles= [-0.4 3 -1.23]

Figures 4 and 5 show the desired work locations for both the normotensive and hypertensive patients.

This transfer is due to the action of the gain matrix K corresponding to each case.

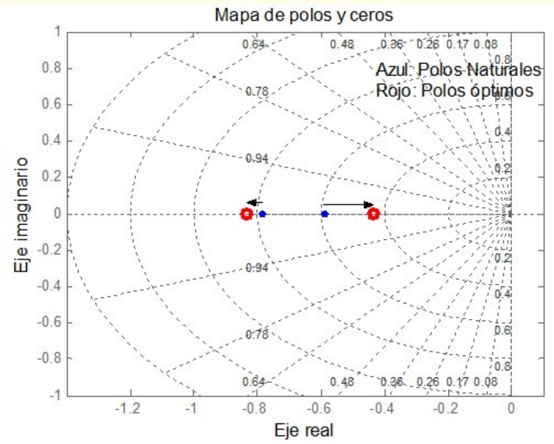

Figure 4: Optimal and natural poles – Normotensive patient.

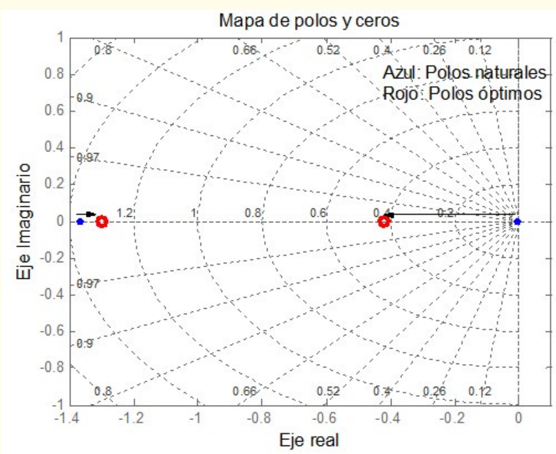

Figure 5: Optimal and natural poles – Hypertensive patient.

This same process can be done with each and every patient. Location in optimal operating positions in the Laplace space can be achieved by chemical compensation through drugs supplied to the patient (Medical treatment) or by electronic adjustment, if the patient has a pacemaker (minor surgery).

The Lyapunov equation of the original normotensive system is,

$V_N(x) = 228.8301*x_1^2 - x_1*x_2 + 0.3667*x_2^2$

And that of the optimized system,

$V_{NK}(x) = 3.4278*x_1^2 - x_1*x_2 + 0.4264*x_2^2$

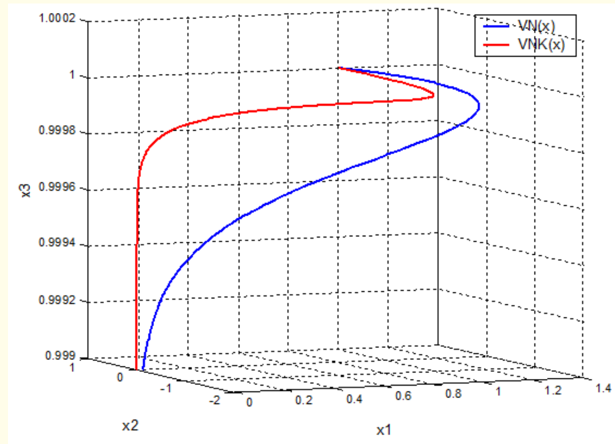

Figure 6: Lyapunov function – normotensive patient.

Figure 6 shows the calculated Lyapunov function for the normotensive patient.

The Lyapunov equations corresponding to the hypertensive patient are,

$V_H(x) = 191.7045*x_1^2 - x_1*x_2 + 0.3670*x_2^2$

And the optimized function,

$V_{HK}(x) = 2.3882*x_1^2 - x_1*x_2 + 0.4499*x_2^2$

Whose functions are in figure 7.

Convergence to an equilibrium point can be increased by selecting other poles and recalculating.

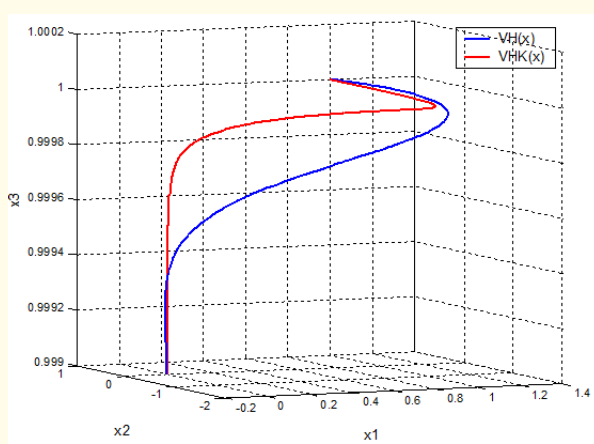

Figure 7: Lyapunov function - Hypertensive patient.

Conclusions

The use of the Pole Location Method with its respective control action and the Stability Circle considered (Lyapunov), produces in the system a convergence to a desired point ensuring stability with a shorter trajectory.

Unfortunately there are no simple analytical methods that allow the designer to define the desired self-values, as well as the control to be performed and the efforts of the control variables.

It is important to note that the K matrix is not unique to a given system, but depends on the desired positions of the closed-loop poles (which determine the speed and damping of the response). It should be noted that the selection of the desired closed-loop poles, or the desired characteristic equation, is a compromise between the speed of response and sensitivity to disturbances and noise in measurement.

As could be seen, working in the space of states allowed to give relevance to the function V(x) making it possible to graph the stability of a dynamical system.

Summary

We present an alternative analysis of the stability of the cardiac system using the tools of Control Engineering. We will seek to determine the Lyapunov function, which is a parameter that indicates the degree of stability of a dynamical system. The model presented in this paper is designed in state space. Elements of Applied Mathematics such as equations of state, transfer function and the dynamic response related to physiological systems will be used to find indicators and factors that from Biomedical Engineering will contribute to the understanding of cardiovascular functionality. The controlling action that is sought will determine in the system a convergence to an equilibrium point before the start of each heartbeat. Usually the variables involved in the heart beats are not taken into account, but if they are not regulated, the work required of the heart is excessive. The energy invested in the actuation of the heart wall of both hypertensive patients and those with normal blood pressure is related to the control action required to avoid instabilities. The results obtained through the use of this new application has been to find a shorter trajectory from one state to another, ensuring greater stability and lower energy expenditure.

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