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Research Article

Functional Response of Biomechanical Prostheses. Controllability Criteria of their Inner Variables

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Abstract

Any mechanical, electrical or physical system, can be described by state equations, which contain all the information of its internal dynamics. In this paper, we present a basic model for understanding the State Function associated with a DC motor intended to drive a myoelectric prosthesis, which could be analyzed in the event of a specific physical impediment of the upper limb. The aim is to find the natural and forced manner of the servo system, treating it from automatic control point of view, obtaining state by state, the response of the system based on the Controllability concept. Modern Control Theory and its tools such as the transfer function, the state space and the bio-mechanical dynamics among others, it will be sought to determine the situation of the variables that will lead to a better performance of the prosthesis. The use of analogous models of the servo system, allow us to know the evolution of the function that guide the response of the system for a given input and a set of given initial conditions. In addition, the graphs in the state space will be new parametric indicators to keep in mind. The results achieved indicate that the use of this analysis developed in this paper collaborates with the understanding of the functionality of the prosthesis driver.

Keywords: Modeling; State Function; Controllability

Introduction

The design of artificial limbs requires a thorough knowledge not only of the mechanics of mechanisms, but also a clear understanding of electromechanical devices, among which drive motors play a key role in the area of prosthetics. The maximum speed, strength and stability of the anatomical limb are still unparalleled with the artificial prosthesis. These limitations are due to the physical constraints of current technology to achieve the properties exhibited by the natural limb. Matching the speed and strength of the muscle with the technological actuator is not an easy task, especially when choosing a drive motor with the right speed-to-torque ratio [1]. The status function that governs the drive of the drive motors adds to the complexity of the proper design of the prosthesis.

Many theories in the physical sciences are known to be based or expressed in terms of optimization. In the field of motor control, optimization also plays a key role. The optimization processes give rise to a specific motor system, from which adaptation, development, evolution, recovery, etc. are investigated. These processes make the system work better and better. In the area of theoretical research, it is natural to look for limits of optimal motor control performance [2].

Several methods are used to control the speed of DC motors [3]. Neenu [4] reports that Proportional – Integral – Derivative (PID) controllers have been widely used for speed position control. The selection of PID parameters using genetic algorithms has led to a more efficient controller [5]. Other authors such as Boumediene

[6], used a particle swarm optimization (PEO) instead of AG. They presented an EPO-based PID controller. They found that the PID-OEP driver offers good performance and minimal uptime.

Sharaf [7] introduced a novel dual-loop PID controller for an industrial permanent magnet DC motor drive powered by solar photovoltaics (PV). However, despite the robustness and seemingly simple structure of the PID control strategy, optimizing the gains of the PID controller remains a difficult task [8].

The basis for being able to carry out all these optimization processes is to know the State Function that drives the functionality of the DC motor. This function obeys two criteria of activity, a natural response, which depends on the states and a forced response, dependent on excitation [9]. These must be known in advance to observe the influence of each of them on the total response of the system.

The natural response tells us what the system does as we allow the internal energy it has stored to dissipate (transient state). The forced response is what the output looks like over time, when eventually, all the stored energy has dissipated (permanent state).

When designing a control system, we must be able to predict its dynamic behavior from the knowledge of the components. The most important characteristic of the dynamic behavior of a control system is absolute controllability, i.e. whether the system is controlole or not.

This definition requires only that the input be capable of transferring any state to another state in a finite time within the state space. The controllability of any system is essential to achieve the best performance of it.

In previous work [10,11] we found the desired energy optimization for a motor used to drive the movement of a robotic arm as efficiently as possible.

The objective of this work is to analyze, from the decomposition of the State Function, the functionality of motors used in biomechanical prostheses of the upper limb (elbow).

Materials and Methods

The elbow is a joint of the upper extremity, which performs the mechanical union between the first segment, the arm and the second segment, the forearm and allows the upper extremity to orient itself in the three planes of space thanks to the shoulder.

Anatomically, the elbow represents a single joint; In reality, there is only one joint cavity. Instead, physiology allows us to distinguish two distinct functions:

- Pronosupination, which sets in motion the upper radioulnar, trochoid joint.
- The flexo-extension, which requires the participation of two joints: The humeroulnar joint and the humeraradial joint.

Being a trochlear has a single degree of freedom of movement that is performed on a transverse axis, where it performs flexion-extension movements in a sagittal plane.

Model of the servomotor

As described by Alvarez Picaza., *et al.* [12], the following direct current motor is presented.

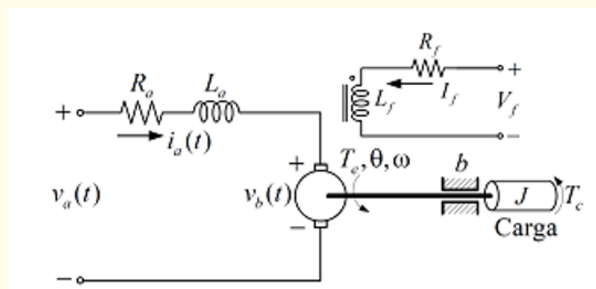

Figure 1: Electrical diagram of the servo motor.

The DC motor in Figure 1 is considered to drive a load through a rigid shaft. If the field current is kept constant at a value I_f or the field flow comes from a permanent magnet, this machine can be controlled only by the voltage $v_a(t)$ applied to the armature.

This machine can be controlled only by the supply voltage v a(t), when the field current I_f remains constant or the flow comes from a permanent magnet. The ratio of the torque electric torque $T_e(t)$ to the current in armature $i_a(t)$ is the torque constant K t, as indicated in equation (1).

$$T_e(t) = K_t i_a(t) \quad \text{------------ (1)}$$

When the motor drives the load, a counter-electromotive force develops in the armature circuit, which opposes the applied voltage v_a(t). The angular velocity of the shaft is directly proportional to the terminal voltage of the motor v_b(t),

$$v_b(t) = K_b \frac{d\theta(t)}{dt} \text{--------------(2)}$$

Where K_b is the speed constant of the motor. The total moment of inertia of the load is J and θ the angular displacement, b is the viscous friction coefficient and T_c is the torque produced by the load.

In the armor circuit, it is verified that,

$$v_a(t) = u(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + K_b \frac{d\theta(t)}{dt} \text{ ------------(3)}$$

Here R a and L_a represent the impedance of the armature winding.

We choose to work in the space of states, whose equations contain all the information of the internal dynamics of the system, allow to easily include the initial conditions and in general are of simple resolution. Figure 2 shows the servo motor model in state space [13].

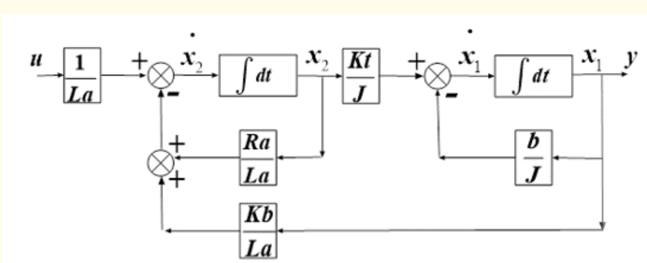

Figure 2: Model of the servo system in state space.

We choose as state variables of this system,

$$x_1 = \dot{\theta}(t) = \omega(t) \quad \text{---------(4)}$$
$$x_2 = i_a(t)$$

Where x_1 is the angular velocity and x_2 is the armature current. Another variable, position, could also be used, but for our model we use only those defined in (4).

The representation in state variables is:

$$\dot{x}_1 = \frac{-b}{J}x_1 + \frac{K_t}{J}x_2 \qquad \text{----------- (5)}$$
$$\dot{x}_2 = \frac{-K_b}{L_a}x_1 + \frac{-R_a}{L_a}x_2 + \frac{1}{L_a}u(t)$$
$$y = x_1$$

The corresponding control system is raised as follows,

$$\begin{matrix} \dot{\mathbf{x}} = \mathbf{A.x} + \mathbf{B.u} & \text{Ecuación de Estado} \\ \mathbf{y} = \mathbf{C.x} + \mathbf{D.u} & \text{Ecuación de Salida} \end{matrix} \qquad \text{------- (6)}$$

Where

x Vector de Estado
u Vector de Entradas
A Matriz de Estado
B Matriz de Entrada
C Matriz de Salida
D Matriz de Transmisión Directa

State function

The matrices of the State and Output Equation of the systems are:

$$\mathbf{A} = \begin{bmatrix} \frac{-b}{J} & \frac{K_t}{J} \\ \frac{-K_b}{L_a} & \frac{-R_a}{L_a} \end{bmatrix}; \ \mathbf{B} = \begin{bmatrix} 0 \\ \frac{1}{L_a} \end{bmatrix} \qquad \text{---------------- (7)}$$
$$\mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}; \ \mathbf{D} = 0$$

To find the State Function of servo motor dynamics we start from the State Equation of the system (6), and analyze it in Laplace space.

$$s\mathbf{X}(s) - \mathbf{X}(0) = \mathbf{AX}(s) + \mathbf{BU}(s) \quad \text{----------- (8)}$$

To finally,

$$\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{X}(0) + (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{BU}(s) \text{--------------- (9)}$$

Anti transforming the system we get,

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0) + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{Bu}(\tau)\,d\tau \quad \text{------------- (10)}$$

The solution of the equation of state posed in (6).

The first addition is the Response to States and here **u**(t) = 0, (transition from the initial state).

The matrix is called the State Transition Matrix; It governs the trajectories of states in a finite time interval $e^{\mathbf{A}t}$ T. It contains all the information of the free movement of the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}.\mathbf{x} + \mathbf{B}.\mathbf{u}$$

The second member of the function is called Input Response and occurs when **x**(t) = 0, (term arising from the input vector).

Note that the response of the system (10) has two components, the natural response which is the zero input response, due to the initial conditions, and the forced response, zero-state response, due to the input. The total answer, then, is the sum of both components.

$$\mathbf{x}(t) = \mathbf{x}_N(t) + \mathbf{x}_F(t) \quad \text{------------ (11)}$$

Controllability

Broadly speaking, controllability studies the possibility of guiding or bringing the states of a system to a desired position by means of the input signal. The equation of state (7) or the pair [A,B] is said to be controllable if for any initial state x(0) = x 0 and any final state x 1, there exists an input u that transfers x 0 to x_1 in a finite time interval. For the system to be controllable, it must be fulfilled that the determinant of the Controllability matrix is nonzero, being the Controllability Matrix.

$$\det(\mathbf{M}) = \left|\mathbf{B} \quad \mathbf{AB} \quad \mathbf{AB}^2 \quad \ldots\ldots \quad \mathbf{AB}^{n-1}\right| \quad \text{----------(12)}$$

different than 0

or

$$\text{rank}(\mathbf{M}) = n \quad \text{-----------(13)}$$

M being the Controllability Matrix.

Results

Data for the servo motor simulation model were obtained from a real motor (RE Maxon® model 40-40 mm) [14] used to drive biomechanical prostheses [15].

The dynamics of the output signal depend on the eigenvalues of matrix A. If all eigenvalues (closed-loop poles) of this matrix have negative real part or lie in the left half-plane of the s-plane, then for any nonzero initial state x(0), the output of the system will tend to the desired value when t → ∞ [16]. The natural poles of the system are,

P= [-1.7739+14.3797i ; -1.7739-14.3797i].

The following values were taken from the manufacturer's catalog:

Ra = Armor strength = 1.16Ω.

The = Armor inductance = 0.329 mH.

Kt= Torque constant= 60.3 mNm/A.

Kb = Speed constant = 158 rpm/V.

b = Coefficient of friction = 3,04 rpm/mNm.

J = Moment of inertia = 138 gcm^2.

The Controllability matrix of the system gives us,

$$\mathbf{M} = \begin{bmatrix} 0 & 1{,}3281 \\ 3{,}0395 & -10{,}7168 \end{bmatrix}$$

$$\det(\mathbf{M}) = -4{,}0369 \text{ different than } 0$$

Figure 3 shows the functions of the servo system drive state variables; They are convergent which means that for these working parameters, the system is controllable and stable.

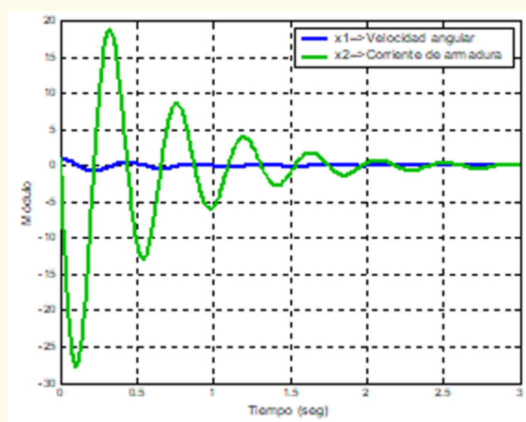

Figure 3: Evolution of system state variables.

According to (11) [17],

$$\mathbf{x}_N(t) = \begin{bmatrix} e^{-1{,}7739t} \\ 0{,}988\,e^{-1{,}7739t} \end{bmatrix}$$

$$\mathbf{x}_F(t) = \begin{bmatrix} 0{,}0063 - 0{,}0063\,e^{-1{,}7739t} \\ -3{,}1896\,e^{-1{,}7739t} \end{bmatrix}$$

Figure 4 shows the surface diagram of the Natural Response $\mathbf{x}_N$(t), where the response to states with zero excitation is observed.

Note that the exponential factor has the form 1/t and denotes the natural frequency, which is an indicator of the rapidity of the response when the energies stored in the inductive and capacitive elements of the servomotor system are exhausted.

Morphologically the figure indicates that the states tend to 0.

A physically stable system is one in which transients decay, that is, the transient response disappears for increasing values over time.

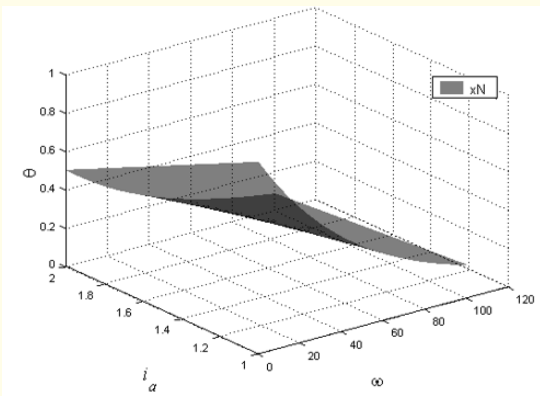

Figure 4: Natural response of system states in state space.

Figure 5 is the Forced Response $\mathbf{x}_F(t)$ of the system and represents the response to input with excitation.

It can be observed how states gain energy.

The forced response of a non-autonomous system is due to inputs.

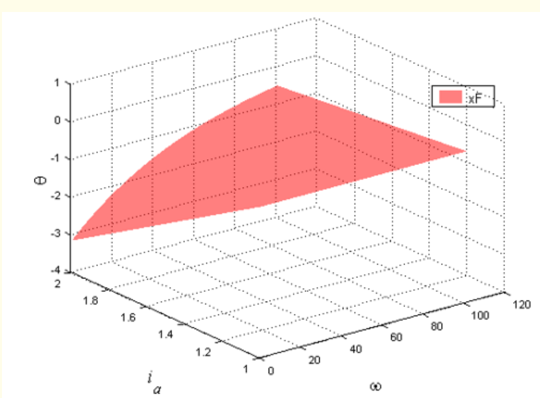

Figure 5: Forced response of system states in state space.

It should be noted that these surface curves correspond to the RE Maxon® 40-40 mm engine.

The same analysis can be performed for any other servo and adjust the parameters according to the required needs.

The functions x_N and x_F allow us to visualize the variations of the states of the different variables involved during the operation of a myoelectric prosthesis.

In linear systems, thanks to the principle of superposition, it is possible to decompose the response into the two parts, free and forced that together provide the total response.

At the moment of start-up, when the engine is stopped its speed is zero, so the back EMF that is proportional to the speed is also zero. This causes the entire supply voltage to fall on the winding of the armature, so that at the moment of starting the current that runs the motor is very high, being able to reach values of up to ten times the nominal current that in stable operating regime. In this process, one of the variables considered i_a, considerably increases the energy of their states.

On the contrary, at the time of shutdown, the values of the states of the other variable w, decreases the energy of their states gained during operation until reaching a null value.

Conclusions

Working in this new space from the equations of state and output allowed us to find the functions, natural and forced that govern the dynamics of the servomotor that activates the myoelectric prosthesis and to be able to graph the energy variation of the states of the different variables. It also allows the analysis of the intrinsic variables of the system to be studied and perform a qualitative analysis of the model. As future work, in a second stage, with the calculation of the Natural Response, we can also infer from the Transition Matrix, the linear transformation that maps the state vector x(0) in t_0 in the state vector $\mathbf{x}(t)$ in t. The regulation of the system depends on the energies invested, either in the states, or in the action of control. The response of the states gives us an idea from a different perspective (state space) of the characteristics of the servo motor used. All this information that is accessed from the use of the control tools, serves as a reference to take into account, a complementary observation when making any adjustment or calibration by the technical professional for each servo motor designated to a particular prosthesis.

Summary

Any mechanical, electrical or physical system can be described through equations of state, which contain all the information of the internal dynamics of it. In this paper we present a basic model of understanding the State Function associated with a direct current motor intended to drive a myoelectric prosthesis, which could be analyzed for some certain physical decrease of upper limb. It aims to find the natural and forced behavior of the servo system, treating it from the point of view of automatic control, obtaining state by state, the response of the system based on the concept of Controllability. Through the use of Modern Control Theory and its tools such as the transfer function, state space and bio-mechanical dynamics among others, we will seek to determine the context situation of the variables that will lead to achieve better performance of the prosthesis. Using models analogous to the servo-motor system, we are interested in knowing the evolution of the function that governs the response of the system for a given input and a set of given initial conditions. In addition, graphs in the state space will be new parametric indicators to take into account. The results obtained indicate that the use of this analysis developed in this paper collaborates with the understanding of the functionality of the prosthesis impeller.

Thanks

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