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Research Article

Probabilistic Shaft Design Using Corrective Factors Methodology Versus Binary Synthesis Methodology

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Abstract

This article compares the traditional static and dynamic (fatigue) methodologies for the mechanical design of a shaft used in the speed reducer of an industrial fan against the probabilistic design methodology based on the binary synthesis method. To make the comparison, the case of the design of the diameter of the intermediate shaft of a speed reducer used between a motor and a fan used to dry grains is analyzed. The main objective is to show the advantage that the probabilistic approach offers over traditional axis design approaches. The effectiveness of the results of the methodologies is validated with the torsional rigidity allowed for the design of shafts. The diameter obtained by the static methodology and the one obtained by the fatigue methodology is smaller than the diameter obtained by the torsional stiffness, on the contrary, the diameter obtained by the binary synthesis probabilistic methodology is greater than the diameter of the torsional stiffness.

Keywords: Probabilistic Shaft Design; Von Mises Theory; Soderberg Theory; Binary Synthesis; Torsional Stiffness

Introduction

Currently, shafts are part of many machines and equipment, such as automobiles, air conditioning and ventilation equipment, electric motors, internal combustion engines, hydraulic pumps, turbines, speed reducers, etc. [1]. Shafts are power transmission elements that are important machine components, their design has the main objective of being safe, for which reason continuous work is being done to improve their reliability and efficiency [2]. Shaft design is said to be a classic mechanical engineering problem, [comparison] where the effect of uncertainty is minimized using factors of safety. But currently, the mechanical design must be a statistical problem, because the variables that determine the efficiency of the designed elements are random [3].

In this article, the design of a shaft from a speed reducer fan used in the drying of xxx grains is carried out through the static approach, the fatigue approach and the probabilistic method of binary synthesis [4]. In the static approach, it can be what is commonly called static analysis because only a comparison of the bending and torsional stresses caused by the loads acting on the element is made and compared to the yield properties of the material [5]. To make the comparison, some failure theory is used, the most common being the Von Mises theory or distortion energy theory (DET) and the Tresca theory or maximum shear stress theory (MSST) [6]. In this case, the Von mises theory is used because it is the proper one for the design of the shaft, since this theory is recommended for ductile materials [7], which is the type of material used in this design. Also, DET is used where it is desired to avoid plastic deformation. In addition, the DET offers a formula adapted to be used in the design of shafts, which facilitates its use [8]. To use this formula, the bending stresses caused by the radial forces and the shear stresses caused by the torques applied to the shaft are calculated. The point where both stresses are maximum should be selected because it is the critical point where the shaft can fail [9]. The DET formula also requires a factor of safety that is deterministic and is most often selected based on ASME standards or sometimes based on experience gained by designers working with similar products [10].

The second approach is made considering the fatigue of the materials since the shaft is rotating and generates alternating bending stresses caused by the bending loads, and since the torque remains constant, it only generates medium stresses. Alternating stresses are repetitive stresses and are less than the magnitude of the yield stresses, but due to repetitiveness they cause the shaft material to crack and cause fatigue and consequently failure [11]. The fatigue method makes use of some theories such as Soderberg, Gerber and Goodman, for this analysis the Soderberg theory was used because the material from which the shaft is made is a ductile material and it is also desired to work in the elastic zone of the shaft material. To use Soderberg's theory, some variables must be calculated, such as the maximum bending and torsion stresses, which is the critical point where the shaft can fail, these data are known since they were used in the static methodology [12]. Because the shaft is turning, repetitiveness of the bending stress is caused, which becomes an alternating stress, on the other hand, the torque is constant because the torque provided by the electric motor remains constant over time. which generates a medium effort. As there is no axial load in the analysis of the axis, the moment can be used instead of the bending effort in the same way that the torque can be used instead of the torsional effort [13]. The yield stress of the material in bending and the yield stress of the material in torsion must also be known. In the same way as in the previous approach, a safety factor is provided. In addition, the theory also requires that the fatigue strength limit Se be calculated, which predicts the design stress available for either no fatigue failure or infinite shaft life [14]. The fatigue strength limit is affected by some modifying factors that reduce its value such as surface factor, size factor, temperature factor, load factor, etc. Finally, the notches must be considered where the wedges that connect the gears with the shaft are positioned because they cause a concentration of efforts. Once all these variables have been calculated, the diameter is calculated, and the third approach is performed.

The third approach is a probabilistic methodology that uses the binary synthesis method that considers each of the variables involved in the design of the axis as random. To carry out this methodology, the failure theory used in the previous approach is used, which is that of Soderberg [15]. In this case, all the variables used in the previous approach are no longer taken as deterministic variables and are taken as random variables. In addition, each variable is considered to have a normal distribution and its coefficient of variation is 10% [16]. To carry out this methodology, two variables are taken and combined using their means and standard deviations to obtain a new function with a normal distribution with its respective mean and standard deviation. The new function obtained is combined with another function and so on. In this approach, the Soderberg theory is not used in the form of an equation, but its graphic form is analyzed. To obtain the diameter of the arrow, the distribution of the stress generated by the failure f(sf) and the distribution of the resistance of the material f(Sf) in addition, the alternating stress and average stress must be known, which is a function of the desired diameter and with the relationship of the alternating stress with the average stress [17]. Due to the above, it is wanted to demonstrate that said probabilistic approach offers an advantage over the first two approaches in the design of an axis and can ensure that it will be a reliable design because the result will be validated with the torsional stiffness that the material can withstand.

To obtain the diameter of the shaft, the distribution of the stress generated by the failure f(Sf) and the distribution of the resistance of the material f(Sf) must be generated, for this a failure theory must be selected, in this case Soderberg's theory will be used because it was used in the previous methodology. Soderberg's theory, as shown in the figure (1), plots on the abscissa axis the yield stress of the material and the mean stress , on the ordinate axis is the resistance limit of the material and the alternating stress, the ratio of and going from the origin to the stress that governs the yield stress failure of the material has the angle Φ. When the binary synthesis method is adapted, the Soderberg diagram changes as shown in the figure 1.

The structure of this article is as follows: in section 2, the axis design problem that will be used for comparison will be presented, the reactions in the supports, torques and bending moments in axis 2 that are used in the three are also calculated. analyzed approaches. The minimum diameter allowed by the torsional stiffness of the material is also calculated [18]. Section 3 describes

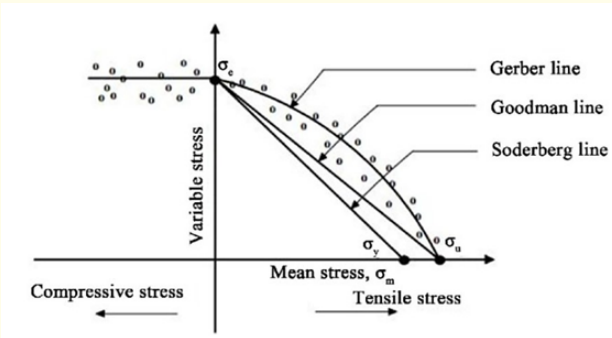

Figure 1: Soderberg's theory Plot.

the static methodology, and it will be applied to the solution of the proposed problem. Section 4 describes the methodology of the dynamic or fatigue approach and its application to shaft design. Section 5 describes the binary synthesis methodology and its solution. Section 6 analyzes and compares the results obtained. Section 7 analyzes the advantages of binary synthesis and the disadvantages of traditional mechanical design methodologies. Finally, in section 8 the conclusions and references are observed.

Data of the analyzed case

In a grain drying process, it is required to move a fan at and the power required to move the fan is . The motor selected for this application is a motor and a turning speed of , so a speed reducer must be designed to reduce the speed to while maintaining the power of . Therefore, a diagram was made of how the speed reducer should be coupled with the motor, the position of the gears on the shafts and their length, see figure 1. The selected motor has an arrow of [19] that connects to shaft number 1 of the speed reducer by means of a flexible coupling. Mounted on shaft 1 is spur gear A which meshes with gear B on shaft number 2. Mounted on shaft number 2 is also mounted a spur gear C which is connected to gear D which is mounted on shaft 3. According to [20], the main characteristics of the 4 gears designed to reduce the speed of the motor are shown in Table 1. The speed reducer was designed in its entirety, from the selection of the motor to the selection of the gears, bearings, and shaft size. Thus, due to the relationship between gears A and B, the initial angular velocity of axis 1 of 1800 rpm is reduced in axis 2 to 900 rpm and finally, due to the relationship between gears C and D, this angular velocity in the axis 3 is reduced to 450 rpm.

Spur gear	Diametral Pitch (in-1)	Pitch diameter (in)	Number of teeth	Face width (1.5)
A	8	2.5	20	1.5
B	8	5	40	1.5
C	8	3	24	1.5
D	8	6	48	1.5

Table 1: Characteristics of the speed reducer gears.

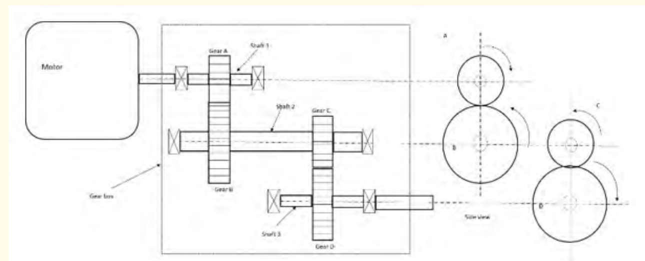

Figure 2: Design of the speed reducer connected with the motor.

In this article the focus will be only on the design of shaft 2 given in figure 3. The material used for the design of the shaft is AISI 1020 normalized steel at 925 °C (1700 °F) air cooled, 50 mm (2 in.) round whose $Sut = 438$ MPa (63,500 psi), $Syt = 319$ MPa (46,300 psi) and a shear modulus of 10,400 ksi. Shaft 2, due to gears B and C, is subjected to bending stresses and constant torsion stress generated by power transmission and to avoid plastic deformation, the shaft design must meet a minimum torsional rigidity of $(0.25°/m)$.

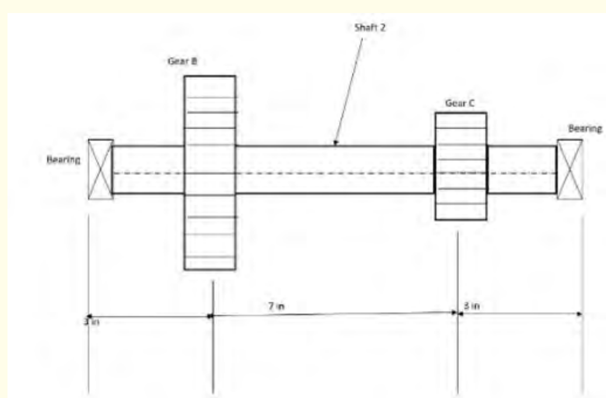

Figure 3: Top view of Shaft.

Due to the bending and torsion efforts to which shaft 2 is subjected, the expected failure mode is due to fatigue, so the design is carried out in accordance with the fatigue resistance limit (Se) (Tamin, M. N., and Hamzah, 2017). According to (Shigley., *et al*. 2002), the factors that affect Se are:

Surface factor ka = 0.8

Size factor kb depends on the size of the diameter to be designed.

Load type factor kc = 1

Temperature factor kd = 1.

On the other hand, due to the concentration of efforts generated by the loads to which the shaft is subjected, the holes for the assembly of the gears and the type of material, the dynamic load factors for bending KFF and for torsion KFT must be considered. However, since the dimensions of the geometry and the diameter of the shaft are not yet known, then according to [16] the initial values of KFF = 2 and KFT = 1.6 are used. Finally, before presenting the design of axis 2 through the three approaches, static, dynamic and binary synthesis, it is necessary to determine 1) the torque; that generate the torques that gears B and C generate and 2) the maximum bending moment; those radial forces and reactions generate. The analysis for the calculation of the torsional stresses is as follows.

Torque estimation shaft 2

The design of the diameter of the shaft by the three design approaches analysis requires knowing which is the critical point of failure of the shaft, and because the failure mode of the shaft is by bending and torsion [21] then the critical point to failure occurs at the point where the bending and shear stresses are maximum. The generated torque is given by

$$T = \frac{63000\,P}{\omega} \qquad \text{---------(1)}$$

In Eq.(1) the constant 63000 is a conversion factor that allows handling the power in horse power (hp), ω is given in revolutions per minute (rpm) and T in pounds/square inch (lb/in2). Thus, under the assumption that there is no power loss in any of the speed reducer bearings, then there will be no power loss, therefore, the power in axis 2 is that given by the 12 hp motor, and the angular speed of axis 2, due to the relationship of gears A and B (see Figure 1), is 900 rpm. Thus, the torque generated on axis 2 due to gear B is

$$T_B = \frac{63000\ (12\ \text{hp})}{(900\ \text{rpm})} = 840\ \text{lb in}$$

Furthermore, since the system is in equilibrium, then the torque in gear C is equal to that in gear B, but in the opposite direction (See Figure 4). Thus, the torque of gear C is Tc = -840 lb.

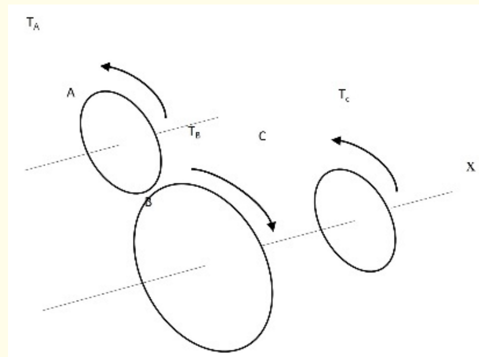

Figure 4: Direction of the torque in the shafts.

Maximum bending moment

Since gears transmit radial and tangential forces, then such forces generate bending moments which in turn generate bending stresses on the shaft. Thus, the maximum bending moment is determined from the bending moment diagram. But, since this diagram is based on the shear force diagram, which is built on the basis of radial, tangential, and reaction forces, then the analysis is as follows.

Calculation of radial and tangential forces

The calculation of the radial and tangential forces acting on the gear teeth B and C depends on the design pressure angle (Φ) of the gears; in this case the pressure angle of gears B and C is Φ=20°. The generated tangential force is a function of the gear torque and radius, and is given by [22].

$$F_t = \frac{T}{r} \qquad \text{--------(2)}$$

The radial force generated is a function of the pressure angle (Φ) and the estimated tangential force and is given by

$$F_r = F_t * \tan\Phi \qquad \text{--------(3)}$$

For gear B we have $F_{tB} = \frac{T}{r_B} = \frac{840\ \text{lb in}}{2.5\ \text{in}} = 336\ \text{lb}$, $F_{rB} = F_{tB}\tan\Phi = (336\ \text{lb})\tan 20° = 122.29\ \text{lb}$.

For gear C we have $F_{tC} = \frac{T}{r_C} = \frac{840\ \text{lb in}}{1.5\ \text{in}} = 560\ \text{lb}$ and $F_{rC} = F_{rC}\tan\Phi = (560\ \text{lb})\tan 20° = 203.82\ \text{lb}$.

Calculation of the reactions in the bearings

The calculation of the reactions that the radial and tangential forces acting on the bearings depend on the axis in which they are generated. Since these forces are on different axes, then they must be considered to act in two different planes; in this case in the x-y plane and in the x-z plane. Thus, the analysis is based on the force diagram given in figure 5 and the bending moment diagram given in figure 6.

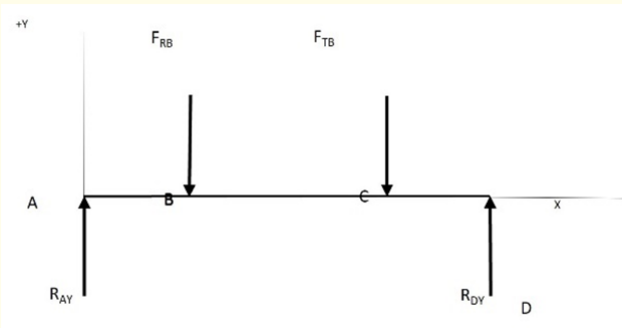

Figure 5: Forces and reactions in the x-y plane.

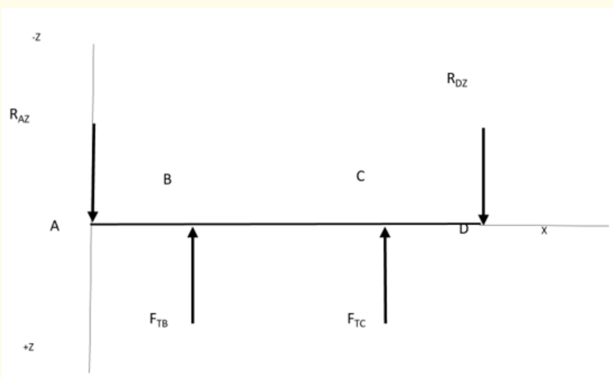

Figure 6: Forces and reactions in the x-z plane.

In the x-y plane the forces and reactions act as shown in figure 5.

$$+\circlearrowleft \sum M_D = 0 = (122.29\text{ lb})(10\text{ in}) + (203.82\text{ lb})(3\text{ in}) - (13\text{ in})(R_{Ay});\ R_{Ay} = 141.10\text{ lb}$$

$$\sum F_y = 0 = 141.10\text{ lb} - 122.29\text{ lb} - 203.82 + R_{Dy};\ R_{Dy} = 185.01\text{ lb}$$

In the x-z plane the forces and reactions act as shown in figure 6.

$$+\circlearrowleft \sum M_D = 0 = (R_{Az})(13\text{ in}) - (336\text{ in})(10\text{ in}) - (560\text{ lb})(3\text{ in});\ R_{Az} = 387.69\text{ lb}$$

$$\sum F_y = 0 = -387.69\text{ lb} + 6\text{ lb} + 560 - R_{Dz};\ R_{Dz} = 508.31\text{ lb}$$

Diagrams of shear forces and maximum bending moments

Using the estimated reaction forces and radial loads, the shear force diagram for the x-y plane is given in figure 7. Based on the forces and distances in the shear force diagram, the corresponding bending moment diagram for the x-y plane is given in figure 8.

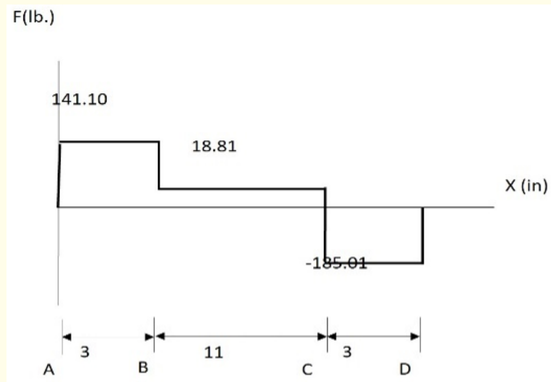

Figure 7: Shear force diagram for the x-y plane.

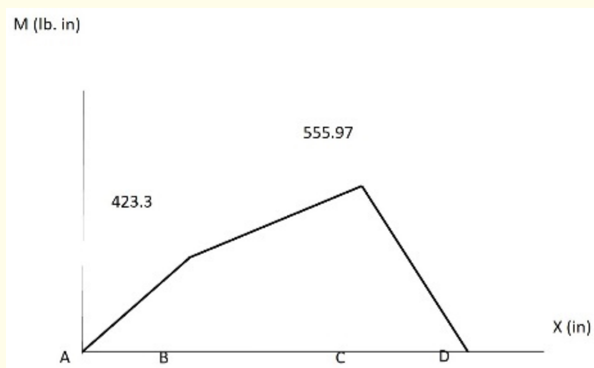

Figure 8: Diagram of bending moments for the x-y plane.

Similarly, for the x-z plane, the shear force diagram and the bending moment diagram are given in figure 9 and figure 10, respectively.

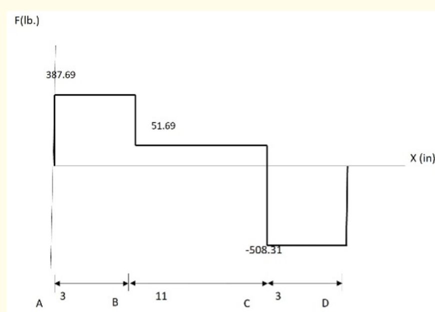

Figure 9: Shear force diagram for the x-z plane.

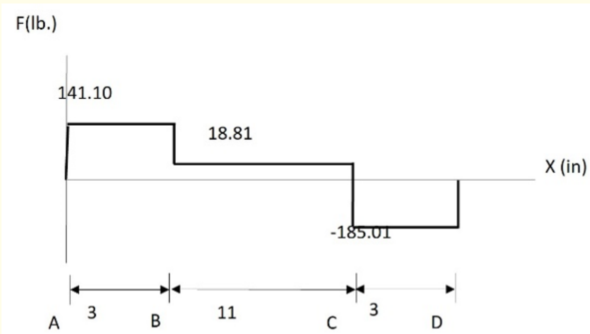

Figure 10: Diagram of bending moments for the x-z plane.

Thus, based on the bending moment diagrams given in Figs.9 and 10, the maximum bending moment acting on axis 2 is

$$M_{MAX} = \sqrt{M_{x-y}^2 + M_{x-z}^2}, \; M_{MAX} = \sqrt{(555.97 \text{ lb in})^2 + (1524.9 \text{ lb in})^2}, \; M_{MAX} = 1623.09 \text{ lb in}$$

The results obtained with the three approaches are validated with a torsional stiffness approach.

Torsional stiffness angle approach

The materials used in the design of shafts must comply with the torsional rigidity which consists of measuring the angle of torsion caused by a torque in a certain length of an arrow. Each material has its modulus of torsional rigidity (G), which is the resistance it offers to be deformed when a torque is applied. Torsional stiffness is not a complete methodology because it does not consider bending or repetitive loads, but it can be used as a constraint when designing a shaft. Torsional rigidity must be considered in the design of a shaft because it can generate a plastic deformation that in turn causes vibration and noise problems that cause the components mounted on the shaft to not work in synchronization [22]. The torsional rigidity also depends on the precision required by the application where the shaft is to be used. In this case, the torsional rigidity would serve to verify if the diameters obtained with the three previous methodologies are adequate.

$$d = \left[\frac{32T}{\pi G\left(\frac{\theta}{L}\right)}\right]^{\frac{1}{4}} \quad \text{-----------(4)}$$

In this case study, a twist angle of 0.0001108 radians in each inch is recommended, substituting in Equation (4)

$$d = \left[\frac{32(840 \text{ lb in})}{\pi(10400\text{x}10^3 \text{ lb/in}^2)(0.0001108 \text{ rad/in})}\right]^{\frac{1}{4}}$$

$$d = 1.65 \text{ in}$$

The diameter obtained is 1.65 in. The results obtained from the three previous methodologies are then compared with the torsional stiffness method. For a diameter obtained by the first three methodologies to be accepted to comply with the torsional rigidity and for this it must be greater than the diameter obtained.

Static design approach

The static method offers the initial steps for the design of an axis [23] since it is based on analyzing the effects of the efforts generated by the forces acting on it and compares them with the effort that can support a material.

Shaft design

For the problem to be solved in this manuscript, the Von mises theory recommended by [10,15] will be used, since it can be derived in a variety of ways, and in this case, it offers a direct way to get the diameter of an arrow.

$$d = \left(\frac{32FS}{\pi S_y}\sqrt{M^2 + {}^3\!/_4 T^2}\right)^{\frac{1}{3}} \quad \text{--------(5)}$$

Using the values of the safety factor, the yield stress of the material, and maximum moment and torque in Equation (5) it is obtained

$$d = \left[\frac{32(2)}{\pi(46300 \text{ lb/in}^2)}\sqrt{(1629.09 \text{ lb in})^2 + \frac{3}{4}(840 \text{ lb in})^2}\right]^{\frac{1}{3}}, \; d = 0.928 \text{ in}$$

Compliance with torsional stiffness

The result obtained with this static method offers an approximation of the diameter dimension that should be used; however, the scope of this method is short because it does not take into consideration that the loads can be cyclical or repetitive and can generate fatigue.

Fatigue method

Fatigue in a mechanical component is caused by the repetitiveness of the loads, these loads being of lesser magnitude than those that can cause plastic deformation in ductile materials or rupture in brittle materials [24]. There can be 3 different cases that can cause fatigue:

- When you have a fixed component that is subjected to loads that vary over time.
- A rotating component that is subjected to loads or moments that are constant over time, but due to rotation, alternation occurs in the stresses.
- A rotating member which is subjected to time-varying loads generates time-varying stresses.

The analysis case presented in this article falls under number 2 because the shaft will be subjected to constant bending and torsion loads and when the shaft rotates, the bending generates alternating loads and when the shaft rotates, the bending generates alternating forces.

$$d = \left[\frac{32FS}{\pi S_y}\sqrt{\left(M_m + \frac{S_y}{S_{ef}}M_a\right)^2 + \frac{3}{4}\left(T_m + \frac{S_{ys}}{S_{et}}T_a\right)^2}\right]^{\frac{1}{3}} \text{ ………………. (5)}$$

To calculate the diameter, it is necessary to obtain the alternating bending moment, the average bending moment, the alternating torque, and the average torque, in addition to the yield stress Sy for bending loads and the for torsion loads and the resistance limits to fatigue for bending (sef) and torsion (set). When the shaft rotates, the bending moment produces alternating stresses because point C can be in tension and compression, as shown in Figure 11. In this case, as can be seen in figure 13, the alternating stress σa is equivalent to the maximum stress σMAX, since moments are to be used then the alternating moment Ma equals the moment MMAX and the mean moment Mm equals zero

$$M_a = M_{MAX} = 1623.09 \text{ lb in}$$
$$M_m = 0$$

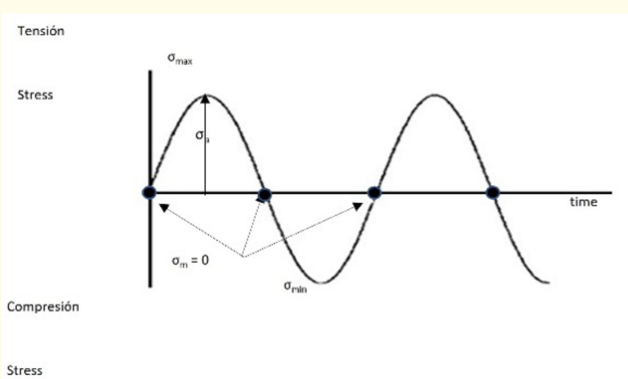

Figure 11: Alternating efforts and average efforts.

Since the torque on the shaft is constant over time (see Figure 12) because the power and angular velocity remain constant, then the average torque Tm is equivalent to Tc, that is, $Tm = Tc = 840$ lb. As there are no variations in the torque, therefore the alternating torque Ta is 0.

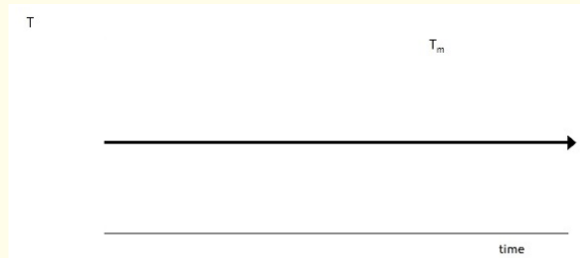

Figure 12: Constant average torque over time.

Since and are 0, then the main formula is as follows

$$d = \left[\frac{32FS}{\pi S_y}\sqrt{\left(\frac{S_y}{S_{ef}}M_a\right)^2 + \frac{3}{4}(T_m)^2}\right]^{\frac{1}{3}} \text{ ----------(6)}$$

Calculating the flexural strength limit value Sef by means of (7)

$$S_{ef} = k_a k_b k_c k_d \frac{S'_{ef}}{K_{FF}} \text{ ………………..(7)}$$

The values of , and were given in the problem so the size factor kb must be calculated

$$k_b = 0.85 \text{ si } \left(\frac{1}{2}\text{in} < d < 2\text{in}\right) \text{ ………………(8)}$$

O

$$k_b = 0.70 \text{ si } (d > 2\text{in}) \text{ …………(9)}$$

As the size factor is obtained based on the diameter and it is the value that is sought in this case study, the diameter obtained in the static method of in is used as a reference, so $kb = 0.85$atigue strength limit for bending is obtained by

$$S'_e = 0.5 S_{ut} \text{ ……………(10)}$$

$$S'_e = 0.5\left(63500\frac{\text{lb}}{\text{in}^2}\right) = 31750 \text{ lb/in}^2$$

Substituting in (7)

$$S_{ef} = (0.8)(0.85)(1)(1)\frac{(31750 \text{ lb/in}^2)}{2}$$
$$S_{ef} = 10795 \text{ lb/in}^2$$

Substituting in (6)

$$d = \left\{\frac{32(2)}{\pi(46300\frac{lb}{in^2})}\sqrt{\left[\frac{(46300\frac{lb}{in^2})(1623.09\ lb\ in)}{10795\ lb/in^2}\right]^2 + \frac{3}{4}(152.78\ lb\ in)^2}\right\}^{\frac{1}{3}}$$

$$d = 1.45\ in$$

The result obtained with this fatigue method is considered to be better than the static method because it takes into account the repetitiveness of the loads, however, this method rules out the variations in the properties of the material, of the factors that affect the limit of resistance to fatigue and the forces involved in the design [25).

Binary synthesis method

There are some reliability methods that use statistics to help solve axis design problems. The use of probabilistic methodologies is since the properties of the materials (the yield stress and the ultimate tensile stress), the loads and the factors that affect the mechanical design are not deterministic and present variations, that is, they are variables. random. In the case of the design of an axis there are several probabilistic methodologies [26], mentions some such as the Monte Carlo simulation method, generation of the system of moments and the binary synthesis of the distribution, which will be used in this article. When the binary synthesis method is adapted, the Soderbergh diagram changes as shown in figure 13.

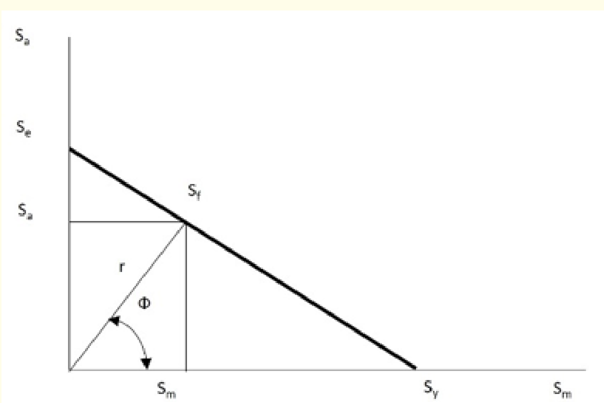

Figure 13: Soderberg diagram.

The adaptation of binary synthesis to the solution of this problem will be described below. To obtain the value that governs the failure and must be calculated. As can be seen in the figure (13), the stress that governs the failure is given by

$$\bar{S_f} = (S_a^2 + S_m^2)^{\frac{1}{2}} \text{ ....(11)}$$

The ratio Sa of Sm to is given by

$$\bar{r} = \frac{\bar{S_a}}{\bar{S_m}} \text{ ....(12)}$$

Equation (11) is left as a function of and is substituted into equation (12) obtaining (13). This is because in this problem is an alternating variable due to bending and is constant because the torque is constant.

$$\bar{S_f} = \left[S_a^2 + \left(\frac{S_a}{r}\right)^2\right]^{\frac{1}{2}} \text{ ....(13)}$$

Taking as a common factor, we get

$$\bar{S_f} = \bar{S_a}\left(1 + \frac{1}{r^2}\right)^{\frac{1}{2}} \text{ ....(14)}$$

Therefore, the values of and must be obtained $Sa = Mc/I$ and $c = d/2$ substituting in (14)

$$S_a = \frac{M\frac{d}{2}}{\frac{\pi d^4}{64}} = \frac{64M}{2\pi d^3} = \frac{32M}{\pi d^3} \text{ ....(15)}$$

Substituting the value of the maximum moment that occurs at point C of the axis, the value of Sa is obtained, but as a function of the diameter

$$S_a = \frac{10.18M}{d^3} = \frac{10.18(1623.09\ lb\ in)}{d^3} = \frac{16523.05\ lb\ in}{d^3}$$

$$S_a = \frac{16523.05\ lb\ in}{d^3}$$

The value of Sm is obtained as follows

$$S_m = \sqrt{3}\,\tau \text{ ....(16)}$$

Since the shear stress is given by $\tau = \frac{Tc}{J}, J = \frac{1}{2}\pi c^4$ and $c = \frac{d}{2}$

$$\tau = \frac{Tc}{J} = \frac{Tc}{\frac{1}{2}\pi c^4} = \frac{2T}{\pi c^3} = \frac{2T}{\pi\frac{d^3}{(2)^3}} = \frac{2(2)^3T}{\pi d^3} = \frac{16T}{\pi d^3} \text{ ....(17)}$$

$$\tau = \frac{5.093T}{d^3} = \frac{5.093(840\ lb\ in)}{d^3}$$

$$\tau = \frac{4278.12\ lb\ in}{d^3}$$

By substituting in (16)

$$S_m = \frac{\sqrt{3}(4278.12 \text{ lb in})}{d^3}$$
$$S_m = \frac{7409.92 \text{ lb in}}{d^3}$$

Next, the fatigue resistance limit Se of formula (18) is obtained using binary synthesis

$$S_e = k_a k_b k_c k_d \frac{S'_e}{K_{FF}} \quad \text{....................(18)}$$

First, we take the variable ka and kb

$$\overline{k_a} = \mu_x = 0.8$$
$$\overline{k_b} = \mu_y = 0.85$$
$$\overline{k_a k_b} = \mu_x \mu_y = (0.8)(0.85) = 0.68$$
$$\sigma_x = \sigma_{k_a} = (0.8)(0.1) = 0.08$$
$$\sigma_y = \sigma_{k_b} = (0.85)(0.1) = 0.085$$

$$\sigma_{k_a k_b} = \left[(\mu_x^2 \sigma_y^2 + \mu_y^2 \sigma_x^2 + \sigma_x^2 \sigma_y^2)(1+\beta^2)\right]^{\frac{1}{2}} \text{.............(19)}$$

If β=-1

$$\sigma_{k_a k_b} = \{[(0.8)^2(0.085)^2 + (0.85)^2(0.08)^2 + (0.08)^2(0.085)^2][1+(-1)^2)]\}^{\frac{1}{2}}$$
$$\sigma_{k_a k_b} = 0.1363$$

Doing binary synthesis of ka kb and kc

$$\overline{k_a k_b} = \mu_x = 0.68$$
$$\overline{k_c} = \mu_y = 1$$
$$(\overline{k_a k_b})(\overline{k_c}) = \mu_x \mu_y = (0.68)(1) = 0.68$$
$$\sigma_x = \sigma_{k_a k_b} = 0.1363$$
$$\sigma_y = \sigma_{k_c} = (1)(0.1) = 0.1$$
$$\sigma_{k_a k_b k_c} = \{[(0.68)^2(0.1)^2 + (1)^2(0.1363)^2 + (0.1363)^2(0.1)^2][1+(-1)^2)]\}^{\frac{1}{2}}$$
$$\sigma_{k_a k_b k_c} = 0.2162$$

Doing binary synthesis of ka kb kc and kd

$$\overline{k_a k_b k_c} = \mu_x = 0.68$$
$$\overline{k_d} = \mu_y = 1$$
$$(\overline{k_a k_b k_c})(\overline{k_d}) = \mu_x = (0.68)(1) = 0.68$$
$$\sigma_x = \sigma_{k_a k_b k_c} = 0.2162$$
$$\sigma_y = \sigma_{k_d} = (1)(0.1) = 0.1$$
$$\sigma_{k_a k_b k_c k_d} = \{[(0.68)^2(0.1)^2 + (1)^2(0.2162)^2 + (0.2162)^2(0.1)^2][1+(-1)^2)]\}^{\frac{1}{2}}$$
$$\sigma_{k_a k_b k_c k_d} = 0.3219$$

Binary synthesis for

$$S_e = \frac{k_a k_b k_c k_d}{K_F} \quad \text{................(20)}$$

$$\overline{k_a k_b k_c k_d} = \mu_x = 0.68$$
$$\overline{K_F} = \mu_y = 2$$

$$\sigma_x = \sigma_{k_a k_b k_c k_d} = 0.3219$$
$$\sigma_y = \sigma_{K_F} = (2)(0.1) = 0.2$$

The following formula is used to obtain the mean

$$\mu_{\frac{k_a k_b k_c k_d}{K_F}} = \frac{\mu_x}{\mu_y} \quad \text{.................(21)}$$

$$\mu_{\frac{k_a k_b k_c k_d}{K_F}} = \frac{\mu_x}{\mu_y} = \frac{0.68}{2} = 0.34$$

To obtain the standard deviation, use the following formula

$$\sigma = \frac{1}{\mu_y}\left(\frac{\mu_x^2 \sigma_y^2 + \mu_y^2 \sigma_x^2}{\mu_y^2 + \sigma_y^2}\right)^{\frac{1}{2}} \quad \text{.......................(22)}$$

$$\sigma_{\frac{k_a k_b k_c k_d}{K_F}} = \frac{1}{2}\left[\frac{(0.68)^2(0.2)^2 + (2)^2(0.3219)^2}{(2)^2 + (0.2)^2}\right]^{\frac{1}{2}}$$
$$\sigma_{\frac{k_a k_b k_c k_d}{K_F}} = 0.1636$$

Binary synthesis for S’e

$$S'_e = 0.5 S_{ut} = 0.5(63500 \frac{\text{lb}}{\text{in}^2})$$

In this case we use the product of a constant

$$\sigma_{S_{ut}} = \sigma_x = (63500 \text{ lb/in}^2)(0.1) = 6350 \text{ lb/in}^2$$
$$S'_e = c\sigma_x = (0.5)(6350 \text{ lb/in}^2)$$
$$S'_e = 3175 \text{ lb/in}^2$$

Binary synthesis for

$$S_e = \left(\frac{\overline{k_a k_b k_c k_d}}{\overline{K_F}}\right)(S'_e) \text{.......................(23)}$$

$$\frac{\overline{k_a k_b k_c k_d}}{\overline{K_F}} = \mu_{\frac{k_a k_b k_c k_d}{K_F}} = \mu_x = 0.34$$
$$\overline{S'_e} = \mu_y = 3175$$
$$\sigma_x = \sigma_{\frac{k_a k_b k_c k_d}{K_F}} = 0.1636$$
$$\sigma_y = (3175)(01) = 317.5$$
$$S_e = \mu_x \mu_y = \left(\frac{\overline{k_a k_b k_c k_d}}{\overline{K_F}}\right)(\overline{S'_e}) = (0.34)(3175) = 1079.5$$
$$\sigma_{\frac{k_a k_b k_c k_d S'_e}{K_F}} = \{[(0.1636)^2(317.5)^2 + (3175)^2(0.1636)^2 + (0.1636)^2(317.5)^2][1+(-1)^2)]\}^{\frac{1}{2}}$$
$$\sigma_{\frac{k_a k_b k_c k_d S'_e}{K_F}} = 741.89$$

Using Solid Works to plot the values of $Se, Syt\ y\ \theta$

$$S_e = 1079.5\ \text{Psi}$$
$$S_{yt} = 46300\ \text{Psi}$$
$$\theta = 65.75°$$

Plot to get the values of $Sa\ y\ Sm$.

With the values of

$$S_a = 1068.5\ \text{Psi}$$
$$S_m = 481.22\ \text{Psi}$$
$$S_a = \frac{16523.05\ \text{lb in}}{d^3}$$

Clearing d3

$$d^3 = \frac{16523.05\ \text{lb in}}{S_a}$$
$$d^3 = \frac{16523.05\ \text{lb in}}{(1068.5\ \text{lb/in}^2)}$$

The diameter obtained with this method is much larger than the diameter obtained with traditional methodologies. To verify which of the three diameters obtained is the appropriate one, it is checked with the torsional rigidity.

Results

The results obtained are shown in table 2.

Method of design	Diameter (in)
Static method	0.928
Fatigue method	1.45
Binary synthesis method	2.49
Torsional rigidity method	1.65

Table 2: Sizes of the diameters obtained by the different technologies.

As can be seen, the diameter obtained by means of the static method and the one obtained by the fatigue method do not comply with the torsional stiffness restriction because they are smaller than the one obtained by means of that methodology. The only diameter that met is the one obtained by means of the binary synthesis method. The result obtained by binary synthesis is rounded to 2.5 to standardize it and to be able to select existing bearings on the market.

Conclusion

The task of designing a shaft is divided into two parts. The first part, which is the constructive design, which consists of the configuration of the geometry and the selection of a possible material to be used. The second part is to check the resistance of the shaft to static and dynamic loads. Verification through mechanical design is done based on failure theories and it is required to find a safe diameter with the material used, the applied loads and a safety factor. As seen above, the proper functioning of a tree depends on several factors. The probabilistic methodology makes use of the current mechanical design methodology to adjust the principal stresses obtained from it and adapt them to the binary synthesis parameters.

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