ACTA SCIENTIFIC MICROBIOLOGY (ISSN: 2581-3226)

Volume 2 Issue 9 September 2019

Review Article

Statistical Quality Control Based on the Process Capability Index and Control Charts with Fuzzy Approach

Khosro alinejad* and Nasir haghkish

Faculty of Industrial Engineering, Urmia University of Technology, Urmia, Iran

***Corresponding Author:** Khosro Alinejad, Faculty of Industrial Engineering, Urmia University of Technology, Urmia, Iran.

Received: July 30, 2019; **Published:** August 19, 2019

DOI: 10.31080/ASMI.2019.02.0349

Abstract

Statistical quality control is a method for monitoring a process to identify the underlying causes of change and corrective actions. Control charts and process capability indices are two important tools for controlling the quality of statistical data. In actual systems and in a lot of circumstances, precise and accurate information is not available and even if it is available it is vague and fuzzy. in this situation fuzzy methods can use a linguistic variable of fuzzy numbers to have a closer look at the state of the production process. In this study, fuzzy control charts are expanded using fuzzy rules, then the actual process capability index (Cpm) evaluated the accuracy and performance of the production process in fuzzy mode method. The results of the study on the quality of water meters in the Rural Water and Wastewater Company of West Azarbaijan province indicated that use of fuzzy rules in comparison with defuzzification data state gives decision makers more choices for decision making and provides a more accurate classification for product quality. Also, the actual process index shows more information about the average location of the process and more sensitivity than the standard deviation and has a flexible analysis of process performance.

Keywords: Quality Control; Fuzzy Logic; Control Charts; Actual Process Capability; Water Meters

Introduction

Since humans started to produce products, it also tried to control quality of them. Quality control refers to using a particular specification of a complete or incomplete product or service to design, produce, maintain and improve the quality of the product or service. Quality control is widely used in controlling statistical processes. Statistical process control is a method for monitoring processes to identify specific causes of changes and revise, using corrective actions. Preventing the production of defective products by the use of statistical methods and in order to control the production process is known as statistical process control, which is generally used to monitor and test a process [1]. Control charts and process capability analysis are two applications of statistical quality control that play an important role in improving the quality of processes and products. Classic charts of control were first proposed by Schwartz, in which quality of a product attributed into a binary function that only contains two values of zero and one, namely, conformity and nonconformity. But the quality of a product always cannot be classified as zero and one. In many processes, statistical data is vague and fuzzy or information about the process is incomplete, in which cases fuzzy control charts are used.

In this method, the fuzzy sets are defined first, then by defining the membership functions and using vague data using triangular or trapezoidal fuzzy numbers, different levels of decision presented for decision makers [2].

The theory of fuzzy sets is used to add more information and flexibility to process analysis. But classical control charts are applicable in cases where data and information are accurate and definite. Another control tool is the process efficiency. This index is a criterion by which the ability of the process for producing products in accordance with customer satisfaction with specification limits can be measured [2]. The process capability index or process efficiency ratio is, in fact, a numerical summary that evaluates and measures the process efficiency in comparison to the Technical specifications range. In other words, this index is used to assess the intrinsic capability of a production process in accordance with their technical specifications [3].

Zadeh introduced a fuzzy set to classify subjective data [4]. For the first time, Bradshaw in 1983 has used fuzzy sets to classify performance of products. Shaw emphasized that fuzzy control offers more favorable results than classical ones [5]. Raz and Wang, applied Fuzzy descriptive control charts using linguistic terminology such as excellent, good, medium, weak, and bad for fuzzy simulated data. The results indicated that control charts based on fuzzy linguistic data are significantly more sensitive than processing simulations [6]. Using the alternating approach for fuzzy control charts and also by the direct fuzzy approach, Gülbay and Kahraman, described fuzzy control charts with -cut and they used fuzzy mode and median respective variables to show fuzzy. The results show that there is no need to defuzzy data and in fuzzy attitude, more favorable results obtained [7]. Sentürk and Erginel, developed triangular fuzzy diagrams using the median variable and -level, and then used their own model to apply fuzzy diagrams to control the internal diameter in process of manufacturing piston used in compressors. The results show that fuzzy control diagrams can provide more flexibility to control process [8]. Kaya and Kahraman, have used the efficiency of analyzing fuzzy measurement processes and fuzzy control charts in an industrial area in Turkey. In this research, they have presented rules for survey the status of samples, then the Cp index has been developed to evaluate the production process. The results indicate that Cp index and the fuzzy control charts have more flexibility and sensitivity for analyzing process [2]. Carot., *et al*. presented a new approach for measuring the efficiency of the control chart process. Results of Cpm index effectiveness are used to discriminate change procedure in the process average, and Cpmk index indicate more power in detecting changes from standard deviation in the process [9]. Wooluru., *et al*. used a case study to examine Cpmk, Cpm, Cp and Cpk indices in classical method. The results indicated that the Cpmk index is more favorable than other indicators [10].

In this research, fuzzy set theory is used to empower the statistical control charts, which increases the ability to control the quality to improve the quality of products and services. In most studies in recent years, for constructing fuzzy control graphs, fuzzy sets are converted to exact and specific numbers using defuzzification operators, and then the control charts are made. First, defuzzification make fuzzy set data get lost and second, the use of different operators results in different results from the diagrams. On the other hand, Kaya and Kahraman have provided rules for analyzing fuzzy control charts that all fuzzy control process states are not considered. Therefore, with regard to the above factors, in this study, first fuzzy rules are developed in the trapezoidal state, which can also provide outputs in a fuzzy form. Then a comparison is made between data defuzzification method with the mean and standard deviation with fuzzy rules. Also, in the researches to evaluate the production process, Cp index (Potential Process Capability Index) is used, which does not provide information about the average location of the technical specifications of LSL and USL. In this research, the Cpm index (actual process index capability) is used to solve this problem, and then it is developed using fuzzy rules. Finally, fuzzy rules and fuzzy process efficiency index are used in a case study on water meters in the water and wastewater company.

Control charts

In Statistical Control of Processes, control charts that also known as Shewhart Chart or Process-Behavior Chart, are tools to determine whether a production or business process is in statistical control or not. Control charts are one of the most widely used statistical control tools and play a key role in improving the quality of processes and products [2]. To control a qualitative characteristic, the mean and its variance over time are examined. The average of the process is controlled by the mean control charts X, Also the process variability can be controlled by control charts in standard deviations or range of changes with S and R charts. In control of statistical quality control, control charts are of great importance, and they are somehow the main part of the quality control process. One of the most important applications of control charts is to estimate the parameters of a production process and to use it to determine the process efficiency and provide useful information to improve the process. If all points are between control limits, the process is in control, otherwise, corrective action on the process should be performed. These diagrams are a way to improve productivity [11]. In the following, the design of fuzzy control diagrams is done in 5 steps.

Fuzzy control charts Design

Steps of designing fuzzy control charts are as follows

Step 1

First, the samples are converted to fuzzy numbers. It is assumed that a quality specification which approximately defined as X, can be defined by a trapezoidal fuzzy number (a, b, c, d).

Step 2

In this step, the specifications of the process are calculated and controlled. By measuring the sample size, the mean is obtained as follows:

$$\tilde{X} = \left(\frac{\sum_{i=1} a_i}{n}, \frac{\sum_{i=1} b_i}{n}, \frac{\sum_{i=1} c_i}{n}, \frac{\sum_{i=1} d_i}{n}\right) = (\bar{X}_a, \bar{X}_b, \bar{X}_c, \bar{X}_d) \quad (1)$$

$i = 1, 2, 3, \ldots, n$

After m sampling, the mean of samples $\tilde{X}$ is calculated as follows

$$\tilde{\bar{X}} = (\mu_a, \mu_b, \mu_c, \mu_d) = (\bar{\bar{X}}_a, \bar{\bar{X}}_b, \bar{\bar{X}}_c, \bar{\bar{X}}_d) = \left(\frac{\sum_j^m \bar{X}_a}{m}, \frac{\sum_j^m \bar{X}_b}{m}, \frac{\sum_j^m \bar{X}_c}{m}, \frac{\sum_j^m \bar{X}_d}{m}\right) \quad (2)$$

i =1, 2, 3, ..., n ; j=1, 2, 3, ..., m

The standard deviation ($\tilde{S}$) is also defined as follows.

$$\tilde{S}_j = \sqrt{\frac{\sum_i^n((X_a,X_b,X_c,X_d)_{ij} - (\bar{X}_a,\bar{X}_b,\bar{X}_c,\bar{X}_d)^2)}{n-1}} = (S_a,S_b,S_c,S_d) \quad (3)$$

And for the average sample $\tilde{S}$ we have.

$$\bar{\tilde{S}}_j = \left(\frac{\sum_j S_{aj}}{m},\frac{\sum_j S_{bj}}{m},\frac{\sum_j S_{cj}}{m},\frac{\sum_j S_{dj}}{m}\right) = (\bar{S}_a,\bar{S}_b,\bar{S}_c,\bar{S}_d) \quad (4)$$

Step 3

Based on the values obtained in the previous section, the upper control limits (UCL) and lower control limits (LCL) in the average mode ($\bar{X}$ are determined as follows, in which A_3 is constant value and determined according to the sample size [12].

$$\tilde{UCL}_{\bar{X}} = \tilde{CL} + A_3\bar{S} = (\bar{\bar{X}}_a,\bar{\bar{X}}_b,\bar{\bar{X}}_c,\bar{\bar{X}}_d) + A_3(\bar{S}_a,\bar{S}_b,\bar{S}_c,\bar{S}_d) = (\tilde{UCL}_1,\tilde{UCL}_2,\tilde{UCL}_3,\tilde{UCL}_4) \quad (5)$$

$$\tilde{CL}_{\bar{X}} = \tilde{CL} = (\bar{\bar{X}}_a,\bar{\bar{X}}_b,\bar{\bar{X}}_c,\bar{\bar{X}}_d) = (\tilde{CL}_1,\tilde{CL}_2,\tilde{CL}_3,\tilde{CL}_4) \quad (6)$$

$$\tilde{LCL}_{\bar{X}} = \tilde{CL} - A_3\bar{S} = (\bar{\bar{X}}_a,\bar{\bar{X}}_b,\bar{\bar{X}}_c,\bar{\bar{X}}_d) - A_3(\bar{S}_a,\bar{S}_b,\bar{S}_c,\bar{S}_d) = (\tilde{LCL}_1,\tilde{LCL}_2,\tilde{LCL}_3,\tilde{LCL}_4) \quad (7)$$

Also, the control limits in the standard deviation ($\tilde{S}$) is calculated as follows, that B_4 and B_3 are constant values determined according to t.he sample size [12].

$$\tilde{UCL}_S = B_4\bar{S} = B_4(\bar{S}_a,\bar{S}_b,\bar{S}_c,\bar{S}_d) = (\tilde{UCL}_1,\tilde{UCL}_2,\tilde{UCL}_3,\tilde{UCL}_4) \quad (8)$$

$$\tilde{CL}_S = \bar{S} = (\bar{S}_a,\bar{S}_b,\bar{S}_c,\bar{S}_d) = (\tilde{CL}_1,\tilde{CL}_2,\tilde{CL}_3,\tilde{CL}_4) \quad (9)$$

$$\tilde{LCL}_S = B_3\bar{S} = B_3(\bar{S}_a,\bar{S}_b,\bar{S}_c,\bar{S}_d) = (\tilde{LCL}_1,\tilde{LCL}_2,\tilde{LCL}_3,\tilde{LCL}_4) \quad (10)$$

Step 4

In this step, the samples placed in the control limits (upper control limits and lower limits) and evaluated according to the following rules.

- **Rule 1:** The calculations show that sample is within the control limits and in this situation, it is in control, and EP = (EP1, EP2, EP3, EP4) is defined as the sample.
- **Rule 2:** The sample is outside the control limits. In this case, the sample is out of control.
- **Rule 3 and 4:** In this case, part of the sample is on the control limit boundary, That is considered to be rather in control or in control.
- **Rule 5:** In this case, a part of the sample is placed in the upper boundary or another part is in the lower boundary, in which, the sample is considered in control or rather in control [2].

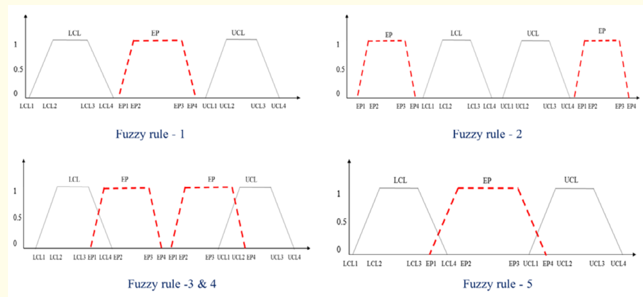

Figure 1: Fuzzy rule (Kaya and Kahraman, 2011).

According to the five rules defined above, these rules do not cover all control process states, therefore in this research, we can extend the following rules in other situations.

- **Rule 6:** Part of the sample is located outside the upper limit, in which case the samples are out of control or rather out of control.
- **Rule 7:** Part of the sample is outside the lower limit, in which case the samples are rather out of control or out of control too.

Step 5

According to the rules defined above, we can define the status of the samples as follows.

$$PC_{EP_j} = \begin{cases} 1 \; (EP_{dj} \leq UCL_1) \text{ and } (EP_{aj} \geq LCL_4) \\ 0 \; (EP_{aj} > UCL_4) \text{ or } (EP_{dj} < LCL_1) \\ 1 - \dfrac{EP_{dj} - UCL_1}{EP_{dj} - EP_{aj}} \; (EP_{dj} > UCL_1) \\ 1 - \dfrac{LCL_4 - EP_{aj}}{EP_{dj} - EP_{aj}} \; (EP_{aj} < LCL_4) \\ \min\left\{\left(1 - \dfrac{EP_{dj} - UCL_1}{EP_{dj} - EP_{aj}}\right), \left(1 - \dfrac{LCL_4 - EP_{aj}}{EP_{dj} - EP_{aj}}\right)\right\} \; (EP_{dj} > UCL_1) \text{ and } (EP_{aj} < LCL_4) \\ 1 - \dfrac{UCL_4 - EP_{aj}}{EP_{dj} - EP_{aj}} \; (EP_{dj} > UCL_4) \\ 1 - \dfrac{EP_{dj} - LCL_1}{EP_{dj} - EP_{aj}} \; (EP_{aj} < LCL_1) \end{cases} \quad (11)$$

According to the obtained values, the status of the processes is determined as follows.

$$\text{Process Control} = \begin{cases} \text{In control} & PC_{\bar{X}} = 1 \wedge PC_S = 1 \\ \text{out of control} & PC_{\bar{X}} = 0 \vee PC_S = 0 \\ \text{Rather in control} & PC_{\bar{X}} \geq \beta \wedge PC_S \geq \beta \\ \text{Rather out of control} & PC_{\bar{X}} < \beta \vee PC_S < \beta \end{cases} \quad (12)$$

In rule 3, 4, 6 and 7 the judgment is the percentage of the area covered. That is, if the sample values (β) are more than 70% outside the control limit, the sample is out of control or rather out of control. If a sample was more than 70% in the control limits, it was classified to be in control or rather in control. In the following, the fuzzy control charts converted to the defuzzy state by using the α-cut method and median.

Designing control charts based on $\bar{\bar{X}}\%$ and $S\%$ in defuzzy state

In order to defuzzy the data and determine control constraints, using the α-cut method and median, the following procedure is performed.

Defuzzy state control charts based on $\bar{\bar{X}}\%$

Given the equation defined in the previous section, we have seen what parameters are needed to calculate the upper and lower bounds. In the following, the manner of calculating the upper and lower limit of a process using α-cut discussed, in which equations are calculated as follows [12]:

$$\widetilde{UCL}^{\alpha}_{\bar{\bar{X}}} = (\bar{\bar{X}}^{\alpha}_{a}, \bar{\bar{X}}_{b}, \bar{\bar{X}}_{c}, \bar{\bar{X}}^{\alpha}_{d}) + A_3(\bar{S}^{\alpha}_{a}, \bar{S}_{b}, \bar{S}_{c}, \bar{S}^{\alpha}_{d}) \quad (13)$$

$$\widetilde{CL}^{\alpha}_{\bar{\bar{X}}} = \widetilde{CL} = (\bar{\bar{X}}^{\alpha}_{a}, \bar{\bar{X}}_{b}, \bar{\bar{X}}_{c}, \bar{\bar{X}}^{\alpha}_{d}) \quad (14)$$

$$\widetilde{LCL}^{\alpha}_{\bar{\bar{X}}} = (\bar{\bar{X}}^{\alpha}_{a}, \bar{\bar{X}}_{b}, \bar{\bar{X}}_{c}, \bar{\bar{X}}^{\alpha}_{d}) - A_3(\bar{S}^{\alpha}_{a}, \bar{S}_{b}, \bar{S}_{c}, \bar{S}^{\alpha}_{d}) \quad (15)$$

So that

$$\bar{\bar{X}}^{\alpha}_{a} = \bar{\bar{X}}_{a} + \alpha(\bar{\bar{X}}_{b} - \bar{\bar{X}}_{a}) \quad (16)$$

$$\bar{\bar{X}}^{\alpha}_{d} = \bar{\bar{X}}_{d} - \alpha(\bar{\bar{X}}_{d} - \bar{\bar{X}}_{c}) \quad (17)$$

And

$$\bar{S}^{\alpha}_{a} = \bar{S}_{a} + \alpha(\bar{S}_{b} - \bar{S}_{a}) \quad (18)$$

$$\bar{S}^{\alpha}_{d} = \bar{S}_{d} - \alpha(\bar{S}_{d} - \bar{S}_{c}) \quad (19)$$

To investigate samples to be in control with defuzzy method in the $\widetilde{\bar{X}}$ mode using the fuzzy median and α-level, regarding to the specified relationships we have [12].

$$UCL^{\alpha}_{mr-\bar{X}} = CL^{\alpha}_{mr-\bar{x}} + A_3\left(\frac{\bar{S}^{\alpha}_{a} + \bar{S}^{\alpha}_{d}}{2}\right) \quad (20)$$

$$CL^{\alpha}_{mr-\bar{X}} = \left(\frac{CL^{\alpha}_{a} + CL^{\alpha}_{d}}{2}\right) \quad (21)$$

$$LCL^{\alpha}_{mr-\bar{X}} = CL^{\alpha}_{mr-\bar{x}} - A_3\left(\frac{\bar{S}^{\alpha}_{a} + \bar{S}^{\alpha}_{d}}{2}\right) \quad (22)$$

The fuzzy median of $\widetilde{\bar{X}}$ for j example in the α- level defined as bellow:

$$S^{\alpha}_{mr-\bar{X},j} = \frac{(\bar{X}_{a_j} + \bar{X}_{d_j}) + \alpha[(\bar{X}_{b_j} - \bar{X}_{a_j}) - (\bar{X}_{d_j} - \bar{X}_{c_j})]}{2} \quad (23)$$

Finally, the process state for each sample is determined as follows [8]:

$$\text{Process control} = \begin{cases} in-control & LCL^{\alpha}_{mr-\bar{X}} \le S^{\alpha}_{mr-x,j} \le UCL^{\alpha}_{mr-\bar{x}} \\ out-of\ \text{control} & for\ \text{otherwise} \end{cases} \quad (24)$$

In relation (24) after defuzzification of process, samples are classified into two classes of in control and out of control.

Defuzzy Control charts based on $S\%$

To defuzzification of data and determining its control constraints in the standard deviation mode ($S\%$) at the alpha-level based on the fuzzy median, these equations are used:

$$\widetilde{UCL}^{\alpha}_{mr-S} = B_4\, f^{\alpha}_{mr-S}(\widetilde{CL}) \quad (25)$$

$$\widetilde{CL}^{\alpha}_{mr-S} = f^{\alpha}_{mr-S}(\widetilde{CL}) \quad (26)$$

$$\widetilde{LCL}^{\alpha}_{mr-S} = B_3\, f^{\alpha}_{mr-S}(\widetilde{CL}) \quad (27)$$

The fuzzy median for the example j in the $S\%$ charts at the α-level is defined as bellow:

$$S^{\alpha}_{mr-S,j} = \frac{(S_{a_j} + S_{d_j}) + \alpha[(S_{b_j} - S_{a_j}) - (S_{d_j} - S_{c_j})]}{2} \quad (28)$$

Finally, the process control status in standard deviation mode for each sample defined as following [8]:

$$\text{Process control} = \begin{cases} in-control & LCL^{\alpha}_{mr-\bar{S}} \le S^{\alpha}_{mr-S,j} \le UCL^{\alpha}_{mr-\bar{S}} \\ out-of\ \text{control} & for\ \text{otherwise} \end{cases} \quad (29)$$

This process also categorizes the samples into two groups of in control and out of control. After determining the status of the process and ensuring that the process is in control, the process capability indicator is used as a process performance criterion in the production process evaluation. In the next section, the process capability indicator of the process in the fuzzy state developed, and it has the advantage to provide several numbers instead of one number, for decision makers, to assess the process.

Fuzzy process capability index

Statistical techniques could be useful at different steps of producing a product, including development activities and activities before production, but X, R, and S control charts do not provide information about the process performance. These activities can be quantity by modifying the process variability, by analyzing the variability according to the needs and specifications limit of the product, and helping to build and develop it for removable variability or to a reduction extra extent. One of the process control capability indices is the C_p index (Process Potential Capability Index). This index defined as the difference between technical specification limits (USL-LSL) and 6σ [13,14].

$$C_p = \frac{USL - LSL}{6\sigma} \tag{30}$$

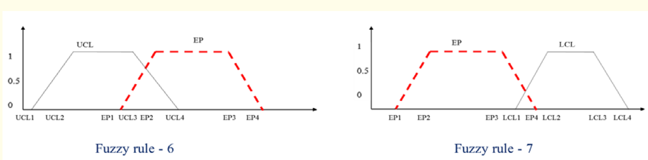

Figure 2: Fuzzy rule.

The process capability indicator as a process performance criterion in practice is very effective in assessing construction process capability. In general, this index is obtained by comparing the width of the process scattering with the width of the acceptable range of process specifications. In most cases, it is not possible to determine the status of the process only by referring to the C_p index (potential process capability). The C_p index does not take the place of process average in relation to the technical specification limits into account, and only measures the distance between the technical characteristics and the 6σ distance. Therefore, this index does not imply the actual performance of the process and indicate that process average movement has no effect on the ability of the process to produce within acceptable specifications. In this case, the indicators C_{pm} (actual capability of process) will be a solution to this problem. This indicator is based on the square error idea and focuses on measuring the ability of the process to gather around the target, which shows the value on the target. This index also provides more information about the location of the average of the process and has more sensitivity to standard deviation. To assess the accuracy and precision of a product, a limit needs to be determined, which has an upper variation limit (USL) and a lower variation limit (LSL). These limit are specifications of desired products and are defined as follows:

$$\tilde{USL} = (u_a, u_b, u_c, u_d),\ \tilde{LSL} = (l_a, l_b, l_c, l_d) \tag{31}$$

$$\tilde{USL} = (u_a, u_b, u_c, u_d),\ \tilde{LSL} = (l_a, l_b, l_c, l_d) \tag{32}$$

Now, using the specification limits above, Cpm index in the fuzzy state defined as below:

$$C_{pm} = \left(\frac{\tilde{USL} - \tilde{LSL}}{6\tau}\right) = \left(\frac{u_a - l_d}{6\tau}, \frac{u_b - l_c}{6\tau}, \frac{u_c - l_b}{6\tau}, \frac{u_d - l_a}{6\tau}\right) \tag{33}$$

In the above equation, τ is the square root of expectation of the square of deviation from target value T, so we have:

$$\begin{aligned} \tau^2 &= E[(X - T)^2] \\ &= E[(X - \mu)^2] + (\mu - T)^2 \\ &= \sigma^2 + (\mu - T)^2 \end{aligned} \tag{34}$$

Then

$$\tau = \sqrt{\sigma^2 + (\mu - T)^2} \tag{35}$$

Where σ^2 is square of the standard deviation and is defined in the fuzzy state as follows:

$$\sigma^2 = \frac{R}{d_2} = (S_a, S_b, S_c, S_d) \tag{36}$$

In this case, d2 is a constant value and is determined according to the sample size, also, variations range of R is obtained from the following equation.

$$\tilde{R} = (\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d) = \left(\frac{\sum R_{a_j}}{m}, \frac{\sum R_{b_j}}{m}, \frac{\sum R_{c_j}}{m}, \frac{\sum R_{d_j}}{m}\right) \tag{37}$$

$$\begin{aligned} &R_{a_j} = X_{\max, a_j} - X_{\min, d_j},\ R_{b_j} = X_{\max, b_j} - X_{\min, c_j} \\ &R_{c_j} = X_{\max, c_j} - X_{\min, b_j},\ R_{d_j} = X_{\max, d_j} - X_{\min, a_j} \end{aligned} \tag{38}$$

The target value of T is also calculated using upper and lower technical characteristic limits. Therefore:

$$T = \frac{\tilde{USL} + \tilde{LSL}}{2} = \left(\frac{u_a + l_d}{2}, \frac{u_b + l_c}{2}, \frac{u_c + l_b}{2}, \frac{u_d + l_a}{2}\right) \tag{39}$$

The $\tilde{C}_{pmk}$ index is defined as follows. This index is more sensitive to process average get far from the target value: It has been shown to be a useful capability index for processes with two-sided specification limits

$$\tilde{C}_{pmk} = \min\{\tilde{C}_{pml}, \tilde{C}_{pmu}\} \tag{40}$$

In which $\tilde{C}_{pml}$ and $\tilde{C}_{pmu}$ are one-way criterions of the process, which determined as:

$$\tilde{C}_{pmu} = \frac{\tilde{USL} - \tilde{\mu}}{3\tau} = \left(\frac{u_a - \mu_d}{3\tau}, \frac{u_b - \mu_c}{3\tau}, \frac{u_c - \mu_b}{3\tau}, \frac{u_d - \mu_a}{3\tau}\right) \quad (41)$$

$$\tilde{C}_{pml} = \frac{\tilde{\mu} - \tilde{LSL}}{3\tau} = \left(\frac{\mu_a - l_d}{3\tau}, \frac{\mu_b - l_c}{3\tau}, \frac{\mu_c - l_b}{3\tau}, \frac{\mu_d - l_a}{3\tau}\right) \quad (42)$$

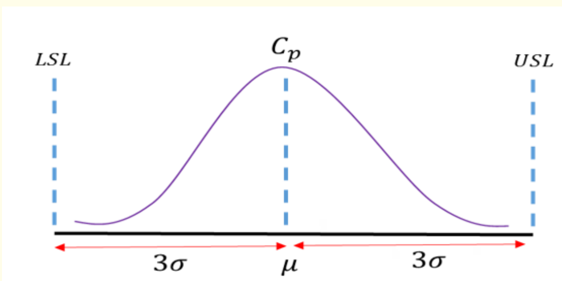

Figure 3: C_p indices.

The focus of CPM index is on measuring the ability of the process to gather around the target and it reflects the degree that process is on target. Extremely high application of this indicator in practice shows that this index is clearly the best indicator of process capability.

Case study

Rural Water and Wastewater Company of West Azarbaijan province has been considered as a case study in this research. The company has a well-equipped meter testing laboratory, which used to evaluate the accuracy of water meter readings and to detect corrupted meters. In the present study, we used the information that has been recorded in this laboratory. The data obtained from the laboratory are fuzzy and examined using fuzzy rules. Then a comparison between the use of fuzzy rules and the method to defuzzy data is made. Information in the laboratory is recorded with the primary and secondary digits (in normal, transitive and minimal estates). The function of the meter calculated on the normal, transfer and minimum flows on a liter basis, then the method of operation of the meters was calculated and the average error percentage of each of the water meters in the three subgroups (normal, transitive and minimum) converted into the trapezoidal fuzzy numbers (Table 1).

Also, to determine the control limit, the mean of the samples using relations (1) and (2), the standard deviation from equations (3) and (4) and the range of changes from (37) and (38) for each sample and each subgroup The table 2 is calculated.

	Least				Transfer				Normal			
	a	b	c	d	a	b	c	d	a	b	c	d
1	98/0	1	02/1	02/1	29/0	30/0	31/0	31/0	0	0	0	0
2	49/0	50/0	52/0	52/0	0	0	0	0	38/0	39/0	40/0	41/0
3	49/0	50/0	51/0	52/0	15/0	15/0	16/0	16/0	77/0	78/0	80/	82/0
	⋮				⋮				⋮			
42	49/0	50/0	51/0	52/0	29/0	30/0	31/0	31/0	77/0	78/0	80/0	82/0
43	48/0	49/0	50/0	51/0	30/0	31/0	33/0	33/0	54/1	57/1	60/1	63/1

Table 1: Fuzzification the percentage of errors in the operation of water meters in a trapezoidal state.

Sample	Range of changes				Mean				Standard deviation			
	R_{aj}	R_{bj}	R_{cj}	R_{dj}	$\bar{X}_{aj}$	$\bar{X}_{bj}$	$\bar{X}_{cj}$	$\bar{X}_{dj}$	S_{aj}	S_{bj}	S_{cj}	S_{dj}
1	0/98	1	1/02	1/02	0/42	0/43	0/44	0/44	0/50	0/51	0/52	0/52
2	0/49	0/50	0/51	0/51	0/29	0/30	0/30	0/31	0/26	0/26	0/27	0/27
3	0/62	0/63	0/65	0/67	0/47	0/48	0/49	0/49	0/31	0/32	0/32	0/33
⋮	⋮				⋮				⋮			
42	0/46	0/48	0/50	0/52	0/52	0/53	0/54	0/54	0/24	0/24	0/25	0/26
43	1/23	1/26	1/30	1/34	0/77	0/79	0/81	0/82	0/67	0/68	0/70	0/71
	$\bar{R}_a$	$\bar{R}_b$	$\bar{R}_c$	$\bar{R}_d$	$\bar{\bar{X}}_a$	$\bar{\bar{X}}_b$	$\bar{\bar{X}}_c$	$\bar{\bar{X}}_d$	$\bar{S}_a$	$\bar{S}_b$	$\bar{S}_c$	$\bar{S}_d$
Mean	1/51	1/54	1/58	1/60	0/85	0/87	0/88	0/89	0/81	0/83	0/84	0/85

Table 2: Range of changes, mean and standard deviations of samples.

Boundaries of the upper and lower limits in the average mode from relations (5) and (6) and (7) could be found.

$$U\tilde{C}L_{\bar{X}} = (2/39, 2/44, 2/50, 2/53)$$

$$\tilde{C}L_{\bar{X}} = (0/85, 0/87, 0/88, 0/89)$$

$$L\tilde{C}L_{\bar{X}} = (0, 0, 0, 0)$$

And in the standard deviation mode, we use the equation (8) and (9) and (10):

$$U\tilde{C}L_{S} = (2/43, 2/48, 2/53, 2/55)$$

$$\tilde{C}L_{S} = (0/85, 0/87, 0/88, 0/89)$$

$$L\tilde{C}L_{S} = (0, 0, 0, 0)$$

Using calculated values, the status of water meters being in control or out of control was evaluated by fuzzy rules. Then, by means of defuzzification in the average (based on equation (23)) and the standard deviation (based on equation (28)) method, a comparison was made with the fuzzy rule method, the results of which are presented in Table 3.

According to the table above, all samples are not in control. Also, in the results obtained in three modes of using fuzzy rules, defuzzy with mean and standard deviation methods, there are differences in the state of the meters, and according to the fuzzy rules, the status of the samples has been investigated. Sample 20, 23 and 30 are completely out of control according to the charts in Figure 4 and regarding to rule 2. Sample 34 and 4 are rather out of control according to the control chart and rule 6, and respectively, PC = 0/21

Sample	Using fuzzy rules		In defuzzy mode and use averages		In defuzzy mode and use standard deviation	
	Status	Rule	$S^{0.65}_{mr-\bar{X},j}$	$0 \le S^{0.65}_{mr-\bar{X},j}$ $S^{\alpha}_{mr-\bar{X},j} \le 2/03$	$S^{0.65}_{mr-S,j}$	$2/06 \ge S^{0.65}_{mr-S,j}$ $S^{\alpha}_{mr-S,j} \ge 0$
1	in control	1	0/44	in control	0/52	in control
2	in control	1	0/30	in control	0/26	in control
3	in control	1	0/48	in control	0/32	in control
4	rather out of control	6	2/06	out of control	2/58	out of control
5	in control	1	0/36	in control	0/40	in control
6	in control	1	1/44	in control	0/98	in control
7	in control	1	0/90	in control	0/60	in control
8	in control	1	0/40	in control	0/10	in control
9	in control	1	0/27	in control	0/25	in control
10	in control	1	0/75	in control	0/28	in control
11	in control	1	1/15	in control	0/77	in control
12	in control	1	0/66	in control	1/14	in control
13	in control	1	0/61	in control	0/53	in control
14	in control	1	0/89	in control	1/29	in control
15	in control	1	1/91	in control	1/27	in control
16	in control	1	0/78	in control	0/55	in control
17	in control	1	0/96	in control	0/64	in control
18	in control	1	0/26	in control	0/46	in control
19	in control	1	0/22	in control	0/26	in control
20	Out of control	2	3/42	out of control	1/70	in control
21	in control	1	0/85	in control	0/64	in control
22	in control	1	0/35	in control	0/18	in control
23	Out of control	2	1/83	in control	2/78	out of control
24	in control	1	0	in control	0	in control
25	in control	1	0/17	in control	0/29	in control
26	in control	1	0/60	in control	0/53	in control
27	in control	1	0/60	in control	0/54	in control
28	in control	1	1/32	in control	1/51	in control

29	in control	1	0/39	in control	0/54	in control
30	out of control	2	1/73	in control	2/86	out of control
31	in control	1	1/06	in control	1/14	in control
32	in control	1	0/91	in control	0/72	in control
33	in control	1	0/50	in control	0/62	in control
34	Rather out of control	6	2/17	out of control	2/51	out of control
35	in control	1	0/18	in control	0/20	in control
36	in control	1	0/34	in control	0/58	in control
37	in control	1	0/96	in control	0/64	in control
38	rather in control	4	2/31	out of control	2/45	out of control
39	in control	1	0/48	in control	0/32	in control
40	in control	1	0/61	in control	0/53	in control
41	in control	1	0/57	in control	0/38	in control
42	in control	1	0/53	in control	0/25	in control
43	in control	1	0/80	in control	0/69	in control

Table 3: Deciding on the status of the samples.

and PC = 0/51 are less than 0.70 respectively. Sample 38 is rather in control regarding the control chart and rule 4, with the PC = 0.82 being more than 0.70. Other samples are in control according to rule 1.

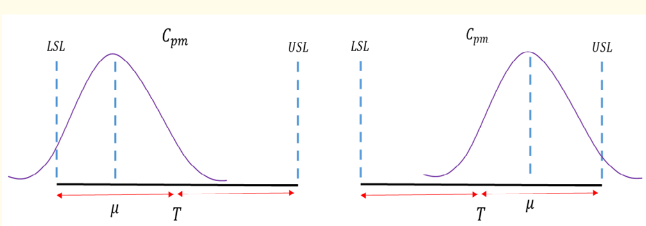

Figure 4: C_{pm} *Process capability index.*

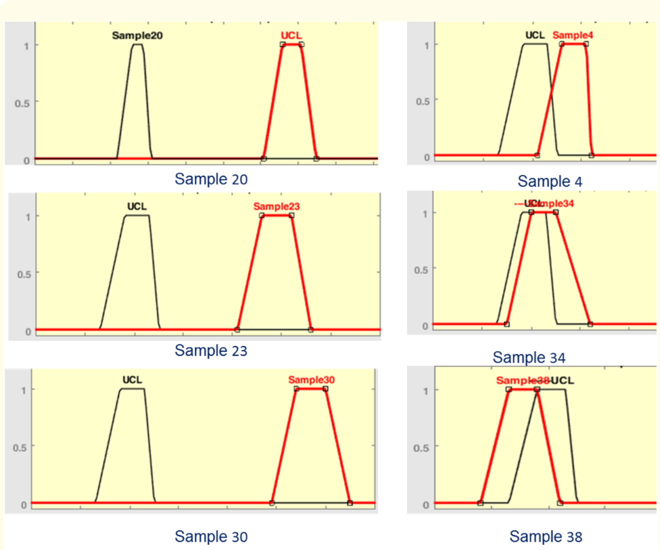

Figure 5: *Quality status of sample process.*

In most samples, the results are the same in all cases, but in a number of examples, different results have been achieved. In general, using fuzzy rules cause more sensitivity to process changes, and decision making about the process state be more cautious.

To reach the process efficiency, samples that are out of control should be removed from the process.to assure process is in control, out of control samples are removed, knowing the distribution of the samples is normal, the actual index of the process is calculated using the equation (33), (40), (41) and (42) as follows.

$\tilde{C}_{pm}$ = (0/692, 0/697, 0/702, 0/711)

$\tilde{C}_{pml}$ = (0/854, 0/879, 0/903, 0/930)

$\tilde{C}_{pmu}$ = (0/487, 0/502, 0/515, 0/534)

$\tilde{C}_{pmk}$ = (0/487, 0/502, 0/515, 0/534)

Given the and values $\tilde{C}_{pmk}$ and $\tilde{C}_{pm}$ values ($\tilde{C}_{pm}$ and $\tilde{C}_{pmk}$ <1) conditions of the process are unfavorable, Therefore this process needs improvement to reduce the amount of process variations and deviations, to increase the process efficiency. Improving the specification performance and recognizing the critical factors of the process, improves the process average and also minimizes the variability, which creates stability in the process. This performance improvement reduces damages and rework, which result in a decrease in a lot of costs. Therefore, in order to decrease the losses caused by the deviation from the target value, the product variability around the target values should be reduced.

Conclusions and Suggestions

In this study fuzzy rules were developed to survey the quality of equipment more accurately. In using fuzzy rules, Judging is based on the percentage of samples that are within the control limits or outside of this range. Fuzzy rules, in comparison with defuzzy methods, instead of offering two states of in-control and out-of-control, provide decision makers and manufacturers other options for better decision making and more assured division (including rather in control or rather out of control) for quality of products. If a process in fuzzy mode placed between two states, the product should be given more attention and precision. Finally, in order to evaluate the accuracy of the production process performance to measure the efficiency of processes according to standard specifications, actual capability index of process C_{pm} in the fuzzy state developed. This index provides more information about the location of the mean of the process and also shows more sensitivity to standard deviation. In general, this index is a number that represents the ability of a process to produce acceptable products and Assesses and measures process behavior in relation to technical specification limits.

The results indicate that using fuzzy state in comparison with the defuzzy state for samples resulted in more accuracy in decision making about situation of the process. Because in the comparison between the condition of using fuzzy rules and defuzzification of data using standard deviations and average methods, results in some cases were totally different. But in general, the results indicate superiority of performance accuracy in the fuzzy method compared to the defuzzy method. Also, based on the fuzzy process real capability index, Cpmk and Cpm indices that are less than one, indicate an inappropriate status of the process. It can be argued that the Cpm index, which used to determine the overall state of the process is the best index because, according to the obtained equation, if the process variance declines, the denominator of the Cpm index decreases, thus the whole index increases. Also, if the difference between the process average and the target value is reduced (At best, the average of the process is equal to the target value) the denominator of this index decreases, and as a result, the whole index increases. It can be concluded that the index is the best Cpm indicator in determining the overall status of the process. In general, this indicator, by considering the mean, the target value and process variance simultaneously, report the overall state of the process. The values obtained for the Cpm and Cpmk index are less than 1 so the process conditions are inappropriate so to reduce the deviation process, the process needs to be improved to reduce losses and overtime in the process.

Comparison between the actual index and the actual process index, the development of fuzzy graphs for the exponential average in fuzzy state can be for future research.

Bibliography

1. Montgomery DC. "Introduction to statistical quality control". John Wiley and Sons (New York) (2009).
2. Kaya İ and Kahraman C. "Process capability analyses based on fuzzy measurements and fuzzy control charts". *Expert Systems with Applications* 38.4 (2011): 3172-3184.
3. Lundkvist P. "Application of Statistical Methods: Challenges Related to Continuous Industrial Processes (Doctoral dissertation, Luleå tekniska universitet) (2015).
4. LA Zadeh. "Fuzzy Sets". *Information and Control* 8.3 (1965): 338-353.
5. Bradshaw CW. "A fuzzy set theoretic interpretation of economic control limits". *European Journal of Operational Research* 13.4 (1983): 403-408.
6. Raz T and Wang JH. "Probabilistic and membership approaches in the construction of control charts for linguistic data". *Production Planning and Control* 1.3 (1990): 147-157.
7. Gülbay M and Kahraman C. "An alternative approach to fuzzy control charts: Direct fuzzy approach". *Information Sciences* 177.6 (2007): 1463-1480.
8. Sentürk S and Erginel N. "Development of fuzzy and control charts using α-cuts". *Information Sciences* 179.10 (2009): 1542-1551.
9. Carot MT., *et al* . "A New Approach for Measurement Of The Efficiency Of C Pm And C Pmk Control Charts". *International Journal for Quality Research* 7.4 (2013).
10. Wooluru Y., *et al* . "The process capability analysis—a tool for process performance measures and metrics—a case study". *International Journal for Quality Research* 806.20 (2014): 399-416.
11. Shah S., *et al* . "Control chart: A statistical process control tool in pharmacy". *Asian Journal of Pharmaceutics* 4.3 (2014).
12. Pandurajan and Varadharajan R. "Construction of α - cut fuzzy and Xbar-R and Xbar-S Control Charts Using Fuzzy Trapezoidal Number". *International Journal of Research and Reviews in Applied Sciences* 9 (2011): 100-111.
13. Kane VE. "Process Capability Indices". *Journal of Quality Technology* 18 (1986): 41-52.
14. Sullivan LP. "Reducing variability: A new approach to quality". *Quality Progress* 17.7 (1984): 15-21.

Volume 2 Issue 9 September 2019