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Research Article

An Integrated Pricing Strategy for an Imperfect Quality Items with Credit Period Based Procurement Cost and Learning Effect on a Screening Process

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Abstract

This study suggested an integrated price and credit period based production model for imperfect quality items which consists of a single supplier and a single vendor. A delay in payment is offered by supplier to the retailer and the procurement cost is subjected to the credit period. A screening procedure is employed on each lot of an imperfect quality items to separate the good and defective items by the vendor. Furthermore supplier s supplier process cost has been assumed to subjected to linearly correlated to the size of batch of items purchased by vendor. The main objective of this study is to get the position of credit period and the number of batches for vendor in the single planning time so that the integrated system receives an optimum cost. This mathematical model has been verified with the help of numerical examples and We have also checked it's stability fragility and feasibility with respect to key parameters through numerically and graphically.

Keywords: Procurement cost, Imperfect quality items, Credit periods, Screening process, Learning Effects.

Abbreviations

AMS Subject Classification: 90B05, 90B30, 90B50.

Introduction

Due to the digitalization of information and globalization, the competition in any kind of business day by day. Today, doing any kind of business has become competitive, uncertain, and unstable. Most companies and organizations want to make static and tensionless business setups, therefore, they try to do long-term cooperative, direct/indirect relationships with their subordinates, supporters, followers, and customers.

Today, doing any kind of business has become competitive, uncertain, and unstable. Most of the firms, organizations, and business houses want to make stable and tensionless business setups, therefore, they try to make long-term cooperative relationships with their subordinates and customers. Guarantee/warranty, buyback, buy one get one free, trade credit financing, lot size dependent procurement cost, etc. are some common and popular policies that are used in business to tie the buyer-seller relationship.

Nowadays, there a lot of literature relating to integrated/coordinated supply chain models like two layers, three layers, multi-layer echelon, closed loop, etc. incorporating various realistic parameters are available. Xie and Wei [1] developed a coordinated policy for a single manufacturer and a single buyer in which they consider sales products depend on local advertising, and a cost local advertising expenditure cost is shared by the manufacturer with the vendor. Giri and Masanta [2] designed a Closed Loop Supply Chain policy for a single vendor and a single manufacturer where they assumed a production process is affected by both the human nature of learning and forgetting. Furthermore, they assumed both the inventories of the manufacturer and the vendor are managed by a stock holding policy. Stochastic lead time is a realistic parameter that measures the time interval between the time of order and obtaining the consignment of order.

Giri and Masanta [3] suggested a CLSC model that consists of one manufacturer, one retailer, and two suppliers, In this study they consider returns of used products as based on a function of random variables with stochastic lead time. Singh developed an insightful production-inventory model for a single manufacturer to tackle the stocking problem of deteriorating items incorporating the issues of raw material quality. Jayaswal., *et al*. [5] studied the effects learning curve on the performance of workers for a given new task and showed that it is a mathematical representation of the same learning process which can be analyzed progressive learning process after frequent repetitions. Nigwal., *et al*. [8] developed, an EOQ model for retailer's price-dependent demand of a single item under a two-stage trade credit financing policy. In this credit financing policy it is assumed that the supplier provides to the vendor a fixed credit period of payment and the retailer also provides a fixed credit period of payment to his customers.

Chung., *et al*. [6] first time adopted the calculus approach to get the optimal solution of lot size Q and number of replenishment n jointly. The solution procedures to determine the optimal solution obtained by our approach can simplify the model of Barron., *et al*. [7]. Khedlekar., *et al*. [9] presented an integrated/coordinated multi-layer supply chain policy for multi-channel and multi-echelon which contains a single manufacturer, distributors, and retailers. The demand is considered as a linear function of time, retail price, and suggested retail price. Nigwal., *et al*. [10] developed a three-stage trade credit financing policy for a three-layer supply chain consisting of a single supplier, manufacturer, and retailer. In this study, they proposed an optimal manufacturing rate and retailing price respectively for the manufacturer and the retailer both under an imperfect production system, and a rework process is also done by the manufacturer.

Khann., *et al*. [11] developed a coordinated vendor buyer inventory model for imperfect quality items with allowable shortages. In this study, they assumed the vendor provides a credit period to the buyer for delay of payment. The objective of this study is to minimize the expected total annual costs incurred by the vendor and the buyer by using an integrated decision-making approach. Quality and environmental concerns in production are essential issues of inventory management. Kazemi N., *et al*. [12] have incorporated Jointly those two relevant factors in a single research study to support decisions, compare the results, and obtain new insights into complications in practice. Sebatjane and Adetunji [13] designed an inventory system in which they considered those items ordered as capable of growing during the course of the inventory replenishment cycle, for example, livestock. Furthermore, in this paper, it has been assumed that a certain part of the items is of poorer quality than desired. It has been also assumed that live newborn items are ordered and fed until they grow up to a customer-preferred weight, after which they are manslaughter.

Hauck Z., *et al*. [14] presented an EOQ model with imperfect quality items. In this study, they assumed that all these items undergo a quality assurance screening process for which both the cost and the blemish detection rate depend on the time devoted to this operation. In this study, a new concept is incorporated by taking screening time and order quantity as decision variables.

Cárdenas-Barrón., *et al*. [15] developed an economic order quantity inventory model for perfect and imperfect quality items, taking into that the account imperfect ones are sent as a single lot to a repair shop for reworking. The demand for the products is taken as nonlinear and price-dependent. The main objective of this study is to optimize jointly the lot size and the retailing price such that the expected total profit per unit of time is optimized.

Das., *et al*. [16] developed a coordinated EPQ model of supplier and retailer where a delay in payment is provided by the supplier to the retailer for a deteriorating item. In this study, they assumed that the retailer's procurement cost linearly depends on the credit period and the supplier's process cost also is a linear function of the amount of quantity ordered by the retailer. In this study position of the credit period and number of replenishments of retailers in a finite time horizon is takenas a decision variable with constant demand in such a way that the coordinated system gets the optimum cost.

In this article, we develop a coordinated EPQ model of supplier and retailer where a delay in payment is provided by the supplier to the retailer for imperfect quality items and A screening procedure is employed on each lot of imperfect quality items to separate the good and defective items by the vendor. Furthermore, in this study, we assumed that the retailer's procurement cost linearly depends on the credit period and the supplier's process cost also is a linear function of the amount of quantity ordered by the retailer. In this study position of the credit period and the number of replenishments of retailers in a finite time horizon are takenas decision variables with price-dependent variable demand in such a way that the coordinated system gets the optimum cost.

Notations and assumptions

Notations	
	Ordering cost per order.
	Setup cost per production run.
:	Demand rate per unit item in unit time for perfect quality items, where .
	Incremental procedure cost to the supply of trading with the vendor.
	Procurement cost per unit item for vendor.
	Production cost per unit item for supplier.
	Inventory holding cost per unit time per unit item for vendor.
	Inventory holding cost per unit time per unit item for supplier.
	Selling price per unit item at vendor 's end.
	Production rate of items per unit time for supplier.
	Fixed amount of items.
	Replenishment time interval of the vendor in unit year.
	Credit period offered by the supplier to the vendor.
	Interest rate per unit time paid by vendor.
	Interest rate per unit time to be paid to the supplier for the remaining stock from to .
	Interest rate for calculating supplier's opportunity interest loss due to delay payment.
	The No. of replenishment of vendor.
	Initial lot size which is taken by the vendor for a one cycle from the supplier.
:	The number of efforts to obtaining of efficiency of screening process by worker.
:	Percentage of defective items per lot.
	Screening cost per unit item.
	Screening time of where,
	Average total cost of the vendor for imperfect quality items.
	Average total cost of the vendor for perfect quality items.
	Average total cost of the supplier for imperfect quality items.
	Average total cost of supplier for perfect quality items.
	Average total cost of integrated system for model with imperfect quality items.
	Average total cost of integrated system for model with perfect quality items.

Table a

Assumptions	
1.	This system has only two members viz supplier and vendor.
2.	The replenishment rate is infinite and instantaneous for vendor.
3.	At initially retailer applies a screening procedure on each lot of products to separate the perfect and imperfect quality items and the learning process is applied only for screening process.
4.	The screening rate is
5.	The credit period lies between to .
6.	The demand rate per unit time for an item is deterministic
7.	The whole business planning period is certain and fixed for one year.
8.	The demand is completely fulfilled by supplier.
9.	The supplier provides a credit period to the vendor to the settle the account.
10.	An interest rate is charged by supplier between the interval to
11.	vendor 's procurement cost is a linear positive correlated to , i.e. , where is the base procurement cost and is the coefficient of on as Das et al.[16].
12.	Suppliers procedure cost is a positive correlated to lot size purchased by vendor i.e. where fixed purchase cost (
13.	In this study M and T are taken as decision variables.

Table b

Mathematical formulation for imperfect quality items

At the time t=0, the supplier initiates the production at a rate P_c, and at the time t=T, he/she supplies the first batch of size $\emptyset_n$ of imperfect quality items to the vendor. The vendor applies a screening process on the received lot to separate the good and defective quality items and simultaneously fulfills the demand of their customers from his/her stock during the period 0 to T of good quality items. After the time t=T, the supplier again supplies the number of items in the time interval T to 2T, and the vendor again applies the screening process and simultaneously fulfills the demand of their customers. The same process is continuously repeated up to the time , where is the total number of lots purchased by the vendor. As per assumptions, the fixed planning interval is equally divided into equal parts. The inventory structure of this integrated system is shown in Figure 1.

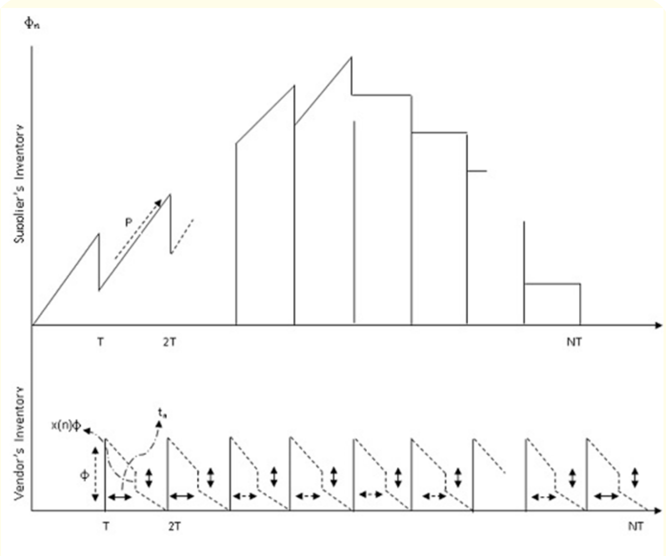

Figure 1

Vendor's model for imperfect quality items

Let be the inventory level of imperfect quality items at time , for vendor and be the demand of customers. Then the vendor's inventory level I(t) at any instant t is can be represented by the following differential equation:

$$\frac{dI(t)}{dt} = -D = -(\alpha - s\beta) \quad (3.1)$$

With the given initial and boundary conditions:

at $t = 0, I(t) = \big(1 - x(n)\big)\emptyset_n$ and $t = T, I(t) = 0$.

solution of the equation (3.1) yields

$$I(t) = -Dt + \big(1 - x(n)\big)\emptyset_n \quad (3.2)$$

$$\text{At } t = T, \emptyset_n = \frac{DT}{(1-x(n))} = \frac{(\alpha - s\beta)\,T}{(1-x(n))} \quad (3.3)$$

After calculating an average screening cost, screening time, average holding cost of perfect and imperfect quality items, average purchasing costs at the vendor's end, and average earned and paid interest by the vendor in the intervals [0, T] and [M, T] respectively.

An average total cost of the vendor can be expressed as

$$RC_i = \frac{O_c}{T} + \frac{ch_r}{T}\left[\big(1 - x(n)\big)\emptyset_n T - \frac{(\alpha - s\beta)\,T^2}{2}\right] + ch_r x(n)\emptyset_n + \frac{c\emptyset_n}{T} + \frac{S_c\emptyset_n}{T} - \frac{sI_d DT}{2}$$
$$+ \frac{cI_p}{T}\left[\big(1 - x(n)\big)\emptyset_n(T - M) - \frac{D(T^2 - M^2)}{2} + \big(1 - x(n)\big)\emptyset_n(T - M)\right] \quad (3.4)$$

Supplier's model for imperfect quality items

The supplier's inventory initiates from and continuously increases up to (where and N is any integer) and after decreases up to NT and at it will be zero. Let be the production rate of the items, be the initial quantity of items and be the demand of items(where). The inventory structure is shown in Figures 1 and 2.

From Figure 2 the supplier's average inventory can be calculated as

$$= \frac{1}{NT}\left[N\emptyset_n NT - \frac{N\emptyset_n N\emptyset_n}{P} - \emptyset_n T\{1 + 2 + 3 + 4 \ldots . +(N - 1)\}\right]$$
$$= N\emptyset_n - \frac{N\emptyset_n^2}{2PT} - \frac{N(N-1)}{2}\emptyset_n T = \frac{\emptyset_n}{2}\left[(N + 1) - \frac{N\emptyset_n}{PT}\right] \quad (3.5)$$

After calculating supplier's average holding cost, setup cost per production run, process cost per setup, production cost, opportunity interest loss (due to providing credit period to the vendor), the supplier's average total cost can be expressed by

$$SC_i = \frac{S_p + NV_d}{NT} + \frac{h_s\emptyset_n}{2}\left[(N + 1) - \frac{N\emptyset_n}{PT}\right] + \frac{\emptyset_n S_p}{T} + \frac{I_0 cM\emptyset_n}{T} \quad (3.6)$$

Since, an integrated coordinated pricing system requires jointly efforts to optimize the cost for all supply chain members, therefore for this integrated supply chain system, the total cost can be expressed by

$$TC_i(T, M) = RC_i + SC_i$$
$$TC_i(T, M) = \frac{O_c}{T} + \frac{ch_r}{T}\left[\big(1 - x(n)\big)\emptyset_n T - \frac{(\alpha - \beta s)T^2}{2}\right] + ch_r x(n)\emptyset_n + \frac{c\emptyset_n}{T} + \frac{S_c\emptyset_n}{T} - \frac{sI_d DT}{2}$$
$$+ \frac{cI_p}{T}\left[\big(1 - x(n)\big)\emptyset_n(T - M) - \frac{D(T^2 - M^2)}{2} + x(n)\emptyset_n(T - M)\right]$$
$$+ \frac{S_p + NV_d}{NT} + \frac{h_s\emptyset_n}{2}\left[(N + 1) - \frac{N\emptyset_n}{PT}\right] + \frac{\emptyset_n P_c}{T} + \frac{I_0 cM\emptyset_n}{T} \quad (3.7)$$

$$TC_i(T, M) = \frac{O_c}{T} + \frac{(c_o + c_1 M)\,h_r}{T}\left[\big(1 - x(n)\big)\emptyset_n T - \frac{(\alpha - \beta s)\,T^2}{2}\right] +$$
$$(c_o + c_1 M)h_r x(n)\emptyset_n + \frac{(c_o + c_1 M)\emptyset_n}{T} + \frac{S_c\emptyset_n}{T} - \frac{sI_d DT}{2}$$
$$+ \frac{(c_o + c_1 M)I_p}{T}\left[\big(1 - x(n)\big)\emptyset_n(T - M) - \frac{D(T^2 - M^2)}{2} + x(n)\emptyset_n(T - M)\right]$$
$$+ \frac{S_p + NV_d}{NT} + \frac{h_s\emptyset_n}{2}\left[(N + 1) - \frac{N\emptyset_n}{PT}\right] + \frac{\emptyset_n P_c}{T} + \frac{(c_o + c_1 M)I_0 M\emptyset_n}{T} \quad (3.8)$$

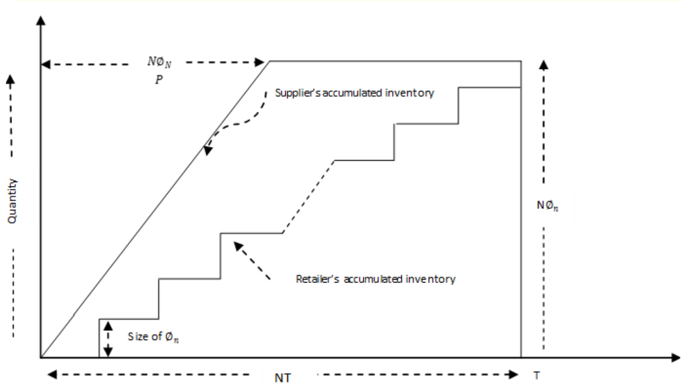

Figure 2

Theorem 3.1

The cost function satisfies jointlythe optimality condition concerning and for an imperfect quality and nonconstant demand of an item.

Proof

Let us compute the Hessian matrix of the cost function concerning and if the determinant of Hessian is positive definite that is if both eigenvalues of the Hessian matrix are positive then the proof will be completed.

Here on differentiating partially of equation (3.8) concerning T and M respectively we get

$$\frac{\partial TC_i(T,M)}{\partial T} = -\frac{O_c}{T^2} - \frac{(c_o+c_1M)h_r(\alpha-\beta s)}{2} - \frac{(c_o+c_1M)\emptyset_n}{T^2} - \frac{S_c\emptyset_n}{T^2} - \frac{sI_d(\alpha-\beta s)}{2} + \frac{(c_o+c_1M)I_p(1-x(n))\emptyset_n M}{T^2}$$
$$-\frac{(c_o+c_1M)I_p(\alpha-s\beta)}{2} - \frac{(c_o+c_1M)I_p(\alpha-\beta s)M^2}{2T^2} + \frac{(c_o+c_1M)I_p x(n)\emptyset_n M}{T^2}$$
$$-\frac{S_p+NV_d}{NT^2} + \frac{h_s\emptyset_n N\emptyset_n}{2T^2} - \frac{\emptyset_n P_c}{T^2} - \frac{(c_o+c_1M)I_0M\emptyset_n}{T^2} \quad (3.9)$$

$$\frac{\partial TC_i(T,M)}{\partial M} = c_1h_r\left[(1-x(n))\emptyset_n - \frac{(\alpha-\beta s)T}{2}\right] +$$
$$c_1h_r x(n)\emptyset_n + \frac{c_1\emptyset_n}{T} + \frac{S_c\emptyset_n}{T} + \frac{c_1I_p}{T}\left[(1-x(n))\emptyset_n(T-M) - \frac{D(T^2-M^2)}{2} + x(n)\emptyset_n(T-M)\right]$$
$$-\frac{(c_o+c_1M)I_p}{T}\left[(1-x(n))\emptyset_n + (\alpha-\beta s)M - x(n)\emptyset_n\right]$$
$$+\frac{(c_o+2c_1M)I_0\emptyset_n}{T} \quad (3.10)$$

Here on differentiating partially again of equation (3.9) concerning T and M respectively we get

$$\frac{\partial^2 TC_i(T,M)}{\partial T^2} = \frac{O_c}{T^3} + \frac{2(c_o+c_1M)\emptyset_n}{T^3} + \frac{2S_c\emptyset_n}{T^3} - \frac{2(c_o+c_1M)I_p(1-x(n))\emptyset_n M}{T^3}$$
$$+\frac{(c_o+c_1M)I_p(\alpha-\beta s)M^2}{T^3} - \frac{2(c_o+c_1M)I_p x(n)\emptyset_n M}{T^3}$$
$$+\frac{2(S_p+NV_d)}{NT^3} - \frac{h_sN\emptyset_n^2}{T^3} + \frac{2\emptyset_n P_c}{T^3} - \frac{2(c_o+c_1M)I_0M\emptyset_n}{T^2} \quad (3.11)$$

$$\frac{\partial^2 TC_i(T,M)}{\partial M^2} = -\frac{c_1I_p}{T}\left[(1-x(n))\emptyset_n + (\alpha-\beta s)M - x(n)\emptyset_n\right]$$
$$-\frac{c_1I_p}{T}\left[(1-x(n))\emptyset_n + x(n)\emptyset_n\right]$$
$$+\frac{(c_o(\alpha-\beta s) + 2c_1(\alpha-\beta s)M)I_p}{T} + \frac{2c_1I_o\emptyset_n}{T} \quad (3.12)$$

$$\frac{\partial^2 TC_i(T,M)}{\partial T\,\partial M} = -\frac{c_1h_r(\alpha-\beta s)}{2} - \frac{c_1\emptyset_n}{T^2} + \frac{(c_o+2c_1M)I_p(1-x(n))\emptyset_n}{T^2}$$
$$-\frac{c_1I_p(\alpha-s\beta)}{2} - \frac{(2c_o+3c_1M)I_p(\alpha-\beta s)}{2T^2} + \frac{(c_o+2c_1M)I_p x(n)\emptyset_n}{T^2}$$
$$-\frac{(c_o+2c_1M)I_0\emptyset_n}{T^2} \quad (3.13)$$

At the values T_0 and M_0

$$HM = \begin{bmatrix} \frac{\partial^2 TC_i(T,M)}{\partial T^2} & \frac{\partial^2 TC_i(T,M)}{\partial T\,\partial M} \\ \frac{\partial^2 TC_i(T,M)}{\partial T\,\partial M} & \frac{\partial^2 TC_i(T,M)}{\partial M^2} \end{bmatrix}$$

$$\mathrm{Det}(HM) = \frac{\partial^2 TC_i(T,M)}{\partial T^2} * \frac{\partial^2 TC_i(T,M)}{\partial M^2} - \left[\frac{\partial^2 TC_i(T,M)}{\partial T\,\partial M}\right]^2 > 0.$$

The profit function will be concave concerning replenishment time and credit period time if We proved it with the help of numerical examples.

Theorem 3.2

There exists an optimum point (T_0, M_0), where the cost function is minimum.

Proof

At the point (T_0, M_0),at which the equations $\frac{\partial TC_i(T,M)}{\partial T} = 0$ and $\frac{\partial TC_i(T,M)}{\partial M} = 0$ must be zero as per the first order condition of classical optimization. On solving these equations, we can obtain the optimum point (T_0, M_0), where at that point cost function is minimum.

Mathematical formulation for perfect quality items

The production system requires preventive and scheduled maintenance to keep it running smoothly and efficiently so that the system produces high quality and efficient and perfect production. The present case assumes that preventive and scheduled maintenance is applied to the production system so that the system works perfectly and consequently produced items are all of perfect quality. Consequently, in this case all the produced items can be considered as perfect quality items.

Vendor's model for perfect quality items

The SCM can be formed with perfect quality items when After calculating an average holding cost of perfect quality items, the average purchasing costs at the vendor's end, an average earned and paid interest by the vendor in the intervals [0, T] and [M, T] respectively.

An average total cost of the vendor can be expressed as

$$RC_p = \lim_{x(n)\to 0}\left[\frac{O_c}{T}+\frac{ch_r}{T}\left[(1-x(n))\emptyset_n T-\frac{(\alpha - s\beta)\,T^2}{2}\right]+ch_r x(n)\emptyset_n+\frac{c\emptyset_n}{T}+\frac{S_c\emptyset_n}{T}-\frac{sI_dDT}{2}+\frac{cI_p}{T}\left[(1-x(n))\emptyset_n(T-M)-\frac{D(T^2-M^2)}{2}+(1-x(n))\emptyset_n(T-M)\right]\right]$$

$$RC_p = \frac{O_c}{T}+(c_o+c_1M)\left[\frac{h_r(\alpha-\beta s)T}{2}+(\alpha-\beta s)\right]-\frac{sI_dDT}{2}+(c_o+c_1M)I_p\left[(\alpha-\beta s)(T-M)-\frac{(\alpha-\beta s)(T^2-M^2)}{2T}\right] \quad (4.1)$$

Supplier's model for perfect quality items

After calculating supplier's average holding cost, setup cost per production run, process cost per setup, production cost, opportunity interest loss (due to providing credit period to the vendor), the supplier's average total cost can be expressed by

$$SC_i = \frac{S_p+NV_d}{NT}+\frac{h_s\emptyset_n}{2}\left[(N+1)-\frac{N\emptyset_n}{PT}\right]+\frac{\emptyset_n c}{T}+\frac{I_0cM\emptyset_n}{T} \quad (4.2)$$

As $x(n) \longrightarrow 0$, $\emptyset_n \longrightarrow DT$, therefore

$$SC_i = \frac{S_p+NV_d}{NT}+\frac{h_sDT}{2}\left[(N+1)-\frac{ND}{P}\right]+DS_p+I_0cMD \quad (4.3)$$

Since an integrated coordinated pricing system requires joint efforts to optimize the cost for all supply chain members, therefore for this integrated supply chain system, the total cost can be expressed by

Theorem 4.1

The cost function satisfies jointly the optimality condition concerning and for an imperfect quality and nonconstant demand of an item.

Proof

Let us compute the Hessian matrix of the cost function concerning and if the determinant of Hessian is positive definite that is if both Eigenvalues of the Hessian matrix are positive then the proof will be completed.

Here on differentiating partially of equation (4.4) concerning and respectively we get

$$\frac{\partial TC_i(T,M)}{\partial T} - \frac{O_c}{T^2}+\frac{(c_o+c_1M)h_r(\alpha-\beta s)}{2}-\frac{sI_d(\alpha-\beta s)}{2}+(c_o+c_1M)I_p\left[(\alpha-\beta s)-\frac{(\alpha-s\beta)}{2}\right]-\frac{S_p+NV_d}{NT^2}+\frac{h_sD(N+1)}{2} \quad (4.5)$$

$$\frac{\partial TC_i(T,M)}{\partial M} = c_1\left[\frac{(\alpha-\beta s)h_r}{2}+(\alpha-\beta s)\right]+c_1I_p\left[(\alpha-\beta s)(T-M)-\frac{(\alpha-\beta s)(T^2-M^2)}{2T}\right]-(c_o+c_1M)\left[(\alpha-\beta s)-\frac{(\alpha-\beta s)M}{2}\right]+I_0c_1M(\alpha-s\beta) \quad (4.6)$$

Here on differentiating partially again of equation (3.9) with respect to and respectively we get

$$\frac{\partial^2 TC_i(T,M)}{\partial T^2} = \frac{2O_c}{T^3}+\frac{S_p+NV_d}{NT^3} \quad (4.7)$$

$$\frac{\partial^2 TC_i(T,M)}{\partial M^2} = c_1I_p\left[\frac{(\alpha-\beta s)M}{T}-(\alpha-\beta s)\right]+c_1\left[\frac{(\alpha-\beta s)M}{2}-(\alpha-\beta s)\right]$$

$$(c_o+c_1M)\frac{(\alpha-\beta s)}{2}+I_0c_1(\alpha-s\beta) \quad (4.8)$$

$$\frac{\partial^2 TC_i(T,M)}{\partial T\,\partial M} = c_1\left[\frac{(\alpha-\beta s)h_r}{2}\right]+c_1I_p\left[\frac{(\alpha-\beta s)}{2}\right] \quad (4.9)$$

At the values T_0 and M_0

$$HM = \begin{bmatrix}\dfrac{\partial^2 TC_i(T,M)}{\partial T^2} & \dfrac{\partial^2 TC_i(T,M)}{\partial T\,\partial M}\\ \dfrac{\partial^2 TC_i(T,M)}{\partial T\,\partial M} & \dfrac{\partial^2 TC_i(T,M)}{\partial M^2}\end{bmatrix}$$

$$Det(HM) = \frac{\partial^2 TC_i(T,M)}{\partial T^2} * \frac{\partial^2 TC_i(T,M)}{\partial M^2} - \left[\frac{\partial^2 TC_i(T,M)}{\partial T\,\partial M}\right]^2 > 0.$$

The profit function will be concave concerning replenishment time and credit period time if We proved it with the help of numerical examples.

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There exists an optimum point (T_0,M_0), where the cost function is minimum.

Proof

At the point (T_0,M_0),at which the equations $\frac{\partial TC_i(T,M)}{\partial T} = 0$ and $\frac{\partial TC_i(T,M)}{\partial M} = 0$must be zero as per the first order condition of classical optimization. On solving these equations we can obtain the optimum point (T_0,M_0), where at that point cost function is minimum.

Numerical examples and sensitivity analysis

The proposed study is investigated here through the examples and theoretical results are obtained by using mathematical software. The values of the different parameters and the results are given as follows:

Numerical example for imperfect quality items

We considered the values of the parameter in appropriate units as follows:

ordering cost O_c=155/order, setup cost S_p=3000/cycle, Base demand ∝=480/item/unit time, Procedure cost V_d=107.28/order, Selling price s=40/unit item, Production rate P=2000/unit time, Holding cost h_r=0.5//unit/unit time, Holding cost h_s=5/unit/unit time, Production cost P_c=10/unit item, Price coefficient β=0.2, Procurement cost c=15.0113/procured amount, Base/fixed amount Q_o=100/cycle, Interest rate I_d=0.03/unit time, Interest rate I_p=0.08/unit time, Interest rate I_o=0.03/unit time, Screening cost s_c=0.5, a=450, g=999, b=1, Screening time t_n=0.5 , No. of repetitions n=15, Optimum values of decision variable are T= 0.213318, M= 0.141833.

Output results are No. of replenishment N=5/unit year, Initial batch size $\emptyset_n$=165.6531, TR_1=13601.78, TS_1=14427.37, TS_1= 28029.15. Also through this example the all Eigen values of Hessian matrix

$$HM=\begin{bmatrix}\frac{\partial^2 TC_i(T,M)}{\partial T^2} & \frac{\partial^2 TC_i(T,M)}{\partial T\,\partial M}\\ \frac{\partial^2 TC_i(T,M)}{\partial T\,\partial M} & \frac{\partial^2 TC_i(T,M)}{\partial M^2}\end{bmatrix}$$

are 54039.53 and 107656118 positive, hence solution will be optimal.

Numerical example for perfect quality items

We considered the values of the parameter in appropriate units as follows:

ordering cost O_c=155/order, setup cost S_p=3000/cycle, Base demand ∝=480/item/unit time, Procedure cost V_d=107.28/order, Selling price s=40/unit item, Production rate P=2000/unit time, Holding cost h_r=0.5/unit/unit time, Holding cost h_s=5/unit/unit time, Production cost P_c=10/unit item, Price coefficient β=0.2, Procurement cost c=15.0113/procured amount, Base/fixed amount Q_o=100/cycle, Interest rate I_d=0.03/unit time, Interest rate I_p=0.08/unit time, Interest rate I_o=0.03/unit time, Screening cost s_c=0.5, a=450, g=999, b=1, Screening time t_n=0.5 , No. of repetitions n=15, Optimum values of decision variable are T= 0.23165, M= 0.104525.

Output results are No. of replenishment N=4/unit year, Initial batch size $\emptyset_n$=109.3387, TR_2=4607.661, TS_1=14427.37, TS_2= 9374.668 , TS_2= 13982.33 . Also through this example the all Eigen values of Hessian matrix

$$HM=\begin{bmatrix}\frac{\partial^2 TC_p(T,M)}{\partial T^2} & \frac{\partial^2 TC_p(T,M)}{\partial T\,\partial M}\\ \frac{\partial^2 TC_p(T,M)}{\partial T\,\partial M} & \frac{\partial^2 TC_p(T,M)}{\partial M^2}\end{bmatrix}$$

are 25632.79 and 1 46286290.3 positive, hence solution will be optimal.

Sensitivity analysis

In this section we organized an analysis to estimate the sensitivity of key parameters, availing valuable insights for decision authority to make effective marketing strategies. Data Table 1 shows the behavior of the cost function concerning several screening repetitions and price-based demand at (base/fixed procurement cost). Data Table 2 shows the behavior of the cost function concerning several screening repetitions and price-based demand at (base/fixed procurement cost). The data table 3 shows the behavior of the cost function concerning the number of screening repetitions and price-based demand at (base/fixed procurement cost). Data Table 4 shows the behavior of the cost function concerning price-based demand at (base/fixed procurement cost). The observations based on Tables 1, 2, 3, and 4 are as follows:

- Production with perfect quality items is more admissible than production with imperfect quality items due to the total optimum cost is minimal for production with perfect quality rather than production with imperfect quality items. However, a comparison of the batch size for each cycle shows adverse results. Furthermore, The production system requires preventive and scheduled maintenance to keep it running smoothly and efficiently so that the system produces high quality and efficient and perfect production, these add the additional expenditure in the cost function.
- The cost function of the system exhibits high sensitivity to the demand parameter. Moderately sensitive aspects include the parameter and selling price. This reveals that increasing demand is positively correlated with an optimal cost function, validating the robustness of the model.
- A higher rate of defective items' production rate in the system increases the number of replenishments and the number of repetitions for the learning process concerning learning efficiency .
- The cost function of manufacturer and vendor demonstrates moderate sensitivity to parameters , and , viz it is shows moderated positively correlated to the parameters , and .
- For any fixed values of and cycle time , batch size , vendor's cost function, and credit period demonstrates moderate sensitivity to parameters , viz.,batch size , vendor's cost function , and show a moderately negative correlation to the parameters .Whereas the manufacturer's cost function shows moderately positively correlated to Furthermore, fluctuates with

Conclusion Suggestions and Extension

The principal aim of this study is to prepare an integrated production model based on credit period and screening procedure for imperfect quality items which consists of a single manufacturer and a retailer. Furthermore, it is also tried to optimize, in a single unit year how many number replenishments are required under the credit period based on procurement cost and price-affected marketing environment. The cost function of this integrated system has been optimized under the decision variables and and analyzed through data tablesconcerning key parameters like , , and

Suggestions

In the cost function of this system, a major expenditure is on the screening process to separate the good and defective items, therefore, the decision authorities/management will have to apply preventive and scheduled maintenance to keep it running smoothly

c	**T**	**M**	**N**	**x(n)**		**n**			
15.0131	0.209663	0.1312	N	0.392187	164.1943	5	13744.17	14524.25	28268.43
15.0111	0.230056	0.111445	N	0.214732	139.4512	7	10705.89	11881.7	22587.59
15.0098	0.245565	0.098325	N	0.049439	122.9684	9	8869.241	10339.37	19208.61
15.0096	0.249094	0.095531	N	0.007392	119.4517	11	8496.537	10031.71	18528.24
15.0095	0.249616	0.095125	N	0.001015	118.938	13	8442.651	9987.375	18430.03
15.0095	0.249688	0.095068	N	0.000138	118.8677	15	8435.291	9981.322	18416.61
15.0132	0.211372	0.132242	N	0.392187	164.8375	5	13675.57	14474.34	28149.91
15.0112	0.232219	0.112468	N	0.214732	140.1709	7	10646.93	11843.61	22490.54
15.0099	0.248127	0.09933	N	0.049439	123.7292	9	8815.425	10307.97	19123.39
15.0097	0.251753	0.09653	N	0.007392	120.2198	11	8443.679	10001.61	18445.29
15.0096	0.25229	0.09612	N	0.001015	119.7071	13	8389.929	9957.464	18347.39
15.0096	0.252364	0.096063	N	0.000138	119.6369	15	8382.587	9951.437	18334.02
15.0133	0.213113	0.133307	N	0.392187	165.494	5	13606.91	14424.48	28031.38
15.0114	0.234429	0.113518	N	0.214732	140.9081	7	10587.85	11805.59	22393.43
15.0100	0.250752	0.100355	N	0.049439	124.5108	9	8761.444	10276.61	19038.06
15.0098	0.254481	0.09755	N	0.007392	121.0095	11	8390.641	9971.559	18362.2
15.0097	0.255033	0.097144	N	0.001015	120.498	13	8337.023	9927.604	18264.63
15.0097	0.255109	0.097085	N	0.000138	120.4279	15	8329.7	9921.602	18251.3
15.0134	0.214887	0.13439	N	0.392187	166.164	5	13538.18	14374.67	27912.85
15.0115	0.236689	0.114585	N	0.214732	141.6634	7	10528.66	11767.61	22296.26
15.0101	0.253444	0.10141	N	0.049439	125.3143	9	8707.29	10245.31	18952.6
15.0099	0.257279	0.098599	N	0.007392	121.8219	11	8337.415	9941.563	18278.98
15.0098	0.257848	0.09819	N	0.001015	121.3116	13	8283.928	9897.795	18181.72
15.0098	0.257926	0.098135	N	0.000138	121.2418	15	8276.622	9891.82	18168.44
15.0135	0.216693	0.135495	N	0.392187	166.8481	5	13469.39	14324.92	27794.32
15.0116	0.238999	0.115678	N	0.214732	142.4376	7	10469.34	11729.68	22199.03
15.0102	0.256206	0.10249	N	0.049439	126.1407	9	8652.957	10214.06	18867.02
15.0100	0.260153	0.09968	N	0.007392	122.6581	11	8283.992	9911.623	18195.61
15.0099	0.260738	0.099265	N	0.001015	122.1492	13	8230.635	9868.041	18098.68
15.0099	0.260818	0.09921	N	0.000138	122.0795	15	8223.346	9862.091	18085.44

Table 2: The behavior of cost function with respect to screening and price based demand.

c	**T**	**M**	**N**	**x(n)**		**n**			
15.0131	0.209663	0.1312	N	0.392187	164.1943	5	13744.17	14524.25	28268.43
15.0111	0.230056	0.111445	N	0.214732	139.4512	7	10705.89	11881.7	22587.59
15.0098	0.245565	0.098325	N	0.049439	122.9684	9	8869.241	10339.37	19208.61
15.0096	0.249094	0.095531	N	0.007392	119.4517	11	8496.537	10031.71	18528.24
15.0095	0.249616	0.095125	N	0.001015	118.938	13	8442.651	9987.375	18430.03
15.0095	0.249688	0.095068	N	0.000138	118.8677	15	8435.291	9981.322	18416.61

15.0132	0.211372	0.132242	N	0.392187	164.8375	5	13675.57	14474.34	28149.91
15.0112	0.232219	0.112468	N	0.214732	140.1709	7	10646.93	11843.61	22490.54
15.0099	0.248127	0.09933	N	0.049439	123.7292	9	8815.425	10307.97	19123.39
15.0097	0.251753	0.09653	N	0.007392	120.2198	11	8443.679	10001.61	18445.29
15.0096	0.25229	0.09612	N	0.001015	119.7071	13	8389.929	9957.464	18347.39
15.0096	0.252364	0.096063	N	0.000138	119.6369	15	8382.587	9951.437	18334.02
15.0133	0.213113	0.133307	N	0.392187	165.494	5	13606.91	14424.48	28031.38
15.0114	0.234429	0.113518	N	0.214732	140.9081	7	10587.85	11805.59	22393.43
15.0100	0.250752	0.100355	N	0.049439	124.5108	9	8761.444	10276.61	19038.06
15.0098	0.254481	0.09755	N	0.007392	121.0095	11	8390.641	9971.559	18362.2
15.0097	0.255033	0.097144	N	0.001015	120.498	13	8337.023	9927.604	18264.63
15.0097	0.255109	0.097085	N	0.000138	120.4279	15	8329.7	9921.602	18251.3
15.0134	0.214887	0.13439	N	0.392187	166.164	5	13538.18	14374.67	27912.85
15.0115	0.236689	0.114585	N	0.214732	141.6634	7	10528.66	11767.61	22296.26
15.0101	0.253444	0.10141	N	0.049439	125.3143	9	8707.29	10245.31	18952.6
15.0099	0.257279	0.098599	N	0.007392	121.8219	11	8337.415	9941.563	18278.98
15.0098	0.257848	0.09819	N	0.001015	121.3116	13	8283.928	9897.795	18181.72
15.0098	0.257926	0.098135	N	0.000138	121.2418	15	8276.622	9891.82	18168.44
15.0135	0.216693	0.135495	N	0.392187	166.8481	5	13469.39	14324.92	27794.32
15.0116	0.238999	0.115678	N	0.214732	142.4376	7	10469.34	11729.68	22199.03
15.0102	0.256206	0.10249	N	0.049439	126.1407	9	8652.957	10214.06	18867.02
15.0100	0.260153	0.09968	N	0.007392	122.6581	11	8283.992	9911.623	18195.61
15.0099	0.260738	0.099265	N	0.001015	122.1492	13	8230.635	9868.041	18098.68
15.0099	0.260818	0.09921	N	0.000138	122.0795	15	8223.346	9862.091	18085.44

Table 2: The behavior of cost function with respect to screening and price based demand.

c	**T**	**M**	**N**	**x(n)**		**n**			
15.0147	0.209479	0.12285	N	0.392187	164.0506	5	13749.11	14521.4	28270.51
15.0125	0.229909	0.104375	N	0.214732	139.3625	7	10709.23	11879.84	22589.08
15.0111	0.245443	0.09211	N	0.049439	122.9071	9	8871.729	10338.04	19209.77
15.0107	0.248977	0.0895	N	0.007392	119.3955	11	8498.863	10030.47	18529.33
15.0107	0.2495	0.08912	N	0.001015	118.8825	13	8444.954	9986.15	18431.1
15.0107	0.249571	0.089065	N	0.000138	118.8123	15	8437.591	9980.099	18417.69
15.0149	0.211185	0.123822	N	0.392187	164.6916	5	13680.53	14471.47	28152
15.0126	0.232069	0.105325	N	0.214732	140.0803	7	10650.29	11841.75	22492.04
15.0112	0.248001	0.093045	N	0.049439	123.6664	9	8817.935	10306.62	19124.56
15.0109	0.251633	0.09043	N	0.007392	120.1621	11	8446.026	10000.36	18446.38
15.0108	0.25217	0.090043	N	0.001015	119.6501	13	8392.254	9956.227	18348.48
15.0108	0.252244	0.089994	N	0.000138	119.5801	15	8384.908	9950.203	18335.11

15.0150	0.212922	0.12481	N	0.392187	165.3456	5	13611.89	14421.59	28033.48
15.0128	0.234275	0.1063	N	0.214732	140.8156	7	10591.24	11803.7	22394.94
15.0113	0.250623	0.094	N	0.049439	124.4466	9	8763.976	10275.25	19039.23
15.0110	0.254357	0.09138	N	0.007392	120.9504	11	8393.01	9970.298	18363.31
15.0109	0.254909	0.09099	N	0.001015	120.4395	13	8339.371	9926.354	18265.73
15.0109	0.254985	0.090945	N	0.000138	120.3697	15	8332.043	9920.356	18252.4
15.0151	0.214691	0.125815	N	0.392187	166.013	5	13543.2	14371.77	27914.96
15.0129	0.236531	0.107294	N	0.214732	141.569	7	10532.07	11765.71	22297.78
15.0114	0.253311	0.09498	N	0.049439	125.2485	9	8709.846	10243.94	18953.78
15.0111	0.257152	0.092355	N	0.007392	121.7613	11	8339.807	9940.289	18280.1
15.0110	0.257721	0.09197	N	0.001015	121.2517	13	8286.297	9896.534	18182.83
15.0110	0.257799	0.09192	N	0.000138	121.182	15	8278.988	9890.561	18169.55
15.0152	0.216494	0.126841	N	0.392187	166.6945	5	13474.43	14322	27796.43
15.0130	0.238837	0.10831	N	0.214732	142.3412	7	10472.78	11727.77	22200.55
15.0115	0.256069	0.09598	N	0.049439	126.0731	9	8655.538	10212.68	18868.21
15.0112	0.260021	0.093356	N	0.007392	122.5959	11	8286.409	9910.335	18196.74
15.0112	0.260606	0.09297	N	0.001015	122.0877	13	8233.028	9866.768	18099.8
15.0112	0.260687	0.09292	N	0.000138	122.0182	15	8225.735	9860.821	18086.56

Table 3: The behavior of cost function with respect to screening and price based demand.

s	T	M	c	N				
50%	0.227389	0.102618	15.0082	N	108.2372	4678.72	9425.527	14104.25
25%	0.229497	0.103561	15.0083	N	108.7815	4643.258	9400.091	14043.35
No change	0.23165	0.104525	15.0084	N	109.3387	4607.661	9374.668	13982.33
25%	0.23385	0.10551	15.0084	N	109.9095	4571.925	9349.259	13921.18
50%	0.236099	0.10651	15.0085	N	110.4942	4536.048	9323.864	13859.91
50%	0.227378	0.099849	15.0100	N	108.2321	4680.366	9424.945	14105.31
25%	0.229486	0.10076	15.0101	N	108.7762	4644.916	9399.504	14044.42
No change	0.231638	0.101695	15.0102	N	109.3333	4609.329	9374.078	13983.41
25%	0.233838	0.10265	15.0103	N	109.9039	4573.604	9348.665	13922.27
50%	0.236087	0.103625	15.0104	N	110.4885	4537.735	9323.268	13861
50%	0.227368	108.2274	15.0116	N	108.2274	4681.986	9424.362	14106.35
25%	0.229475	108.7714	15.0118	N	108.7714	4646.544	9398.918	14045.46
No change	0.231628	109.3283	15.0119	N	109.3283	4610.967	9373.487	13984.45
25%	0.233827	109.8987	15.0120	N	109.8987	4575.252	9348.071	13923.32
50%	0.236075	110.4832	15.0121	N	110.4832	4539.393	9322.669	13862.06

Table 4: The behavior of cost function with respect to price based demand.

and efficiently so that the system produces high quality, efficient and perfect production.

Decision authority/Management will have to appoint efficient employees for screening units to reduce the number of repetitions and learning periods.

Decision authority/Management should have reduced the values of,,, and increased the values of , and parameters so that the cost function becomes minimum.

Extension

- One can extend this study by incorporating a rework process on defective items to convert perfect-quality items.
- One can extend this study allowing with shortage of deteriorating items and applying learning effects on the screening process to separate the deteriorating items.
- One can extend this study by taking speed screening as a decision variable which can be jointly with the order quantity.
- One can also extend this study by incorporating applying limitations/constraints on the credit period.

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