Acta Scientific COMPUTER SCIENCES

Volume 5 Issue 3 March 2023

Review Article

Local Fractional Shehu Transform and its Application to Solve Linear Local Fractional Differential Equations

Djeloul Ziane[1], Rachid Belgacem[2]* and Ahmed Bokhari[2]

[1]*Laboratory of mathematics and its applications (LAMAP), University of Oran1 Ahmed Ben Bella, Oran, 31000, Algeria*

[2]*Department of Mathematics, Faculty of Exact Sciences and Informatics, Hassiba Benbouali University of Chlef, Algeria*

***Corresponding Author:** Rachid Belgacem, Department of Mathematics, Faculty of Exact Sciences and Informatics, Hassiba Benbouali University of Chlef, Algeria.

Received: January 02, 2023

Published: February 07, 2023

© All rights are reserved by **Rachid Belgacem., *et al.***

Abstract

In this paper we shall present a method for solving local fractional differential equations. This method is based on the combination of the Shehu transform and the local fractional derivative (we can call it the local fractional Shehu transform), where we have presented some important results and properties. We concluded this work by providing illustrative examples, through which we focused on solving some linear local fractional differential equations in order to obtain non- differential analytical solutions. From the results obtained, it can be concluded that this suggested method is effective when applied this type of local fractional partial deferential equations.

Keywords: Local Fractional Calculus; Local Fractional Laplace Transform; Shehu Transform Method; Local Fractional Differential Equations

Introduction

Transformations defined by integrals play an important role in the resolution of ordinary differential equations, partial differential equations and in the resolution of integral differential equations with integer order or fractional order. It also intervenes in mathematical physics, probability calculus, automatics, engineering, etc...

Among the most famous transformations, we find the Laplace transform method [14], the Fourier transform method [23], the Mellin transform method [24], and there are other transformations that have appeared in the recent period, we cite for example, the Sumudu transform method [25], the Natural transform method [26], the Ezaki transform method [27], the Aboodh transform method [28], the Shehu transform method [10] and others.

The work in this paper is based on the Shehu transformation method, as this transformation that have appeared recently and is defined by an integral due to its association with the well-known Laplace transform [14]. It was recently discovered by Maitama and Zhao in 2019 [10], and has been used by many researchers in the field of mathematics to solve both ordinary and partial differential equations of integer order ([1,2,10,20-22]), both ordinary and partial differential equations of fractional order ([6,9,11,12]), integro-differential equation [3], and integral equation [4].

The main objective of the present work is to combine the local fractional derivative with the Shehu transform in order to resolve linear differential equations with local fractional derivative. We supported this work with illustrative examples showing how to apply this transform with the use of local fractional derivative.

The structures of the paper are as follows. In Section 2 some basic definitions and properties of the local fractional calculus and local fractional Laplace transform method. In section 3, we present some important results. In section 4, we apply the local fractional Shehu transform method (LFST) to solve the proposed example. Finally, we conclude with the conclusion.

Basic of local fractional calculus

In this section, we give the concepts of local fractional derivatives and integrals and polynomial functions on cantor sets.

Definition 1

([13,15,16]) If there exists the relation

$$\left|\Phi(\tau)-\Phi(\tau_0)\right| < \gamma^{\sigma},$$

With $|\tau-\tau_0|<\delta$, for $\delta>0$, and $\delta\in\mathrm{R}$. Now $\Phi(\upsilon)$ is called local fractional continuous at $\tau=\tau_0$, denote.

By $\lim_{\tau\to\tau_0}\Phi(\tau)=\Phi(\tau_0)$ Then $\Phi(\tau)$ is called local fractional continuous on the interval (a,b) , denoted by $\Phi(\tau)\in C_{\sigma}(a,b)$

Definition 2

([13,15,16]) Setting $\Phi(\tau)\in C_{\sigma}(a,b)$, the local fractional derivative of $\Phi(\tau)$ of order σ at $\tau=\tau_0$ is defined as

$$\Phi^{(\sigma)}(\tau)=\left.\frac{d^{\sigma}\Phi(\tau)}{d\tau^{\sigma}}\right|_{\tau=\tau_0}=\frac{\Delta^{\sigma}(\Phi(\tau)-\Phi(\tau_0))}{(\tau-\tau_0)^{\sigma}},$$

Where

$$\Delta^{\sigma}(\Phi(\tau)-\Phi(\tau_0)) \cong \Gamma(1+\sigma)[(\Phi(\tau)-\Phi(\tau_0))].$$

Definition 3

([13,15,16]) The local fractional integral of $\Phi(\tau)$ of order σ in the interval $[a,b]$ is defined as

$$\begin{aligned}{}_aI_b^{(\sigma)}\Phi(\tau)&=\frac{1}{\Gamma(1+\sigma)}\int_a^b\Phi(\zeta)(d\zeta)^{\sigma},\\ &=\frac{1}{\Gamma(1+\sigma)}\lim_{\Delta\zeta\to0}\sum_{i=0}^{N-1}f(\zeta_i)(\Delta\zeta_i)^{\sigma},\end{aligned}$$

Where $\Delta\zeta_i=\zeta_{i+1}-\zeta_i$, $\Delta\zeta=\max\{\Delta\zeta_0,\Delta\zeta_1,\Delta\zeta_2,\cdots\}$ and $[\zeta_i,\zeta_{i+1}]$, $\zeta_0=a$, $\zeta_N=b$, is a partition of the interval $[a,b]$

Definition 4

([7,13,19]) The local fractional Laplace transform of $\Phi(\tau)$ of order σ is defined as

$$L_{\sigma}\{\Phi(\tau)\}= F_{\sigma}(s)=\frac{1}{\Gamma(1+\sigma)}\int_0^{\infty}E_{\sigma}(-s^{\sigma}\tau^{\sigma})\Phi(\tau)(d\tau)^{\sigma}.$$

If $L_{\sigma}\{\Phi(\tau)\}=F_{\sigma}(s)$ the inverse formula is given as follows

$$\Phi(\tau)= L_{\sigma}^{-1}\{F_{\sigma}(s)\}=\frac{1}{(2\pi)^{\sigma}}\int_{\beta-i\infty}^{\beta+i\infty}E_{\sigma}(s^{\sigma}\tau^{\sigma})F_{\sigma}(s)(ds)^{\sigma},$$

Where $\Phi(\tau)$ s local fractional continuous, $s^{\sigma}=\beta^{\sigma}+i^{\sigma}\infty^{\sigma}$, and $\mathrm{Re}(s)=\beta>0$.

Theorem 1

([15]) If $L_{\sigma}\{\Phi(\tau)\}=F_{\sigma}(s)$ then one has

$$L_{\sigma}\{\Phi^{(\sigma)}(\tau)\}=s^{\sigma}L_{\sigma}\{\Phi(\tau)\}-\Phi(0)$$

Proof

See [15].

Theorem 2

([15]) If $L_{\sigma}\{\Phi(\tau)\}=F_{\sigma}(s)$ then one has

$$L_{\sigma}\{{}_0I_{\tau}^{\sigma}\Phi(\tau)\}=\frac{1}{s^{\sigma}}L_{\sigma}\{\Phi(\tau)\}.$$

Proof

See [15].

Theorem 3

([15]) If $L_{\sigma}\{\Phi(\tau)\}= F_{\sigma}(s)$ and $L_{\sigma}\{\Psi(\tau)\}=\Omega_{\sigma}(s)$, then one has

$$L_{\sigma}\{(\Phi(\tau)*\Psi(\tau))_{\sigma}\}=F_{\sigma}(s)\Omega_{\sigma}(s)$$

Where

$$(\Phi(\tau)*\Psi(\tau))_{\sigma}=\frac{1}{\Gamma(1+\sigma)}\int_0^{\infty}\Phi(x)\Psi(\tau-x)(dx)^{\sigma}.$$

Proof

See [15].

Theorem 4

([17]) Suppose that $\Phi(\tau)\in C_{\sigma}[a,b]$ then there is a function

$$\Pi(\tau)={}_aI_{\tau}^{(\sigma)}\Phi(\tau)$$

The function has its derivative with respect to $(d\tau)^{\sigma}$,

$$\frac{d^{\sigma}\Pi(\tau)}{(d\tau)^{\sigma}}=\Phi(\tau),\quad a\le\tau\le b.$$

Proof

See [17].

Main Result

In this section, we present the local fractional Shehu transformation (LFST) method and some properties are discussed.

If there is a new transform operator ${}^{LF}S_{\sigma}$: $\Phi(\tau)\to\Omega_{\sigma}(\upsilon,\nu)$, namely,

$${}^{LF}S_{\sigma}\{\Phi(\tau)\}={}^{LF}S_{\sigma}\left\{\sum_{k=0}^{\infty}a_k\frac{\tau^{k\sigma}}{\Gamma(1+k\sigma)}\right\}=\sum_{k=0}^{\infty}a_k\left(\frac{\upsilon}{\nu}\right)^{(k+1)\sigma}.$$

For example if $\Phi(\tau) = E_\sigma(i^\sigma \tau^\sigma)$ we obtain

$$ {}^{E}S_\sigma\left\{E_\sigma(i^\sigma\tau^\sigma)\right\} = {}^{E}S_\sigma\left\{\sum_{k=0}^{\infty}\frac{i^{k\sigma}\tau^{k\sigma}}{\Gamma(1+k\sigma)}\right\}, $$
$$ = \sum_{k=0}^{\infty} i^{k\sigma}\left(\frac{\nu}{\upsilon}\right)^{(k+1)\sigma}, $$

And if $\Phi(\tau) = \frac{\tau^\sigma}{\Gamma(1+\sigma)}$, we get

$$ {}^{E}S_\sigma\left\{\frac{\tau^\sigma}{\Gamma(1+\sigma)}\right\} = \frac{\nu^{2\sigma}}{\upsilon^{2\sigma}}. $$

As the generalized result, we give the following definition.

Definition 5

The local fractional Shehu transform of $\Phi(\tau)$ of order σ is defined as

$$ {}^{E}S_\sigma\{\Phi(\tau)\} = \Omega_\sigma(\upsilon,\nu) = \frac{1}{\Gamma(1+\sigma)}\int_0^\infty E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)\Phi(\tau)(d\tau)^\sigma,\ 0<\sigma\le 1. $$

The inverse transformation can be obtained as follows

$$ {}^{E}S^{\,1}\{\quad(\ ,\ )\}\qquad(\ ) $$

Theorem 5

(linearity) If ${}^{E}S_\sigma\{\Phi(\tau)\} = \Omega_\sigma(\upsilon,\nu)$ and ${}^{E}S_\sigma\{\Psi(\tau)\} = \Pi_\sigma(\upsilon,\nu)$ then one has

$$ {}^{E}S_\sigma\{\lambda\Phi(\tau)+\mu\Psi(\tau)\} = \lambda\Omega_\sigma(\upsilon,\nu)+\mu\Pi_\sigma(\upsilon,\nu) $$

Where λ and μ are constant.

Proof

Using definition 5, we obtain

$$ {}^{E}S_\sigma\{\lambda\Phi(\tau)+\mu\Psi(\tau)\} = \frac{1}{\Gamma(1+\sigma)}\int_0^\infty E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)\{\lambda\Phi(\tau)+\mu\Psi(\tau)\}(d\tau)^\sigma, $$
$$ = \frac{1}{\Gamma(1+\sigma)}\int_0^\infty\left[E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)(\lambda\Phi(\tau)) + E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)(\mu\Psi(\tau))\right](d\tau)^\sigma, $$
$$ = \frac{\lambda}{\Gamma(1+\sigma)}\int_0^\infty E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)\Phi(\tau)(d\tau)^\sigma + \frac{\mu}{\Gamma(1+\sigma)}\int_0^\infty E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)\Psi(\tau)(d\tau)^\sigma, $$
$$ = \lambda\Omega_\sigma(\upsilon,\nu)+\mu\Pi_\sigma(\upsilon,\nu) $$

This ends the proof.

Theorem 6

(local fractional Shehu-Laplace and Laplace-Shehu duality)

If $L_\sigma\{\Phi(\tau)\} = F_\sigma(s)$ and ${}^{E}S_\sigma\{\Phi(\tau)\} = \Omega_\sigma(\upsilon,\nu)$ then one has

$$ {}^{E}S_\sigma\{\Phi(\tau)\} = F_\sigma\left(\frac{\upsilon}{\nu}\right) $$
$$ L_\sigma\{\Phi(\tau)\} = \Omega_\sigma(s\nu,\nu) $$

Proof

First, we proof the formula (1).

Using the definition 5, we find

$$ {}^{E}S_\sigma\{\Phi(\tau)\} = \frac{1}{\Gamma(1+\sigma)}\int_0^\infty E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)\Phi(\tau)(d\tau)^\sigma, $$
$$ = \frac{1}{\Gamma(1+\eta)}\int_0^\infty E_\sigma\left(-\left(\frac{\upsilon}{\nu}\right)^\sigma\tau^\sigma\right)\Phi(\tau)(d\tau)^\sigma, $$
$$ = F_\sigma\left(\frac{\upsilon}{\nu}\right) $$

Now we prove the second formula, we have

$$ \Omega_\sigma(\upsilon,\nu) = F_\sigma\left(\frac{\upsilon}{\nu}\right) $$

By substituting $s = \frac{\upsilon}{\nu}$, we obtain

$$ F_\sigma(s) = \Omega_\sigma(s\nu,\nu) $$

Therefore, we get

$$ L_\sigma\{\Phi(\tau)\} = \Omega_\sigma(s\nu,\nu) $$

This and the proof.

Theorem 7

(local fractional Shehu transform of local fractional derivative)

If ${}^{E}S_\sigma\{\Phi(\tau)\} = \Omega_\sigma(\upsilon,\nu)$ then one has

$$ {}^{E}S_\sigma\{D_{0+}^{\sigma}\Phi(\tau)\} = \frac{\upsilon^\sigma}{\nu^\sigma}\Omega_\sigma(\upsilon,\nu) - \Phi(0)\quad 0<\sigma\le 1, $$

And

$$ {}^{E}S_\sigma\{D_{0+}^{n\sigma}\Phi(\tau)\} = \frac{\upsilon^{n\sigma}}{\nu^{n\sigma}}\Omega_\sigma(\upsilon,\nu) - \sum_{k=0}^{n-1}\left(\frac{\upsilon}{\nu}\right)^{(n-k-1)\sigma}\Phi^{(k\sigma)}(0)\quad 0<\sigma\le 1. $$

Proof

We proof the first formula. Using the definition 5 and the integration by parts [Guy], we get the following

$$^{E}S_\sigma\{\Phi^{(\sigma)}(\tau)\}=\frac{1}{\Gamma(1+\sigma)}\int_0^\infty E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)\Phi^{(\sigma)}(\tau)(d\tau)^\sigma,$$
$$=\frac{1}{\Gamma(1+\sigma)}\left(\left[-\Gamma(1+\sigma)\Phi(0)\right]+\frac{\upsilon^\sigma}{\nu^\sigma}\lim_{x\to\infty}\int_0^x E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)\Phi(\tau)(d\tau)^\sigma\right),$$
$$=-\Phi(0)+\frac{\upsilon^\sigma}{\nu^\sigma}\left(\frac{1}{\Gamma(1+\sigma)}\int_0^\infty E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)\Phi(\tau)(d\tau)^\sigma\right),$$
$$=\frac{\upsilon^\sigma}{\nu^\sigma}\Omega_\sigma(\upsilon,\nu)-\Phi(0)$$

To demonstrate the validity of the second formula, we use mathematical induction.

If $n=1$, we obtain

$$^{E}S_\sigma\{\Phi^{(\sigma)}(\tau)\}=\frac{\upsilon^\sigma}{\nu^\sigma}\Omega_\sigma(\upsilon,\nu)-\Phi(0)$$

So, according to the first formula of Theorem 7, we note that the formula holds when $n=1$.

Assume inductively that the formula holds for n, so that

$$^{E}S_\sigma\{D_{0+}^{n\sigma}\Phi(\tau)\}=\frac{\upsilon^{n\sigma}}{\nu^{n\sigma}}\Omega_\sigma(\upsilon,\nu)-\sum_{k=0}^{n-1}\left(\frac{\upsilon}{\nu}\right)^{(n-k-1)\sigma}\Phi^{(k\sigma)}(0)$$

It remains to show that (A32) is true for $n+1$. Let $D_{0+}^{n\sigma}\Phi(\tau)=\Psi(\tau)$ (where $^{E}S_\sigma\{\Psi(\tau)\}=\Pi_\eta(\upsilon,\nu)$), we have

$$^{E}S_\sigma\left[D_{0+}^{(n+1)\sigma}\Phi(\tau)\right]={}^{E}S_\sigma\left[D_{0+}^{\sigma}\Psi(\tau)\right]=\frac{\upsilon^\sigma}{\nu^\sigma}\Pi_\eta(\upsilon,\nu)-\Psi(0),$$
$$=\frac{\upsilon^\sigma}{\nu^\sigma}\left[\frac{\upsilon^{n\sigma}}{\nu^{n\sigma}}\Omega_\sigma(\upsilon,\nu)-\sum_{k=0}^{n-1}\left(\frac{\upsilon}{\nu}\right)^{(n-k-1)\sigma}\Phi^{(k\sigma)}(0)\right]-\Psi(0),$$
$$=\frac{\upsilon^{(n+1)\sigma}}{\nu^{(n+1)\sigma}}\Omega_\sigma(\upsilon,\nu)-\sum_{k=0}^{n-1}\left(\frac{\upsilon}{\nu}\right)^{(n-k)\sigma}\Phi^{(k\sigma)}(0)-D_{0+}^{n\sigma}\Phi(0)$$
$$=\frac{\upsilon^{(n+1)\sigma}}{\nu^{(n+1)\sigma}}\Omega_\sigma(\upsilon,\nu)-\sum_{k=0}^{n}\left(\frac{\upsilon}{\nu}\right)^{(n-k)\sigma}\Phi^{(k\sigma)}(0)$$

Therefore the formula is true for $n+1$.

Thus by the principle of mathematical induction, for all $n\geq 1$, the second formula of this theorem holds.

Theorem 8

(Local fractional Shehu transform of local fractional integral)

If $^{E}S_\sigma\{\Phi(\tau)\}=\Omega_\sigma(\upsilon,\nu)$ then one has

$$^{E}S_\sigma\left\{{}_0I_\tau^{(\sigma)}\Phi(\tau)\right\}=\frac{\nu^\sigma}{\upsilon^\sigma}\Omega_\sigma(\upsilon,\nu)$$

Proof

Let $P(\tau)={}_0I_\tau^{(\sigma)}\Phi(\tau)$ According to the (theorem 3.2.9, [XJ]), we get

$$D_0^\sigma P(\tau)\qquad(\upsilon)$$

And

$$P(0)=0.$$

Taking the local fractional Shehu transform on both sides of this equation, we have

$$^{E}S_\sigma\{D_{0+}^{\sigma}P(\tau)\}={}^{E}SH_\sigma\{\Phi(\tau)\}.$$

Which give

$$\frac{\upsilon^\sigma}{\nu^\sigma}\left({}^{LF}S_\sigma\{P(\tau)\}\right)=\Omega_\sigma(\upsilon,\nu),$$

Because $P(0)=0$, and $^{E}S_\sigma\{\Phi(\tau)\}=\Omega_\sigma(\upsilon,\nu)$

Thus we get

$$^{E}S_\sigma\left\{{}_0I_\tau^{(\sigma)}\Phi(\tau)\right\}=\frac{\nu^\sigma}{\upsilon^\sigma}\Omega_\sigma(\upsilon,\nu)$$

Theorem 9

(local fractional convolution)

If $^{E}S_\sigma\{\Phi(\tau)\}=\Omega_\sigma(\upsilon,\nu)$ and $^{E}S_\sigma\{\Psi(\tau)\}=\Pi_\sigma(\upsilon,\nu)$, then one has

$$^{E}S_\sigma\{(\Phi(\tau)*\Psi(\tau))_\sigma\}=\Omega_\sigma(\upsilon,\nu)\Pi_\sigma(\upsilon,\nu)$$

Where

$$(\Phi(\tau)*\Psi(\tau))_\sigma=\frac{1}{\Gamma(1+\sigma)}\int_0^\tau\Phi(t)\Psi(\tau-t)(dt)^\sigma.$$

Proof

Using the definition 5, gives

$$^{E}S_\sigma\{(\Phi(\tau)*\Psi(\tau))_\sigma\}=\frac{1}{\Gamma^2(1+\sigma)}\int_0^\infty E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)(d\tau)^\sigma\int_0^\tau\Phi(t)\Psi(\tau-t)(dt)^\sigma,$$
$$=\frac{1}{\Gamma^2(1+\sigma)}\int_0^\infty E_\sigma\left(-\frac{\upsilon^\sigma(\tau-t)^\sigma}{\nu^\sigma}\right)E_\sigma\left(-\frac{\upsilon^\sigma t^\sigma}{\nu^\sigma}\right)(d\tau)^\sigma\int_0^\tau\Phi(t)\Psi(\tau-t)(dt)^\sigma.$$

We make the change $\omega=\tau-x$ and $\rho=x$, we get

$$^{E}{}_{\sigma}\{(\Phi(\tau)*\Psi(\tau))_{\sigma}\}$$
$$=\frac{1}{\Gamma^{2}(1+\sigma)}\int_{0}^{\infty}E_{\sigma}\left(-\frac{\upsilon^{\sigma}\rho^{\sigma}}{\nu^{\sigma}}\right)\Phi(\rho)(d\rho)^{\sigma}\int_{0}^{\infty}E_{\sigma}\left(-\frac{\upsilon^{\sigma}\omega^{\sigma}}{\nu^{\sigma}}\right)\Psi(\omega)(d\omega)^{\sigma},$$
$$=\,^{E}S_{\sigma}\{\Phi(\tau)\}^{E}S_{\sigma}\{\Psi(\tau)\}.$$

This and the proof.

Shehu transform of some special functions

In all of the following results, we relied on the first formula of definition 5 , and some of the results found in references ([5,18]).

If $\Phi(\tau)=1,$ we get

$$^{E}S_{\sigma}\{1\}=\frac{1}{\Gamma(1+\sigma)}\int_{0}^{\infty}E_{\sigma}\left(-\frac{\upsilon^{\sigma}\tau^{\sigma}}{\nu^{\sigma}}\right)(d\tau)^{\sigma},$$
$$=\lim_{x\to\infty}\left[\frac{-\nu^{\sigma}}{\upsilon^{\sigma}}E_{\sigma}\left(-\frac{\upsilon^{\sigma}\tau^{\sigma}}{\nu^{\sigma}}\right)\right]_{0}^{x},$$
$$=\frac{\nu^{\sigma}}{\upsilon^{\sigma}}.$$

If $\Phi(\tau)=\frac{\tau^{\sigma}}{\Gamma(1+\sigma)}$ $(0<\sigma\leq 1)$ using the integral by parts [Guy], we get the following

$$^{E}S_{\sigma}\{\tau^{\sigma}\}=\frac{1}{\Gamma(1+\sigma)}\int_{0}^{\infty}E_{\sigma}\left(-\frac{\upsilon^{\sigma}\tau^{\sigma}}{\nu^{\sigma}}\right)\tau^{\sigma}(d\tau)^{\sigma},$$
$$=\frac{1}{\Gamma(1+\sigma)}\lim_{x\to\infty}\left(\int_{0}^{x}\left(\frac{-\nu^{\sigma}}{\upsilon^{\sigma}}E_{\sigma}\left(-\frac{\upsilon^{\sigma}\tau^{\sigma}}{\nu^{\sigma}}\right)\right)^{(\sigma)}\frac{\tau^{\sigma}}{\Gamma(1+\sigma)}(d\tau)^{\sigma}\right),$$
$$=\frac{\nu^{\sigma}}{\upsilon^{\sigma}}\frac{1}{\Gamma(1+\sigma)}\lim_{x\to\infty}\left(\int_{0}^{x}E_{\sigma}\left(-\frac{\upsilon^{\sigma}\tau^{\sigma}}{\nu^{\sigma}}\right)(d\tau)^{\sigma}\right).$$

Because $\lim_{x\to\infty}\left[\frac{-\nu^{\sigma}}{\upsilon^{\sigma}}E_{\sigma}\left(-\frac{\upsilon^{\sigma}\tau^{\sigma}}{\nu^{\sigma}}\right)\frac{\tau^{\sigma}}{\Gamma(1+\sigma)}\right]_{0}^{x}=0.$

Therefore

$$^{E}S_{\sigma}\{\tau^{\sigma}\}=\frac{\nu^{\sigma}}{\upsilon^{\sigma}}\lim_{x\to\infty}\left[\frac{-\nu^{\sigma}}{\upsilon^{\sigma}}E_{\sigma}\left(-\frac{\upsilon^{\sigma}\tau^{\sigma}}{\nu^{\sigma}}\right)\right]_{0}^{x},$$
$$=\frac{\nu^{2\sigma}}{\upsilon^{2\sigma}}.$$

If $\Phi(\tau)=E_{\sigma}(a^{\sigma}\tau^{\sigma}),$ using the definition 5, we get

$$^{E}S_{\sigma}\{E_{\sigma}(a^{\sigma}\tau^{\sigma})\}=\frac{1}{\Gamma(1+\sigma)}\int_{0}^{\infty}E_{\sigma}\left(-\frac{\upsilon^{\sigma}\tau^{\sigma}}{\nu^{\sigma}}\right)E_{\sigma}(a^{\sigma}\tau^{\sigma})(d\tau)^{\sigma},$$
$$=\frac{1}{\Gamma(1+\sigma)}\int_{0}^{\infty}E_{\sigma}\left(-\frac{(\upsilon-a\nu)^{\sigma}\tau^{\sigma}}{\nu^{\sigma}}\right)(d\tau)^{\sigma},$$
$$=\lim_{x\to\infty}\left[\frac{-\nu^{\sigma}}{\upsilon^{\sigma}-a^{\sigma}\nu^{\sigma}}E_{\sigma}\left(-\frac{(\upsilon-a\nu)^{\sigma}\tau^{\sigma}}{\nu^{\sigma}}\right)\right]_{0}^{x},$$
$$=\frac{\nu^{\sigma}}{\upsilon^{\sigma}-a^{\sigma}\nu^{\sigma}}.$$

If $\Phi(\tau)=\tau^{\sigma}E_{\sigma}(a^{\sigma}\tau^{\sigma}),$ we get

$$^{E}S_{\sigma}\{\tau^{\sigma}E_{\sigma}((a\tau)^{\sigma})\}=\frac{1}{\Gamma(1+\sigma)}\int_{0}^{\infty}E_{\sigma}\left(-\frac{\upsilon^{\sigma}\tau^{\sigma}}{\nu^{\sigma}}\right)\tau^{\sigma}E_{\sigma}((a\tau)^{\sigma})(d\tau)^{\sigma},$$
$$=\frac{1}{\Gamma(1+\sigma)}\int_{0}^{\infty}E_{\sigma}\left(-\frac{(\upsilon-a\nu)^{\sigma}\tau^{\sigma}}{\nu^{\sigma}}\right)\tau^{\sigma}(d\tau)^{\sigma},$$
$$=\frac{\nu^{\sigma}}{(\upsilon-a\nu)^{\sigma}}\frac{1}{\Gamma(1+\sigma)}\int_{0}^{\infty}E_{\sigma}\left(-\frac{(\upsilon-a\nu)^{\sigma}\tau^{\sigma}}{\nu^{\sigma}}\right)(d\tau)^{\sigma},$$
$$=\frac{\nu^{\sigma}}{(\upsilon-a\nu)^{\sigma}}\lim_{x\to\infty}\left[\frac{-\nu^{\sigma}}{(\upsilon-a\nu)^{\sigma}}E_{\sigma}\left(-\frac{(\upsilon-a\nu)^{\sigma}\tau^{\sigma}}{\nu^{\sigma}}\right)\right]_{0}^{x}.$$

Because $\lim_{x\to\infty}\left[\frac{-\nu^{\sigma}}{(\upsilon-a\nu)^{\sigma}}E_{\sigma}\left(-\frac{(\upsilon-a\nu)^{\sigma}\tau^{\sigma}}{\nu^{\sigma}}\right)\tau^{\sigma}\right]_{0}^{x}=0.$

Therefore, we get

$$^{E}S_{\sigma}\{\tau^{\sigma}E_{\sigma}((a\tau)^{\sigma})\}=\frac{\nu^{2\sigma}}{(\upsilon-a\nu)^{2\sigma}}.$$

If $\Phi(\tau)=\sin_{\sigma}((a\tau)^{\sigma})$ $(0<\sigma\leq 1)$ we get

$$^{LF}S_{\sigma}\{\sin_{\sigma}((a\tau)^{\sigma})\}=\frac{1}{\Gamma(1+\sigma)}\int_{0}^{\infty}E_{\sigma}\left(-\frac{\upsilon^{\sigma}\tau^{\sigma}}{\nu^{\sigma}}\right)\frac{E_{\sigma}(i^{\sigma}(a\tau)^{\sigma})-E_{\eta}(-i^{\sigma}(a\tau)^{\sigma})}{2i^{\sigma}}(d\tau)^{\sigma},$$
$$=\frac{1}{2i^{\sigma}}\frac{1}{\Gamma(1+\sigma)}\int_{0}^{\infty}\left[E_{\sigma}\left(-\frac{(\upsilon-ai)^{\sigma}\tau^{\sigma}}{\nu^{\sigma}}\right)-E_{\sigma}\left(-\frac{(\upsilon+ai)^{\sigma}\tau^{\sigma}}{\nu^{\sigma}}\right)\right](d\tau)^{\sigma},$$
$$=\frac{1}{2i^{\sigma}}\lim_{x\to\infty}\left[-\frac{\nu^{\sigma}}{(\upsilon-ai)^{\sigma}}E_{\sigma}\left(-\frac{(\upsilon-ai)^{\sigma}\tau^{\sigma}}{\nu^{\sigma}}\right)+\frac{\nu^{\sigma}}{(\upsilon+ai)^{\sigma}}E_{\sigma}\left(-\frac{(\upsilon+ai)^{\sigma}\tau^{\sigma}}{\nu^{\sigma}}\right)\right]_{0}^{x}.$$

After the calculations we find

$$^{E}S_{\sigma}\{\sin_{\sigma}(a^{\sigma}\tau^{\sigma})\}=\frac{a\nu^{2\sigma}}{\upsilon^{2\sigma}+a^{2\sigma}\nu^{2\sigma}}.$$

If $\Phi(\tau)=\cos_{\sigma}((a\tau)^{\sigma})$ $(0<\sigma\leq 1)$ knowing that $\cos_{\sigma}((a\tau)^{\sigma})=\frac{E_{\sigma}(i^{\sigma}(a\tau)^{\sigma})+E_{\sigma}(-i^{\sigma}(a\tau)^{\sigma})}{2},$ and by following the same previous steps, we get

$$ {}^{LF}S_\sigma\left\{\cos_\sigma((a\tau)^\sigma)\right\}=\frac{\nu^\sigma\upsilon^\sigma}{\upsilon^{2\sigma}+a^{2\sigma}\nu^{2\sigma}}. $$

If $\Phi(\tau)=E_\sigma\left((a\tau)^\sigma\right)\sin_\sigma\left((b\tau)^\sigma\right)\ (0<\sigma\ \ 1)$ we obtain

$$ {}^{LF}S_\sigma\left\{E_\sigma\left((a\tau)^\sigma\right)\sin_\sigma\left((b\tau)^\sigma\right)\right\} $$

$$ =\frac{1}{\Gamma(1+\sigma)}\int_0^\infty E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)E_\sigma\left((a\tau)^\sigma\right)\frac{E_\sigma\left(i^\sigma(b\tau)^\sigma\right)-E_\eta\left(-i^\sigma(b\tau)^\sigma\right)}{2i^\sigma}(d\tau)^\sigma, $$

$$ =\frac{1}{2i^\sigma}\frac{1}{\Gamma(1+\sigma)}\int_0^\infty\left[E_\sigma\left(-\frac{(\upsilon-(a+b)\nu)^\sigma}{\nu^\sigma}\tau^\sigma\right)-E_\sigma\left(-\frac{(\upsilon-(a-b)\nu)^\sigma}{\nu^\sigma}\tau^\sigma\right)\right](d\tau)^\sigma, $$

$$ =\frac{1}{2i^\sigma}\lim_{x\to\infty}\left[\begin{array}{l}-\frac{\nu^\sigma}{(\upsilon-(a+b)\nu)^\sigma}E_\sigma\left(-\frac{(\upsilon-(a+b)\nu)^\sigma}{\nu^\sigma}\tau^\sigma\right)\\ +\frac{\nu^\sigma}{(\upsilon-(a-b)\nu)^\sigma}E_\sigma\left(-\frac{(\upsilon+(a+b)\nu)^\sigma}{\nu^\sigma}\tau^\sigma\right)\end{array}\right]_0^x. $$

After the calculations we find

$$ {}^{LF}S_\sigma\left\{E_\sigma\left((a\tau)^\sigma\right)\sin_\sigma\left((b\tau)^\sigma\right)\right\}=\frac{b^\sigma\nu^{2\sigma}}{(\upsilon-a\nu)^{2\sigma}+b^{2\sigma}\nu^{2\sigma}}. $$

If $\Phi(\tau)=E_\sigma\left((a\tau)^\sigma\right)\cos_\sigma\left((b\tau)^\sigma\right)$, and by applying the definition 5, we get

$$ {}^{LF}S_\sigma\left\{E_\sigma\left((a\tau)^\sigma\right)\cos_\sigma\left((b\tau)^\sigma\right)\right\} $$

$$ =\frac{1}{\Gamma(1+\sigma)}\int_0^\infty E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)E_\sigma\left((a\tau)^\sigma\right)\frac{E_\sigma\left(i^\sigma(b\tau)^\sigma\right)+E_\sigma\left(-i^\sigma(b\tau)^\sigma\right)}{2}(d\tau)^\sigma $$

$$ =\frac{1}{2}\frac{1}{\Gamma(1+\sigma)}\int_0^\infty\left[E_\sigma\left(-\frac{(\upsilon-(a+b)\nu)^\sigma}{\nu^\sigma}\tau^\sigma\right)+E_\sigma\left(-\frac{(\upsilon-(a-b)\nu)^\sigma}{\nu^\sigma}\tau^\sigma\right)\right. $$

$$ =\frac{1}{2}\lim_{x\to\infty}\left[\begin{array}{l}-\frac{\nu^\sigma}{(\upsilon-(a+b)\nu)^\sigma}E_\sigma\left(-\frac{(\upsilon-(a+b)\nu)^\sigma}{\nu^\sigma}\tau^\sigma\right)\\ -\frac{\nu^\sigma}{(\upsilon-(a-b)\nu)^\sigma}E_\sigma\left(-\frac{(\upsilon+(a+b)\nu)}{\nu^\sigma}\tau^\sigma\right)\end{array}\right]_0^x. $$

And by doing some calculations, we get the final result

$$ {}^{LF}S_\sigma\left\{E_\sigma\left((a\tau)^\sigma\right)\cos_\sigma\left((b\tau)^\sigma\right)\right\}=\frac{\nu^\sigma(\upsilon^\sigma-a\nu^\sigma)}{(\upsilon-a\nu)^{2\sigma}+b^{2\sigma}\nu^{2\sigma}}. $$

If $\Phi(\tau)=\sinh_\sigma\left((a\tau)^\sigma\right)\ (0<\sigma\leq 1)$ we obtain

$$ {}^{LF}S_\sigma\left\{\sinh_\sigma((a\tau)^\sigma)\right\} $$

$$ =\frac{1}{\Gamma(1+\sigma)}\int_0^\infty E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)\sinh_\sigma((a\tau)^\sigma)(d\tau)^\sigma, $$

$$ =\lim_{x\to\infty}\left[\frac{-\nu^\sigma}{\upsilon^\sigma}E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)\sinh_\sigma((a\tau)^\sigma)\right]_0^x $$

$$ +\frac{\nu^\sigma}{\upsilon^\sigma}\frac{a^\sigma}{\Gamma(1+\sigma)}\lim_{?\to\infty}\int_0^? E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)\cosh_\sigma((a\tau)^\sigma)(d\tau)^\sigma, $$

$$ =-\frac{a^\sigma\nu^{2\sigma}}{\upsilon^{2\sigma}}\lim_{?\to\infty}\left[E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)\cosh_\sigma((a\tau)^\sigma)\right]_0^x $$

$$ +\frac{a^{2\sigma}\nu^{2\sigma}}{\upsilon^{2\sigma}}\frac{1}{\Gamma(1+\sigma)}\lim_{?\to\infty}\int_0^x E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)\sinh_\sigma((a\tau)^\sigma)(d\tau)^\sigma, $$

By performing simple operations, we find

$$ {}^{LF}S_\sigma\left\{\sinh_\sigma((a\tau)^\sigma)\right\}=\frac{a^\sigma\nu^{2\sigma}}{\upsilon^{2\sigma}-a^{2\sigma}\nu^{2\sigma}}. $$

If $\Phi(\tau)=\cosh_\sigma\left((a\tau)^\sigma\right)\ (0<\sigma\leq 1)$ we obtain

$$ {}^{LF}S_\sigma\left\{\cosh_\sigma((a\tau)^\sigma)\right\} $$

$$ {}^{LF}S_\sigma\left\{\cosh_\sigma((a\tau)^\sigma)\right\}=\frac{1}{\Gamma(1+\sigma)}\int_0^\infty E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)\cosh_\sigma((a\tau)^\sigma)(d\tau)^\sigma, $$

$$ =\lim_{x\to\infty}\left[\frac{-\nu^\sigma}{\upsilon^\sigma}E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)\cosh_\sigma((a\tau)^\sigma)\right]_0^x $$

$$ +\frac{\nu^\sigma}{\upsilon^\sigma}\frac{a^\sigma}{\Gamma(1+\sigma)}\lim_{?\to\infty}\int_0^? E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)\sinh_\sigma((a\tau)^\sigma)(d\tau)^\sigma, $$

$$ =\frac{\nu^\sigma}{\upsilon^\sigma}-\frac{a^\sigma\nu^{2\sigma}}{\upsilon^{2\sigma}}\lim_{x\to\infty}\left[E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)\sinh_\sigma((a\tau)^\sigma)\right]_0^x $$

$$ +\frac{(a\nu)^{2\sigma}}{\upsilon^{2\sigma}}\frac{1}{\Gamma(1+\sigma)}\lim_{x\to\infty}\int_0^x E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)\cosh_\sigma((a\tau)^\sigma)(d\tau)^\sigma, $$

$$ =\frac{\nu^\sigma}{\upsilon^\sigma}+\frac{(a\nu)^{2\sigma}}{\upsilon^{2\sigma}}\frac{1}{\Gamma(1+\sigma)}\int_0^\infty E_\sigma\left(-\frac{\upsilon^\sigma\tau^\sigma}{\nu^\sigma}\right)\cosh_\sigma((a\tau)^\sigma)(d\tau)^\sigma. $$

By performing simple operations, we find.

Illustrative examples

In this section, we will apply the local fractional Shehu transform (LFST) to some suggested local fractional differential equations.

Example 1

First, we consider the following local fractional differential equation of order

$$\frac{d^{\sigma}\omega(\tau)}{d\tau^{\sigma}}+\omega(\tau)=-1,\ 0<\sigma\leq 1^{\cdot}$$

With the initial condition

$$\omega(0)=0.$$

Taking the local fractional Shehu transform on both sides of given equation, we have

$$\frac{\upsilon^{\sigma}}{\nu^{\sigma}}\ {}^{LF}S_{\sigma}\{\omega(\tau)\}-\omega(0)+{}^{LF}S_{\sigma}\{\omega(\tau)\}=-\ {}^{LF}S_{\sigma}\{1\}.$$

Then

$$\left(\frac{\upsilon^{\sigma}}{\nu^{\sigma}}+1\right)\ {}^{E}\ S_{\sigma}\{\omega(\tau)\}=-\frac{\nu^{\sigma}}{\upsilon^{\sigma}}.$$

Which give

$$\begin{aligned}{}^{E}\ S_{\sigma}\{\omega(\tau)\}&=-\frac{\nu^{2\sigma}}{\upsilon^{\sigma}\left(\upsilon^{\sigma}+\nu^{\sigma}\right)},\\ &=\frac{\nu^{\sigma}}{\upsilon^{\sigma}+\nu^{\sigma}}-\frac{\nu^{\sigma}}{\upsilon^{\sigma}}.\end{aligned}$$

By applying the inverse transformation, yields

$$\omega(\tau)=E_{\sigma}(-\tau^{\sigma})-1.$$

Example 2

Next, we consider the following local fractional differential equation

$$\frac{d^{\sigma}\omega(\tau)}{d\tau^{\sigma}}-2\omega(\tau)=2,\ 0<\sigma\leq 1.$$

With the initial condition

$$\omega\quad=$$

Taking the local fractional Shehu transform, we have

$$\frac{\upsilon^{\sigma}}{\nu^{\sigma}}\ {}^{E}\ S_{\sigma}\{\omega(\tau)\}-2\ {}^{E}\ S_{\sigma}\{\omega(\tau)\}=2\frac{\nu^{\sigma}}{\upsilon^{\sigma}}.$$

By following the same steps as the previous example, we obtain

$${}^{E}\ S_{\sigma}\{\omega(\tau)\}=\frac{\nu^{\sigma}}{\upsilon^{\sigma}-2\nu^{\sigma}}-\frac{\nu^{\sigma}}{\upsilon^{\eta}}.$$

Take the inverse transformation, we get

$$\omega(\tau)=2E(2\tau^{\sigma})-1.$$

This result represents the exact solution to our equation.

Example 3

Finally, we consider the following local fractional differential equation of order $2\sigma,\ (0<\sigma\leq 1)$

$$\frac{d^{2\sigma}\omega(\tau)}{d\tau^{2\sigma}}+\omega(\tau)=-\frac{\tau^{\sigma}}{\Gamma(1+\sigma)},$$

Subject to the initial conditions

$$\omega(0)=0,\ \frac{d^{\sigma}\omega(0)}{d\tau^{\sigma}}=0.$$

Taking local fractional Shehu transform, we have

$$\frac{\upsilon^{2\sigma}}{\nu^{2\sigma}}\ {}^{E}\ S_{\sigma}\{\omega(\tau)\}+{}^{E}\ S_{\sigma}\{\omega(\tau)\}=-\frac{\nu^{2\sigma}}{\upsilon^{2\sigma}}.$$

By following the same steps as the previous example, we obtain

$${}^{E}\ S_{\sigma}\{\omega(\tau)\}=\frac{\nu^{2\sigma}}{\upsilon^{2\sigma}+\nu^{2\sigma}}-\frac{\nu^{2\sigma}}{\upsilon^{2\sigma}}.$$

Take the inverse transformation, we get

$$\omega(\tau)=\sin_{\sigma}(\tau^{\sigma})-\frac{\tau^{\sigma}}{\Gamma(1+\sigma)}.$$

This result represents the exact solution to our equation.

Conclusion

In this work, we proposed the local fractional Shehu transform based on the local fractional calculus and its results were discussed, where we presented some important results and properties with their proofs. To prove the effectiveness of this method, we have applied it to solve some linear local fractional differential equations, where we found the results to be accurate and from the type of no differential functions. Based on the results of the suggested examples, we can say that this method is practical and effective in solving other forms of linear local fractional differential equations.

Conflict of Interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgement

The authors would like to thank the to the General Directorate of Scientific Research and Technological Development (DGRSDT) of Algeria in for supporting partially the research project No. C00L03UN020120220002.

Bibliography

1. S Aggarwal., *et al*. "Application of Shehu Transform for Handling Growth and Decay Problems". *Global Journal of Engineering Science And Researches* 6.4 (2019): 190-198.
2. A N ALbukhuttar., *et al*. "Applications of A Shehu Transform to the Heat and Transport Equations". *International Journal of Psychosocial Rehabilitation* 24.5 (2020).
3. S Aggarwal., *et al*. "Primitive of First Kind Volterra Integro-Differential Equation Using Shehu Transform" 5.8 (2020): 31-38.
4. S Aggarwal., *et al*. "Transform for Solving Abel's Integral Equation". *Journal of Emerging Technologies and Innovative Research* 6.5 (2019): 101-110.
5. J Ahmad., *et al*. "Analytic solutions of the Helmholtz and Laplace equations by using local fractional derivative operators". *Waves Wavelets and Fractals* 1 (2015): 22-26.
6. R Belgacem., *et al*. "Transform and Application to Caputo-Fractional Differential Equations". *International Journal of Analysis and Applications* 17.6 (2019): 917-927.
7. Ji-H He. "Asymptotic Methods for Solitary Solutions and Compactons". *Abstract and Applied Analysis* 916793 (2012): 130.
8. G Jumarie. "Table of some basic fractional calculus formulae derived from a modified Riemann--Liouville derivative for non-differentiable functions". *Applied Mathematics Letters* 22 (2009): 378-385.
9. H Khan., *et al*. "Analytical Solutions of (2+Time Fractional Order) Dimensional Physical Models, Using Modified Decomposition Method". *Applied Science* 10.122 (2020): 1-20.
10. S Maitama and W Zhao. "New Integral Transform: Shehu Transform a Generalization of Sumudu and Laplace Transform for Solving differential equations". *International Journal of Analysis and Applications* 17.2 (2019): 167-190.
11. S Maitama and W Zhao. "New homotopy analysis transform method for solving multidimensional fractional diffusion equations". *Arab Journal of Basic and Applied Sciences* 27.1 (2019): 27-44.
12. S Qureshi and P Kumar. "Using Shehu Integral Transform to Solve Fractional Order Caputo Type Initial Value Problems". *Journal of Applied Mathematics and Computational Mechanics* 18.2 (2019): 75-83.
13. HM Srivastava., *et al*. "Local Fractional Sumudu Transform with Application to IVPs on Cantor Sets". *Abstract and Applied Analysis* (2014): 1-7.
14. M R Spiegel. "Theory and problems of Laplace transform, New York, USA". Schaum's Outline Series, McGraw--Hill., (1965).
15. X-J Yang. "Local Fractional Functional Analysis and Its Applications". Asian Academic, Hong Kong, (2011).
16. X-J Yang. "Advanced Local Fractional Calculus and Its Applications". World Sci. Pub., New York, NY, USA, (2012).
17. X-J Yang., *et al*. "Problems of local fractional definite integral of the one-variable non-differentiable function". *World Sci-Tech R&D, (in Chinese)* 31.4 (2009): 722-724.
18. X-J Yang. "Generalized Sampling Theorem for Fractal Signals". *Advanced Expression Manipulation* 1.2 (2012): 88-92.
19. C G Zhao., *et al*. "The Yang-Laplace Transform for Solving the IVPs with Local Fractional Derivative". *Abstract and Applied Analysis* (2014): 1-5.
20. D Ziane., *et al*. "Combination of Two Powerful Methods for Solving Nonlinear Partial Differential Equations". *Earthline Journal of Mathematical Sciences* 3.1 (2020): 121-138.
21. D Ziane., *et al*. "A new modified Adomian decomposition method for nonlinear partial differential equations". *Open Journal of Mathematical Analysis* 3.2 (2019): 81-90.
22. D Ziane., *et al*. "Local Fractional Aboodh Transform and its Applications to Solve Linear Local Fractional Di§erential Equations". *Advances in the Theory of Nonlinear Analysis and its Application* 6.2 (2022): 217-228.
23. S Benzoni. "Analyse de Fourier". Universite de Lyon/Lyon 1, France (2011).
24. D. Lomen, Application of the Mellin Transforin to Boundary Value Problems". *Proceedings of the Iowa Academy of Science* 69.1 (1962): 436-442.

25. G K Watugala. "Sumudu transform: a new integral transform to solve deferential equations and control engineering problems". *International Journal of Education in Mathematics, Science and Technology* 24.1 (1993): 35-43.

26. ZH Khan and W A Khan. "N-transform properties and applications". *NUST Journal of Engineering Sciences* 1 (2008): 127-133.

27. TM Elzaki and SM Ezaki. "On the ELzaki Transform and Ordinary Differential Equation with Variable Coefficients". *The Advances in Theoretical and Applied Mathematics* 6.1 (2011): 41-46.

28. KS Aboodh. "The new integrale transform "Aboodh transform". *Global Journal of Pure and Applied Mathematics* 9.1 (2013): 35-43.