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Mini Review

Compose Sequences

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Abstract

Through compose sequences, specially orthogonal sequences, we can construct other new sequences with more length and more security. In our short communication we show how we can use this idea for getting, in some times, new orthogonal sequences from other know orthogonal sequences.

Keywords: Orthogonal Sequences; Walsh Sequences; M-Sequences

Introduction

The best and easy orthogonal sequences are.

M-Sequences

These sequences are generated by characteristic prime polynomial g(x) and if the degree of g(x) is k then the generated sequence $\{a_n\}$ by g(x) is a periodic orthogonal sequence with the length which equal to the period T = 2^k-1, and each period contains $2^{k-1}$of "1" and 2^{k-1}-1of "0" and the difference between the number of agreements and the number of disagreement between two different cyclic permutations is 1 [1-4].

Example 1

- Using the prime polynomial g(x) = x^2+ x + 1 as a characteristic polynomial of the recurring sequence $\{a_n\}$, where $a_{n+2} = a_{n+1} + a_n$ with the initial vector (0 1), we get the periodic orthogonal sequence: 0 1 1 …. With the period 2^2-1 =3.
- Using the prime polynomial g(x) = x^3+ x^2 + 1 as a characteristic polynomial of the recurring sequence $\{b_n\}$, where $b_{n+3} = b_{n+2} + b_n$ with the initial vector (011), we get the periodic orthogonal sequence: 0 1 1 1 0 1 0 … With the period 2^3-1 =7.

Walsh sequences

Walsh sequences have the length 2^k and they can generated using the recurring formula as the following: , $H_{2^0} = [1],\ \ H_{2^k} = \begin{bmatrix} H_{2^{k-1}} & H_{2^{k-1}} \\ H_{2^{k-1}} & \overline{H}_{2^{k-1}} \end{bmatrix}$, where in $\overline{H}$ we replace each 1 by 0 and each 0 by 1, thus, we have $H_{2^1} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, H_{2^2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ so and so, and the rows of the matrix except the zero row form a set of orthogonal sequences and each nonzero row contains $2^{k-1}$of units and the same number of zeros [5-7].

Results and Discussion

Suppose G is a set of binary vectors of length n: $G = \{X; X = (x_0, x_1, ..., x_{n-1})\ \ x_i \in F_2, i = 0,1,..., n-1\}$.

Let's 1* = -1 and 0* = 1, The set G is said to be orthogonal if the following two conditions are satisfied:

$1.\ \forall X \in G, \left|\sum_{i=0}^{n-1} x_i^*\right| \le 1$; $2.\ \forall X, Y \in G, X \ne Y \Rightarrow \left|\sum_{i=0}^{n-1} x_i^*.y_i^*\right| \le 1$, That is the absolute value of agreements minus the number of disagreements is less than or equals one.

Compose sequence $\{a_n\}$ with the sequence $\{b_n\}$ is the sequence $\{c_n\}$ and $c_n = b_n \circ a_n = b_n(a_n)$ where we replace each "1" in b_n by one period (or one line) of the sequence $\{a_n\}$ and replace each "0" in b_n by the complement of one period (or the complement of one line) of the sequence $\{a_n\}$, for example [3,4].

Example 2

If the sequence $\{a_n\}$ is: 1 1 0 ..... and $\{b_n\}$ is: 1011... and each of them is periodic then the sequence $\{c_n\}$ is: 1 1 0 0 0 1 1 1 0 1 1 0 ...., thus if on period (or one line) of the sequence $\{a_n\}$ contains n_1 of "1.s" and m_1 of "0.s", and each period (or one line) of $\{b_n\}$ contains n_2 of "1.s" and m_2 of "0.s" then each one period (or one line) of $\{c_n\}$ contains $n_2(n_1) + m_2(m_1)$ of "1.s" and $n_2(m_1) + m_2(n_1)$ of "0.s" [8].

Compose two M-Sequences. Suppose $\{a_n\}$ and $\{b_n\}$ are two binary M-Sequences and $A = \{A_1, A_2, ..., A_{2^{k_1}-1}\}$, $B = \{B_1, B_2, ..., B_{2^{k_2}-1}\}$ are their sets of all permutations of one period of each of them respectively.

And suppose the sequences $\{c_n\} = \{b_n(A_j)\}$, and $C = \{C_1, C_2, ..., C_{2^{k_2}-1}\}$ is the set of all C_i where $C_i = B_i(A_j)$ $i = 1,2,...,2^{k_2}-1$, then the length of each C_i is $l = (2^{k_2}-1)(2^{k_1}-1)$ and contains:

$$2^{k_2-1}.2^{k_1-1} + (2^{k_2-1}-1)(2^{k_1-1}-1) = 2^{(k_2+k_1)-1} - (2^{k_2-1}+2^{k_1-1}) + 1 \quad \text{of units}$$
$$2^{k_2-1}(2^{k_1-1}-1) + 2^{k_1-1}(2^{k_2-1}-1) = 2^{(k_2+k_1)-1} - (2^{k_2-1}+2^{k_1-1}) \quad \text{of zeros}$$

If the difference between the number of agreements and the number of disagreement in two different cyclic permutations ($B_i \,\&\, B_j, i \neq j$) of B is 1 then the difference between the number of agreements and the number of disagreement in two different $C_i \,\&\, C_j, i \neq j$ of C is $2^{k_1}-1$, For example, from example1 we have:

B_1= 0 1 1 1 0 1 0 $\Rightarrow$ C_1= $B_1(A_1)$ = 100 011 011 011 100 011 100
B_2= 1 1 1 0 1 0 0 $\Rightarrow$ C_2= $B_2(A_1)$ = 011 011 011 100 011 100 100

We can check that the difference between the agreement and disagreement in C_1 and C_2 is $2^2 - 1 = 3$.

Thus, the set C is non-orthogonal set.

Result 1. Compose two M-Sequences don't give a set of orthogonal sequences.

Compose two Walsh Sequences. Suppose two sets of non-zero Walsh sequence $W \; \{w_1, w_2, ...w_{2^{l_1}-1}\}$ and $U = \{u_1, u_2, ...u_{2^{l_2}-1}\}$ which each sequence has the lengths 2^{l_1} *and* 2^{l_2} respectively and each w_i has 2^{l_1-1} of units and the same number of zeros and each u_j has 2^{l_2-1} of units and the same number of zeros and suppose the set of sequences C = $W(u_j) = \{C_1 = w_1(u_j), C_2 = w_2(u_j), ..., C_{2^{l_1}-1} = w_{2^{l_1}-1}(u_j)\}$ then the length of each of C_i is: $l = 2^{l_1+l_2}$ and contains $2^{(l_1+l_2)-1}$ and the same number of zeros. The difference between the number of agreements and the number of disagreements in two different $w_i \,\&\, w_j, i \neq j$ of W is zero and the same for $u_i \,\&\, u_j, i \neq j$ of U then the difference between the number of agreements and the number of disagreement in two different $C_i \,\&\, C_j, i \neq j$ of C is zero thus the set C is an orthogonal set. For example, Suppose $W = \{w_1, w_2, w_3\}$ where: w_1\{0 1 0 1\}, w_2= \{0 0 1 1\}, w_3=\{0 1 1 0\} and $U = \{u_1 = \{0\,1\}$ then: $C_1 = w_1(u_1) = 1\,0\,0\,1\,1\,0\,0\,1$, $C_2 = w_2(u_1) = 1\,0\,1\,0\,0\,1\,0\,1$, and $C_3 = w_3(u_1) = 1\,0\,0\,1\,0\,1\,1\,0$ we can see that the set C is an orthogonal set.

Result 2. Compose two Walsh Sequences gives a set of orthogonal sequences.

Compose M-Sequences with Walsh Sequences. Suppose $\{a_n\}$ is a binary M-Sequences and $A = \{A_1, A_2, ..., A_{2^{k_1}-1}\}$ is its set of all cyclic permutations of one period of it, the set of non-zero Walsh sequence $W = \{w_1, w_2, ...w_{2^{l_1}-1}\}$ and the set of sequences $C = \{C_1, C_2, ..., C_{2^{l_1}-1}\}$,

Where

$C_i = w_i(A_j), i = 1,....,2^{l_1}-1$ which has the length $2^{l_1}(2^{k_1}-1)$ and contains.

$2^{l_1-1}.2^{k_1-1} + 2^{l_1-1}(2^{k_1-1}-1) = 2^{(l_1+k_1)-1} - 2^{l_1-1}$ of unites and the same number of zeros and the difference between the number of agreements and the number of disagreement in two different $w_i \,\&\, w_j, i \neq j$ of W is zero then the difference between the number of agreements and the number of disagreement in two different $C_i \,\&\, C_j, i \neq j$ of C is zero thus the set C is an orthogonal set.

For example.

Suppose $W = \{w_1, w_2, w_3\}$ where: w_1\{0 1 0 1\}, w_2= \{0 0 1 1\}, w_3=\{0 1 1 0\} and as in xample1 A_1= 011
w_1= 0 1 0 1 $\Rightarrow$ C_1= $w_1(A_1)$ = 1 0 0 0 1 1 1 0 0 0 1 1
w_2= 0 0 1 1 $\Rightarrow$ C_2= $w_2(A_1)$ = 1 0 0 1 0 0 0 1 1 0 1 1

We can see that C_1 and C_2 are orthogonal sequences and the set C is an orthogonal set.

Result 3. Compose M-Sequences with a Walsh Sequences gives a set of orthogonal sequences.

Compose Walsh sequences with a M-Sequences. Suppose $\{a_n\}$ is a binary M-Sequences and $A = \{A_1, A_2, \ldots, A_{2^{k_1}-1}\}$ is its set of all cyclic permutations of one period of it, the set of non-zero Walsh sequence $W = \{w_1, w_2, \ldots w_{2^{l_1}-1}\}$ and the set of sequences $D = \{D_1, D_2, \ldots, D_{2^{k_1}-1}\}$, where $D_i = A_i(w_j)$ $i = 1, \ldots, 2^{k_1} - 1$ which has the length $(2^{k_1} - 1)2^{l_1}$ and contains:

$$2^{k_1-1}.2^{l_1-1} + (2^{k_1-1} - 1)2^{l_1-1} = 2^{(k_1+l_1)-1} - 2^{l_1-1} \quad \oint \ units$$
$$2^{k_1-1}.2^{l_1-1} + (2^{k_1-1} - 1)2^{l_1-1} = 2^{(k_1+l_1)-1} - 2^{l_1-1)} \quad \oint \ zeros$$

The difference between the number of agreements and the number of disagreement in two different A_i & $A_j, i \neq j$ of A is one then the difference between the number of agreements and the number of disagreement in two different D_i & $D_j, i \neq j$ of D is 2^{l_1}, thus the set C is non orthogonal set. For example.

Suppose $A = \{A_1, A_2, A_3\}$ where: A_1\{0 1 1\}, A_2= \{1 1 0\}, A_3=\{1 0 1\} and as in xample1 w_1= 0101
A_1= 0 1 1 $\Rightarrow$ D_1= $A_1(w_1)$ = 1 0 1 0 0 1 0 1 0 1 0 1
A_2= 1 1 0 $\Rightarrow$ D_2= $A_2(w_1)$ = 0 1 0 1 0 1 0 1 1 0 1 0

It is very clear that D_1 and $D_2$are non-orthogonal and satisfies the theoretical study and A(W) is not orthogonal set.

Result 4. Compose Walsh sequences with a M-Sequences gives a set of non-orthogonal sequences.

Conclusion

- Compose two M-Sequences gives a set of non-orthogonal sequences.
- Compose two Walsh Sequences gives a set of orthogonal sequences.
- Compose M-Sequences with a Walsh Sequences gives a set of orthogonal sequences.

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