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Review Article

Analytic Solutions to the System of Ion Sound and Langmuir Waves Via First Integral Method

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Abstract

The goal of this paper is to look into a nonlinear system of partial differential equations (PDEs) with unknown functions that are both complex and real-valued. We use the first integral method based on commutative algebra theory to construct new solutions to the system of ion sound and Langmuir waves. All algebraic computations in this work are performed using the Maple software. This method is significant, efficient, and applicable to a diverse set of nonlinear differential equations.

Keywords: The System of Ion Sound and Langmuir Waves; First Integral Method; Traveling Wave Solutions; Commutative Algebra; Nonlinear Partial Differential Equations

Introduction

Finding solitary solutions of nonlinear equations in mathematical physics and applied mathematics has become increasingly important in solitary theory. Over the last several decades, new exact solutions may have aided in the discovery of new solutions. A variety of powerful methods, such as Simplest equation method [14,18], (G′/G)-expansion method [6,7], generalized tanh function method [19,20], tanh method [4,15], sine-cosine method [5,13], F-expansion method [2,10] and Extended mapping method [1,16] have been proposed to obtain exact solutions. All of these methods have one thing in common: they all require the assistance of a computer algebra system, such as Maple, to solve the solutions of nonlinear evolution equations. The first integral method is ideal for finding new analytic solutions to various partial differential equations. The first integral method has the advantage of being purely algebraic and using symbolic computation during solution [8,9]. This method was first proposed by Feng [8,9] in solving Burgers-KdV equation which is based on the ring theory of commutative algebra. Recently, this useful method is widely used by many such as in [3,12,17] and by the reference therein. The objective of this paper is to use the first integral method to find new exact soliton solutions for the system of ion sound and Langmuir waves. This system describes the ion sound wave under the action of the ponder motive force due to a high-frequency field, as well as the Langmuir wave [11,21]. The paper is arranged as follows: First integral method is described in Section 2. In Section 3, in order to illustrate the method, the system for the ion sound and Langmuir waves are investigated and abundant exact solutions are obtained which include new hyperbolic wave solution. Finally, in Section 4, we give the conclusions.

Description of the First integral method:

Let the nonlinear partial differential equation in the form:

$$F\left(u, u_t, u_x, u_{xx}, u_{xt}, \ldots\ldots\right) = 0 \qquad \text{----------(1)}$$

Where represent the exact solution of equation (1). We use the travelling wave solutions in the form:

$$u(x,t) = f(\xi) \qquad \text{----------(2)}$$

Where and c is constant. Now, by using chain rule we get:

$$\frac{\partial}{\partial t}(.) = -c\frac{d}{d\xi}(.), \frac{\partial}{\partial x}(.) = \frac{d}{d\xi}(.), \frac{\partial^2}{\partial x^2}(.) = \frac{d^2}{d\xi^2}(.), \qquad \text{----------(3)}$$

Using (3) to transfer the nonlinear partial differential equation (1) into nonlinear ordinary differential equation in the form:

$$Q(f(\xi),\frac{df}{d\xi},\frac{d^2f}{d\xi^2},\ldots\ldots) = 0 \qquad ----------(4)$$

We define new independent variables:

$$X(\xi) = f(\xi), Y(\xi) = f_\xi(\xi) \qquad --------- (5)$$

This leads to a system of ordinary differential equations:

$$\begin{cases} X_\xi(\xi) = Y(\xi), \\ Y_\xi(\xi) = F_1(X(\xi), Y(\xi)) \end{cases} \qquad ----------(6)$$

From the qualitative theory of ordinary differential equations, if one can find two first integrals to system (6)under the same conditions, then the analytic solutions to (6) can be solved directly. However, in general, it is difficult to realize this even for a single first integral, because for a given plane autonomous system, there is no general theory telling us how to find its first integrals in a systematic way. We use the division theorem to obtain one first integral to equation (6) which reduces equation (4) to a first order integrable ordinary differential equation. For convenience, first let us recall the division theorem for two variables in the complex domain (5, 6).

Division theorem

Suppose that P(w , z) and Q(w , z) are polynomials of two variables C[w , z] and P(w , z) in complex domain and is irreducible in C[w , z]. If Q(w , z) vanishes at all zero points of , then there exists a polynomial G(w , z) in C[w , z] such that:

Q(w,z)=P(w,z)G(w,z)

Applications

Now, we using the first intgral method to solve the system of ion sound and Langmuir waves.

The system of ion sound and Langmuir waves write as following [21]:

$$iE_t + \frac{1}{2}E_{xx} - VE = 0,$$
$$V_{tt} - V_{xx} - 2(|E|^2)_{xx} = 0 \qquad ----------(7)$$

The normalized electric field of the Langmuir oscillation is , and the normalized density perturbation is . Here, represents the spatial domain and represents time. The system of equations for the ion sound wave under the action of the ponderomotive force caused by a high frequency field and for the Langmuir wave is strongly nonlinear, and obtaining its solitary wave solutions is difficult [11].

We use the travelling wave transformation for equation (7) as follows:

$$E(x,t) = e^{iz}E(\xi), V(x,t) = V(\xi)\ z = \gamma x + \lambda t, \xi = \alpha x + \beta t \qquad ----------(8)$$

Where γ, λ, α and β are constants. Then, put equation (8) with derivatives in equation (7) we get:

$$i(\beta + \alpha\gamma)E' = 0, \qquad ----------(9)$$

$$\alpha^2 E'' - (2\lambda + \gamma^2)E - 2EV = 0, \qquad --------(10)$$

$$(\beta^2 - \alpha^2)V'' - 2\alpha^2(E^2)'' = 0 \qquad --------(11)$$

Equation (11) integrating twice to ξ and make constant of integration equal to zero, we have:

$$V(\xi) = \frac{2\alpha^2}{\beta^2 - \alpha^2}E^2 \qquad --------(12)$$

Put equation (12) in equation (10) and also equation (9) get:

$$\alpha^2(\gamma^2 - 1)E'' - (\gamma^2 - 1)(2\lambda + \gamma^2)E - 4E^3 = 0 \qquad --------(13)$$

Using (5) in (13) we obtain:

$$X'(\xi) = Y(\xi)\ (14) \qquad --------(14)$$

$$Y'(\xi) = \frac{(2\lambda+\gamma^2)}{\alpha^2}X(\xi) + \frac{4}{\alpha^2(\gamma^2-1)}X^3(\xi) \qquad --------(15)$$

Now, we are applying the division theorem to seek the first integral method to equation (13). Suppose that X(ξ) and Y(ξ) are the nontrivial solutions to equation (14) and (15), and $q(X,Y)=\sum_{i=0}^{m} a_i(X)\,Y^i=0$ is an irreducible polynomial in the complex domain C[X,Y] such that:

$$q[X(\xi), Y(\xi)] = \sum_{i=0}^{m} a_i(X)Y^i = 0 \qquad -----------(16)$$

Where a_i (x)(i=0,1,…….,m)are polynomials of and all relatively prime in C[X,Y], a_m (X)≠0., Equation(16) is also called the first integral to (14) and (15). Note that dq/dξ is a polynomial in X and Y and q[X(ξ),Y(ξ)]=0 implies dq/dξ=0, due to the Division Theorem, there exists a polynomial (g(X)+h(X)Y) in C[X,Y] such that:

$$\frac{dq}{d\xi} = \frac{dq}{dX}\frac{dX}{d\xi} + \frac{dq}{dY}\frac{dY}{d\xi} = (g(X) + h(X)Y)\sum_{i=0}^{m} a_i(X)Y^i \qquad --------(17)$$

Let m=2 in (17), due to the division theorem, there exists a polynomial g(X) +h(X) Y in the complex domain C[X,Y] such that

$$\frac{dq}{d\xi} = \frac{\partial q}{\partial X}\frac{\partial X}{\partial \xi} + \frac{\partial q}{\partial Y}\frac{\partial Y}{\partial \xi} = (g(X) + h(X)Y)\sum_{i=0}^{2} a_i(X)\,Y^i \qquad \text{--------(18)}$$

By comparing the coefficients of Y^i (i=3,2,1,0) on both sides of equation (18) we get:

$$a'_2(X) = a_2(X)h(X) \qquad \text{--------(19)}$$

$$a'_1(X) = a_1(X)h(X) + a_2(X)g(X) \qquad \text{--------(20)}$$

$$a'_0(X) = a_1(X)g(X) + a_0(X)h(X) - \frac{2(2\lambda+\gamma^2)}{\alpha^2}\ a_2(X)\,X(\xi) -$$

$$\frac{8}{\alpha^2(\gamma^2-1)} a_2(X)\,X^3(\xi)\,\frac{(2\lambda+\gamma^2)}{\alpha^2}\ a_1(X)\,X(\xi) + \qquad \text{--------(22)}$$

$$\frac{4}{\alpha^2(\gamma^2-1)} a_1(X)\,X^3(\xi) = a_0(X)g(X)$$

Since a_2 (X) is polynomial of X. Then, from (19) we conclude that a_2 (X) is constant and h(X)=0, To simplify, we take a_2 (X) =1 , and balance the degrees of g(X), a_1 (X) and a_0 (X) we conclude that deg g(X)=0 only. Now, we discuss this case: If deg g(X) = 0, suppose that g(X)=A_1, then we can find a_1 (X) and a_0 (X) :

$$a_1(X) = A_1X + B_0 \qquad \text{-------(23)}$$

$$a_0(X) = \frac{1}{2}\left(A_1{}^2 - \frac{2(2\lambda+\gamma^2)}{\alpha^2}\right)X^2 - \frac{2}{\alpha^2(\gamma^2-1)}\ X^4 + A_1B_0X + d \qquad \text{-------(24)}$$

Where A_0 and B_0 are arbitrary integration constants. Substituting a_0 (X) and g(X) in (22) and setting all the coefficients of powers X to be zero, we obtain a system of nonlinear algebraic equations:

$$\frac{B_0}{\alpha^2(\gamma^2-1)} = 0 \qquad \text{--------(25)}$$

$$\frac{1}{2}\left(A_1{}^2 - \frac{2(2\lambda+\gamma^2)}{\alpha^2}\right) = \frac{(2\lambda+\gamma^2)}{\alpha^2} \qquad \text{--------(26)}$$

$$\frac{(2\lambda+\gamma^2)}{\alpha^2} = A_1{}^2 \qquad \text{--------(27)}$$

Solving the last algebraic equations, we obtain:

$$A_1 = 0, B_0 = 0, d = 0, \lambda = \frac{-\gamma^2}{2} \qquad \text{--------(28)}$$

Using the condition (28) into (18) we obtain:

$$Y(\xi) = \sqrt{\frac{2}{\alpha^2(\gamma^2-1)}}\ X^2 \qquad \text{--------(29)}$$

Combining (29) with (15), we obtain the exact solution to (13) as follows:

$$E(\xi) = -\sqrt{\frac{\alpha^2(\gamma^2-1)}{2}}\,\frac{1}{\xi} \qquad \text{------------(30)}$$

Substituting (30) into (12) we get:

$$V(\xi) = \frac{\alpha^4(\gamma^2-1)}{\beta^2-\alpha^2}\ \frac{1}{\xi^2} \qquad \text{--------(31)}$$

Where is an arbitrary constant. Then the exact solutions to equation (7) can be written as:

$$E(x,t) = -\sqrt{\frac{\alpha^2(\gamma^2-1)}{2}}\,\frac{1}{\alpha x+\beta t}\,e^{i\left(\gamma x-\frac{\gamma^2}{2}t\right)}, \qquad \text{----------32)}$$

$$v(x,t) = \frac{\alpha^4(\gamma^2-1)}{\beta^2-\alpha^2}\ \frac{1}{(\alpha x+\beta t)^2}$$

These solutions are all new exact solutions. The solutions above obtained when , for the solution steps become complicated and cannot obtain solutions.

The hyperbolic wave and behavior of the solutions E(x,t) and V(x,t) are shown in Figure (1) for some fixed values of the parameters, (γ=2,α=1,β=2).

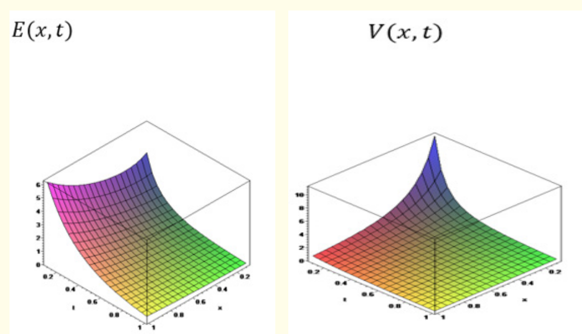

Figure 1: Waves solutions for equations (32).

Conclusions

We extend the application of the first integral method to solve the system of ion sound and Langmuir waves in this work. This method's performance has been found to be reliable and effective. The Maple software was used to perform complex and time-consuming algebraic calculations. The proposed method is applicable to other nonlinear problems in mathematics.

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