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Review Article

Automated Design of the References for Compensated Multivariable Linear Systems

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Abstract

The current paper is based on the method we established for the synthesis of the reaction matrices used by compensators that reduces multivariable linear systems to extended systems that are cyclical. The driving of the extended systems (original multivariable system plus compensator) has been reduced to a reaction that depends on the system outputs and a set of references calculated by the algorithm described in our paper.

Keywords: Linear Systems; References; Calculation

Introduction

The control of multivariable linear systems has been greatly improved with the introduction of the compensator based design [2-4]. The current paper derives the values of the references and produces the associated program for their automated calculation. This paper is dedicated to my mentors, prof. dr. Elena Kreindler-Wexler, dr. Lupas Laurentiu and prof. dr. eng. Vlad Ionescu.

Description of the program for calculating the references

Input matrices $A(N,N), B(N,IR), C(M,N)$ describing the dynamics of the system to be compensated are introduced in [1]. The task is to calculate the references [1]: $w_{r,1} = TR(0)^{\#}_{r,m} y_{steady}$ where $TR(0)^{\#}_{r,m}$ is the pseudoinverse of $TR(0)_{r,m}$ and:

$$TR(0) = C' \frac{1}{a^1_{0e}} [\alpha^1_{1e} I + .. + \alpha^1_{n-1,e} (A^1_e)^{n-2} + (A^1_e)^{n-1}] B'$$

$$n = N + L$$

$$C' = \begin{bmatrix} C_{m,N} & 0_{m,L} \end{bmatrix}, B' = \begin{bmatrix} B_{N,r} \\ 0_{L,r} \end{bmatrix}$$

$$m = M, r = IR$$

----------(2.1)

The "e" subscript stands for "extended system" (system including the compensator). y_{steady} is the steady value of outputs. The dynamic dimensions of the matrices used by the program are [1]:

$$A(N,N), B(N,IR), C(M,N), BETA(N+L),$$
$$AE(N+L,N+L), BE(N+L,IR+L),$$
$$CE(M+L,N+L), BE1(N+L,M+L),$$
$$CE1(N+L,N+L), A2(N+L,N+L),$$
$$ALFA2(N+L), ACK(IR,N), BAKC(N,N),$$
$$Y(N,N), QC(N,N*IR), QO(N,N*M),$$
$$T(N), G(N), R(N,N), E(N+L,L+(L+1)IR),$$
$$PSE(L+(L+1)IR, N+L),$$
$$F(N+L, L+(L+1)IM), PSF(L+(L+1)M, N+L),$$
$$P(N,N), CK1(IR+L, M+L),$$
$$CK2(IR+L, M+L), CK(IR+L, M+L),$$
$$OK1(IR+L, M+L), OK2(IR+L, M+L),$$
$$OK(IR+L, M+L), D(N+L),$$
$$X(\max\{L+(L+1)IR, L+(L+1)M\}), H(IR), ETA(M),$$
$$CL(N,IR), OL(N,M), AK1(IR,M),$$
$$AS(IR*M), AK(IR,M)$$

Figure 1: The diagram of the compensated system.

Looking at figure 1 we see that we need to compute the references for the compensator: w_1, w_2, w_3. The dynamics of the compensated system are described by the equations:

$$\dot{x}_l = A_e x_e + B_e u_e$$
$$y_e = C_e x_e$$
$$A_e = \begin{bmatrix} A & 0_{n,l} \\ 0_{l,n} & 0_{l,l} \end{bmatrix}, B_e = \begin{bmatrix} B & 0_{n,l} \\ 0_{l,r} & I_{l,l} \end{bmatrix}, C_e = \begin{bmatrix} C & 0_{m,l} \\ 0_{l,n} & I_{l,l} \end{bmatrix}, x_e = \begin{bmatrix} x_{n,1} \\ \bar{x}_{l,1} \end{bmatrix},$$

$$y_e = \begin{bmatrix} y_{m,1} \\ \bar{y}_{l,1} \end{bmatrix}, u_e = \begin{bmatrix} u_{r,1} \\ \bar{u}_{l,1} \end{bmatrix}$$

$$l = L, m = M, r = IR, n = N \qquad \text{----------(2.2)}$$

The above system splits into two separate systems:

$$\dot{x} = Ax + Bu$$
$$y = Cx \qquad \text{---------- (2.3)}$$

$$\dot{\bar{x}} = I_{l,l} u$$
$$\bar{y} = I_{l,l} \bar{x} \qquad \text{----------(2.4)}$$

According to [1], there exists K such that the extended system obtained by using the output feedback $u_l = Ky_l + w_l$ will have the dynamics described by:

$$\dot{x}_e = (A_e + B_e K C_e) x_e + B_e w_e$$
$$y_e = C_e x_e \qquad \text{----------(2.5)}$$

And the matrix $A_e^1 = A_e + B_e K C_e$ will have the desired eigenvalues and $w_e = \begin{bmatrix} w_{r,1} \\ 0_{l,1} \end{bmatrix}$

$$\dot{x}_e = A_e^1 x_e + B_e w_e$$
$$y_e = C_e x_e \qquad \text{---------- (2.6)}$$

This means that:

$$Y_e(s) = C_e (sI_{n+l} - A_e^1)^{-1} B_e W(s) = T_{m+l,r+l}(s) * W_{r+l,1}(s) \qquad \text{----------(2.7)}$$

Since $W(s) = \begin{bmatrix} W_{r,1}(s) \\ 0_{l,1} \end{bmatrix}$ and $Y(s) = \begin{bmatrix} Y_{m,1}(s) \\ \bar{Y}_{l,1}(s) \end{bmatrix}$:

$$Y_{m,1}(s) = TR_{m,r}(s) * W_{r,1}(s) \qquad \text{----------(2.8)}$$

The steady state of the outputs is:

$$Y_{steady}(s) = Y(\infty) = \lim_{s \to 0} sY(s) = TR_{m,r}(0) \lim_{s \to 0} sW_{r,1}(s) = TR_{m,r}(0) W_{steady} \qquad \text{---------- (2.9)}$$

So:

$$y_{steady} = TR(0) * w \qquad \text{----------(2.10)}$$

The references are:

$$w_{r,1} = TR(0)^{\#}_{r,m} * y_{steady} \quad \text{----------(2.11)}$$

Since:

$$(sI - A)^{-1} = \frac{1}{X_A(s)}[(s^{n-1} + \alpha_{n-1}s^{n-2} + ... + \alpha_1)I + .. + (s + \alpha_{n-1})A^{n-2} + A^{n-1}] \quad \text{----------(2.12)}$$

It follows that:

$$(sI_{n+l,n+l} - A_e^1)^{-1}|_{s=0} = \frac{1}{a_{0,e}^1}[\alpha_{1,e}^1 I + .. + \alpha_{n-1,e}^1 (A_e^1)^{n-2} + (A_e^1)^{n-1}] \quad \text{----------(2.13)}$$

$$T(0) = C_e \frac{1}{a_{0,e}^1}[\alpha_{1,e}^1 I + .. + \alpha_{n-1,e}^1 (A_e^1)^{n-2} + (A_e^1)^{n-1}]B_e \quad \text{----------(2.14)}$$

$$TR(0) = C' \frac{1}{a_{0,e}^1}[\alpha_{1,e}^1 I + .. + \alpha_{n-1,e}^1 (A_e^1)^{n-2} + (A_e^1)^{n-1}]B'$$

$$C' = \begin{bmatrix} C_{m,N} & 0_{m,L} \end{bmatrix}, B' = \begin{bmatrix} B_{N,r} \\ 0_{L,r} \end{bmatrix}, n = N + L \quad \text{---------- (2.15)}$$

The references are:

$$w_{r,1} = TR(0)^{\#} * y_{steady} \quad \text{----------(2.16)}$$

The above algorithm avoids the calculation of the transfer matrix $T(s)$ altogether. The program furnishes the matrix A_e^1 (denoted as $A2(N+L, N+L)$ in the program) and the coefficients $\alpha_{i,e}^1$ of the characteristic polynomial $X_{A_e^1}(s)$, denoted as $ALFA2(N+L)$ in the program [5]. The algorithm has the added strength that if the desired steady outputs y_{steady} vary, then the only thing needed in re-calculating the references is to re-calculate the product $TR(0)^{\#} y_{steady}$, $TR(0)$being time invariable.

Description of the subroutines that needed to be constructed and the steps leading to the desired result, the code needed to be appended to the MAIN program from link [5]:

DIMENSION C'(2,6),B'(6,3),W(3,1),YSTDY(2,1),TR(2,3),TRPI(3,2)

CPRIME(C,M,N,L,C') constructs the matrix $C' = \begin{bmatrix} C_{m,N} & 0_{m,L} \end{bmatrix}$ of dimensions (M,N+L). Here $0_{m,L}$ represents a matrix made up of m rows of zeroes and L columns of zeroes.

BPRIME(B,L,N,R,B') constructs the matrix $B' = \begin{bmatrix} B_{N,r} \\ 0_{L,r} \end{bmatrix}$ of dimensions (N+L,IR).

TR(B',C',TR,A2,ALFA2,N,L,R) constructs the matrix $TR(0) = C' \frac{1}{a_{0,e}^1}[\alpha_{1,e}^1 I + .. + \alpha_{n-1,e}^1 (A_e^1)^{n-2} + (A_e^1)^{n-1}]B'$ of dimensions (M,IR).

The invocation of subroutine PS(TR,M,R,TRPI) constructs the pseudoinverse $TR(0)$ of dimensions (IR,M).

The invocation of the subroutine PROMAT(TRPI,YSTDY,W,IR,M,1) constructs the desired references $w_{r,1} = TR(0)^{\#} * y_{steady}$, a vector of dimension W=W(IR,1). Here, the vector YSTDY=YSTDY(M,1) can be taken as a set of arbitrary numbers. The description of subroutines PROMAT and PS as well as the values for matrices B,C,ALFA2,A2 and the dimensions L,M,N,IR can be found on the link referred by [5], see below:

$N = 5, M = 2, IR = 3$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The matrices A2, ALFA2 are produced at the end of the program MAIN (also from [5]).

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