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Research Article

On Super Root Square Mean Labeling of Some Cycle Related Graphs

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Abstract

Let f: V (G) → {1, 2, 3, . . ., p + q} be an injective function.

For each edge e = uv let f *(e = uv) defined by

$$f^*(e=uv) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil \text{ or } \left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor,$$

Then f is called a super root square mean labeling if f (V) ∪{f ∗(e): e ∈ E(G)} = {1, 2, 3, . . ., p + q}. A graph which admits super root square mean labeling is called super root square mean graph. In this paper, we determined the super root square mean labeling of the Sunlet, Middle Cycle, Polygonal Chain, Alternate Polygonal Chain and Kayak Paddle Graphs.

Keywords: Graph Labeling; Super Root Square Mean Labeling; Cycle Related Graphs

Introduction

In this paper, we will be dealing with graphs that are finite, simple (has no loops or parallel edges), connected and undirected graph G = (V, E) with p vertices and q edges. Graph labeling can be defined as an assignment of integers to the vertices or edges, or both, subject to certain conditions [2]. Graph Labeling is used nowadays in rigorous applications in many disciplines like coding theory, x-ray crystallography, radar, astronomy, circuit design, communication network addressing and data base management [5].

One of the types of graph labeling techniques is super root square meanlabeling. In 2015, the notion of super root square mean labeling was introduced by K. Thirugnanasambandam and K. Venkatesan [9]. Let f: V (G) → {1, 2, 3, . . ., p + q} be an injective function. For each edge e = uv let f *(e = uv) defined by

$$f^*(e=uv) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil \text{ or } \left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor,$$

Then f is called a super root square mean labeling if f (V) {f*(e): e ∈E(G)} = {1, 2, 3, . . ., p + q}. A graph which admits super root square meanlabeling is called super root square mean graph. Super root square meanlabeling behavior of several graphs are studied by K. Thirugnanasambandam and K. Venkatesan [9], R. Chitra devi and S. Saravana Kumar [1], S. S. Sandhya, S. Somasundaram and S. Anusa [6-8] and R. Gopi [3]. Foran extensive survey of graph labeling as well as bibliographic references, werefer to Gallian [2].

Basic concepts

Definition 2.1

A Polygonal Chain is G_n^m m blocks are polygons C_n [4].

Definition 2.2

The graph is m blocks of C_n connected with a chord [4].

Note

We name the graph as Alternate Polygonal Chain Graph.

Results and Discussions

Theorem 3.1

If a graph G is a Super Root Square Mean Graph, thenadjacent vertices must be labeled with nonconsecutive natural numbers.

Proof. Suppose that a graph G is a Super Root Square Mean Graph and two consecutives numbers were labeled to all adjacent vertices of G. By definition, a graph G = (V, E) with p vertices and q edges is said to be a Super Root Square Mean Graph if it is possible to label the vertices $x \in V(G)$ with distinct elements f (x) from the set $\{1, 2, \ldots, p + q\}$ in such a way that when each edge e = uv is labeled by f *(e = uv) defined by

$$f^*(e = uv) = \left\lceil \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rceil \text{ or } \left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor,$$

Then f(V) $\{f^*(e): e \in E(G)\} = \{1, 2, 3, \ldots, p + q\}$. Now, let the adjacent vertices of a graph G be labeled by consecutive numbers, a and a+1, respectively. Hence, the induced edge labeling f * that is in the ceiling would be

$$\begin{aligned} f^*(e = uv) &= \left\lceil \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rceil \\ &= \left\lceil \sqrt{\frac{a^2 + (a+1)^2}{2}} \right\rceil \\ &= a + 1. \end{aligned}$$

While the induced edge labeling f^* that is in the floor function would be

$$\begin{aligned} f^*(e = uv) &= \left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor \\ &= \left\lfloor \sqrt{\frac{a^2 + (a+1)^2}{2}} \right\rfloor \\ &= a. \end{aligned}$$

Observe that the resulting edge labels is a + 1 and a, respectively, i.e, in the ceiling and floor function. Hence, the vertex and edge labels are not distinct. Thus, if we union the set of vertex labels and set of edge labels we get, $f(V)\{f^*(e): e \in E(G)\} \neq \{1, 2, 3, \ldots, p + q\}$, a contradiction to our assumption that a graph G is a Super Root Square Mean Graph. Therefore, adjacent vertices of a Super Root Square Mean Graph G must be labeledwith nonconsecutive natural numbers.

Observation 3.2

If a graph G is a Super Root Square Mean Graph andlet the numbers (a and b) labeled to all adjacent vertices, then their incident edge labeled by number (c) is always a < c < b or a > c > b. Moreover, the first and last numbers of the given set which are 1 and p + q (sum of order and size) are always labeled to the set of vertices.

Theorem 3.3

The Sunlet Graph Sl_n is Super Root Square Mean Graph for all $n \geq 3$.

Proof. Let $V(Sl_n) = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$ be the vertex set and $E(Sl_n) = \{u_i u_{i+1}, u_1 u_n$ for $1 \leq i \leq n - 1$ and $u_i v_i$ for $1 \leq i \leq n\}$ be the edge set of Sunlet Graph Sl_n, $n \geq 3$ as shown in figure 1. To prove the theorem, we will consider the following cases.

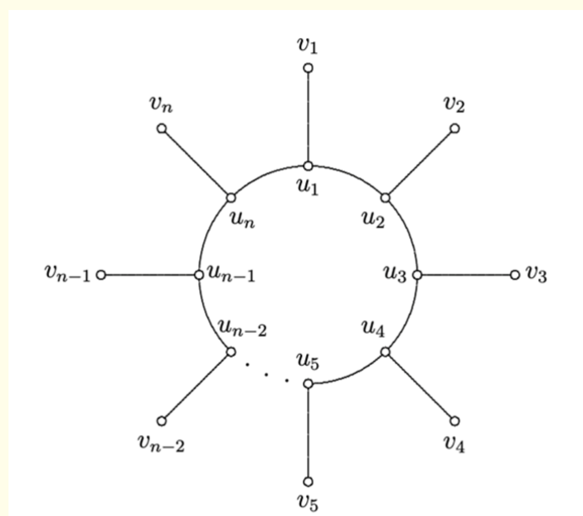

Figure 1: A sunlet graph SL_n.

Case 1

n is odd, $(n \geq 3)$.

Subcase 1

n = 3.

If n = 3 then the graph Sl_3 $T_{2\circ}K_1$ which is a Super Root Square Mean Graph [3].

Subcase 2

n ≥ 5

Define the function f: V (Sl_n) → {1, 2, 3, . . ., p + q} by:

$$f(u_i) = \begin{cases} 3 & i = 1 \\ 8i - 11 & 2 \leq i \leq \frac{n+1}{2} \\ 8n - 8i + 12 & \frac{n+3}{2} \leq i \leq n \end{cases}$$

$$f(v_i) = \begin{cases} 1 & i = 1 \\ 8i - 9 & 2 \leq i \leq \frac{n+1}{2} \\ 8n - 8i + 10 & \frac{n+3}{2} \leq i \leq n \end{cases}$$

The edge labels of Sl_n are the following:

$$f^*(u_iu_{i+1}) = \begin{cases} 4 & i = 1 \\ 8i - 7 & 2 \leq i \leq \frac{n+1}{2} \\ 8n - 8i + 8 & \frac{n+3}{2} \leq i \leq n - 1 \end{cases}$$

$$f^*(u_1u_n) = 6$$

$$f^*(u_iv_i) = \begin{cases} 2 & i = 1 \\ 8i - 10 & 2 \leq i \leq \frac{n+1}{2} \\ 8n - 8i + 11 & \frac{n+3}{2} \leq i \leq n \end{cases}$$

In view of the above labeling, f(V (Sl_n))f(E(Sl_n)) = {1, 2, 3, . . ., p + q}.

Case 2

n is even, (n ≥ 4).

Subcase 1

n = 4.

If n = 4 then the graph Sl_4 $Q_{2o}K_1$ which is a Super Root Square Mean Graph [3].

Subcase 2

n ≥ 6.

Define the function f: V (Sl_n) → {1, 2, 3, . . . , p + q} by:

$$f(u_i) = \begin{cases} 2i + 1 & 1 \leq i \leq 2 \\ 8i - 8 & 3 \leq i \leq \frac{n+2}{2} \\ 8n - 8i + 9 & \frac{n+4}{2} \leq i \leq n \end{cases}$$

$$f(v_i) = \begin{cases} i^3 & 1 \leq i \leq 2 \\ 8i - 10 & 3 \leq i \leq \frac{n+2}{2} \\ 8n - 8i + 11 & \frac{n+4}{2} \leq i \leq n \end{cases}$$

The edge labels of Sl_n are the following:

$$f^*(u_iu_{i+1}) = \begin{cases} 3 & i = 1 \\ 8i - 4 & 2 \leq i \leq \frac{n}{2} \\ 8n - 8i + 5 & \frac{n+2}{2} \leq i \leq n - 1 \end{cases}$$

$$f^*(u_1u_n) = 6$$

$$f^*(u_1v_i) = \begin{cases} 2 & i = 1 \\ 8i - 9 & 2 \leq i \leq \frac{n+2}{2} \\ 8n - 8i + 10 & \frac{n+4}{2} \leq i \leq n \end{cases}$$

In view of the above labeling, f(V (Sl_n)) f(E(Sl_n)) = {1, 2, 3, . . . , p + q}. Therefore, the Sunlet Graph Sl_n is a Super Root Square Mean Graph for all n ≥ 3.

Theorem 3.4

The Middle Cycle Graph M(C_n) is Super Root Square Mean Graph for all n ≥ 3.

Proof. Let V (M (C_n)) = {v_1, v_2, . . ., v_n, u_1, u_2, . . ., u_n} be the vertex set and E(M (C_n)) = {u_iu_{i+1}, u_1u_n, u_iv_{i+1}, u_nv_1 for 1 ≤ i ≤ n − 1 and v_iu_i for 1 ≤ i ≤ n} be the edge set of the Middle Cycle Graph M(C_n), n ≥ 3 as shown in figure 2. To prove the theorem, we will consider the following cases.

Case 1

n is odd, (n ≥ 3).

Subcase 1

The super root square mean labeling of Middle Cycle Graph M (C_3) shownin figure 3.

Subcase 2

n ≥ 5.

Define the function f: V (M(C_n)) → {1, 2, 3, . . ., p + q} by:

$$f(v_i) = \begin{cases} 1 & i = 1 \\ 10i - 14 & 2 \leq i \leq \frac{n+1}{2} \\ 5n & i = \frac{n+3}{2} \\ 10n - 10i + 13 & \frac{n+5}{2} \leq i \leq n \end{cases}$$

$$f(u_i) = \begin{cases} 3 & i = 1 \\ 10i - 8 & 2 \leq i \leq \frac{n+1}{2} \\ 10n - 10i + 10 & \frac{n+3}{2} \leq i \leq n - 1 \\ 8 & i = n \end{cases}$$

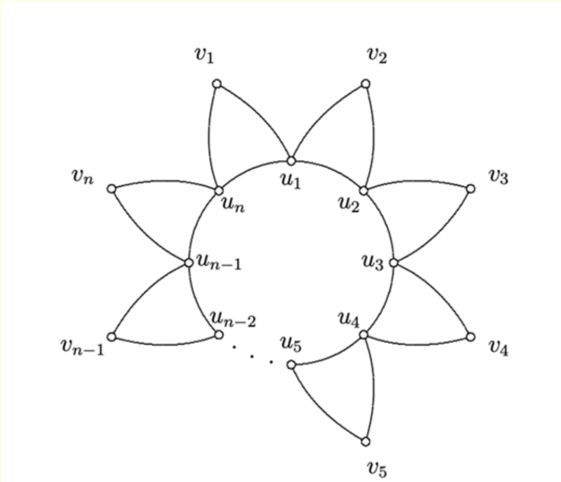

Figure 2: A middle cycle graph $M(C_n)$.

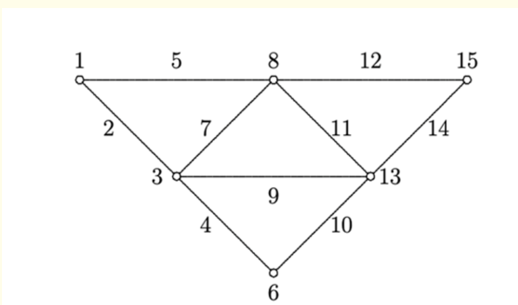

Figure 3: A middle cycle graph $M(C_3)$ and its super root square mean labeling.

The edge labels of $M(C_n)$ are the following:

$$f^*(u_iu_{i+1}) = \begin{cases} 9 & i = 1 \\ 10i - 2 & 2 \le i \le \frac{n-1}{2} \\ 5n - 4 & i = \frac{n+1}{2} \\ 10n - 10i + 5 & \frac{n+3}{2} \le i \le n - 1 \end{cases}$$

$$f^*(u_1u_n) = 7$$

$$f^*(v_iu_i) = \begin{cases} 2 & i = 1 \\ 10 & i = 2 \\ 10i - 11 & 3 \le i \le \frac{n+1}{2} \\ 5n - 2 & i = \frac{n+3}{2} \\ 10n - 10i + 11 & \frac{n+5}{2} \le i \le n \end{cases}$$

$$f^*(u_iv_{i+1}) = \begin{cases} 10i - 6 & 1 \le i \le \frac{n+1}{2} \\ 10n - 10i + 7 & \frac{n+3}{2} \le i \le n - 1 \end{cases}$$

$$f^*(u_nv_1) = 5$$

In view of the above labeling, $f(V(M(C_n)))f(E(M(C_n))) = \{1, 2, 3, \ldots, p + q\}$.

Case 2

n is even, $(n \geq 4)$.

Subcase 1

$n = 4$.

The super root square mean labeling of Middle Cycle Graph $M(C_4)$ shownin figure 4.

Subcase 2

$n \geq 6$.

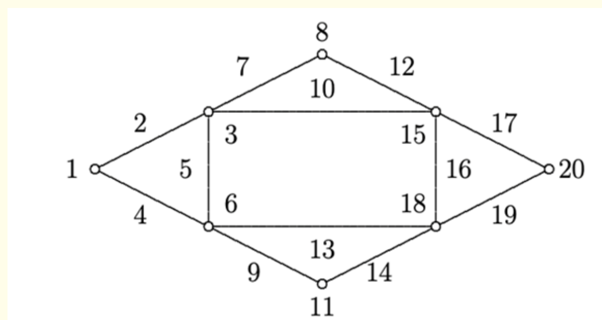

Figure 4: A middle cycle graph $M(C_4)$ and its super root square mean labeling.

Define the function $f\colon V(M(C_n)) \to \{1, 2, 3, \ldots, p + q\}$ by:

$$f(v_i) = \begin{cases} 1 & i = 1 \\ 10i - 12 & 2 \le i \le \frac{n}{2} \\ 5n & i = \frac{n+2}{2} \\ 10n - 10i + 11 & \frac{n+4}{2} \le i \le n \end{cases}$$

$$f(u_i) = \begin{cases} 3 & i = 1 \\ 10i - 5 & 2 \le i \le \frac{n}{2} \\ 5n - 2 & i = \frac{n+2}{2} \\ 10n - 10i + 7 & \frac{n+4}{2} \le i \le n - 1 \\ 6 & i = n \end{cases}$$

The edge labels of $M(C_n)$ are following:

$$f^*(u_iu_{i+1}) = \begin{cases} 10i & 1 \le i \le \frac{n-2}{2} \\ 5n-4 & i = \frac{n}{2} \\ 10n-10i+3 & \frac{n+2}{2} \le i \le n-1 \end{cases}$$

$$f^*(u_1u_n) = 5$$

$$f^*(v_iu_i) = \begin{cases} 10i-8 & 1 \le i \le \frac{n}{2} \\ 10n-10i+9 & \frac{n+2}{2} \le i \le n \end{cases}$$

$$f^*(u_iv_{i+1}) = \begin{cases} 7 & i = 1 \\ 10i-4 & 2 \le i \le \frac{n-2}{2} \\ 5n-3 & i = \frac{n}{2} \\ 10n-10i+4 & \frac{n+2}{2} \le i \le n-1 \end{cases}$$

$$f^*(u_nv_1) = 4$$

In view of the above labeling, $f(V(M(C_n)))f(E(M(C_n))) = \{1, 2, 3, \ldots, p + q\}$.

Therefore, the Middle Cycle Graph $M(C_n)$ is a Super Root Square MeanGraph for all $n \geq 3$.

Theorem 3.5

The Polygonal Chain Graphis Super Root Square Mean Graph for all $m \geq 1$, $n \geq 3$.

Proof. Let $V(G_n^m) = \{v_2^1, v_{2,\ldots,}^1 v_{n,}^1 v_2^2, v_{2,\ldots,}^2 v_{n,}^2 v_2^m, v_{2,\ldots,}^m v_n^m\}$ be the vertex set and $E(G_n^m) = \{e_j^i$ for $1 \le i \le m, 1 \le j \le n\}$ be the edge set of the Polynomial Chain Graph (G_n^m), $m \geq 1$, $n \geq 3$ as shown in figure 5. Note that when n is odd, $v_{(n+3)/2}^i = v_1^{i+1}$. Likewise, when n is even, $v_{(n+2)/2}^i = v_1^{i+1}$.

To prove the theorem, we will consider the following cases.

Case 1

n is odd, $(n \geq 3)$ and $m \geq 1$.

Subcase 1

$m = 1$ and $n \geq 3$.

If $m = 1$ and $n \geq 3$ then the graph C_n, $n \geq 3$ which is a Super Root Square Mean Graph [9].

Subcase 2

$m \geq 2$ and $n = 3$.

If $m \geq 2$ and $n = 3$ then the graph T_n, $n \geq 2$ which is a Super Root Square Mean Graph [8].

Subcase 3

$m \geq 2$ and $n \geq 5$.

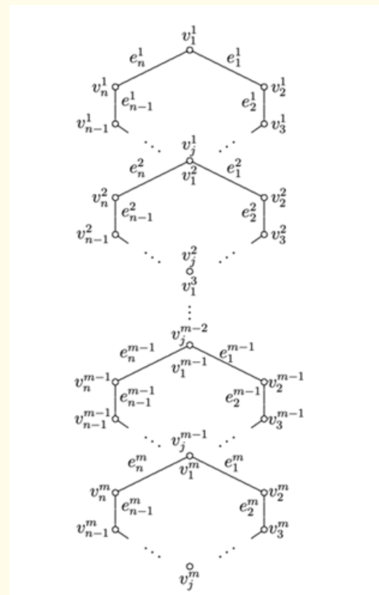

Figure 5: A polygonal snake G_n^m.

Define the function f: V () → {1, 2, 3, …, p + q} by:

$$f(v_j^i) = \begin{cases} (2n-1)(i-1)+1 & 1 \le i \le m,\ j = 1 \\ (2n-1)(i-1)+4j-5 & 1 \le i \le m,\ 2 \le j \le \frac{n+1}{2} \\ (2n-1)(i-1)+4n-4j+6 & 1 \le i \le m,\ \frac{n+3}{n} \le j \le n \end{cases}$$

The edge labels of are the following:

$$f^*(e_j^i) = \begin{cases} (2n-1)(i-1)+2 & 1 \le i \le m,\ j = 1 \\ (2n-1)(i-1)+4j-3 & 1 \le i \le m,\ 2 \le j \le \frac{n+1}{2} \\ (2n-1)(i-1)+4(n-j+1) & 1 \le i \le m,\ \frac{n+3}{2} \le j \le n \end{cases}$$

In view of the above labeling, f(V (M ())f(E(M ()) = {1, 2, 3, . . . ,p + q}.

Case 2

n is even, (n ≥ 4) and m ≥ 1.

Subcase 1:

m = 1 and n ≥ 4.

If m = 1 and n ≥ 4 then the graph $G_{1,n}$ C_n, n ≥ 4 which is a Super Root Square Mean Graph [9].

Subcase 2

m ≥ 2 and n ≥ 4.

Define the function f: V () → {1, 2, 3, . . . , p + q} by:

$$f(v_j^i) = \begin{cases} (2n-1)(i-1)+1 & 1 \le i \le m,\ j=1 \\ (2n-1)(i-1)+4j-4 & 1 \le i \le m,\ 2 \le j \le \frac{n+2}{2} \\ (2n-1)(i-1)+4n-4j+5 & 1 \le i \le m,\ \frac{n+4}{n} \le j \le n \end{cases}$$

The edge labels of are the following:

$$f^*(e_j^i) = \begin{cases} (2n-1)(i-1)+4j-2 & 1 \le i \le m,\ 1 \le j \le \frac{n}{2} \\ (2n-1)(i-1)+4n-4j+3 & 1 \le i \le m,\ \frac{n+2}{2} \le j \le n \end{cases}$$

In view of the above labeling, f(V (M ())f(E(M ()) = {1, 2, 3, . . ., p + q}.

Therefore, the Polygonal Chain Graph is a Super Root Square Mean Graph for all m ≥ 1 and n ≥ 3.

Theorem 3.6

The Alternate Polygonal Chain Graph is Super Root Square Mean Graph for all m ≥ 1, n ≥ 3.

Proof. Let V () {} be the vertex set and E () = {for 1 ≤ i ≤ m, 1 ≤ j ≤ n} be the edge set of the Alternate Polynomial Chain Graph , m ≥ 1, n ≥ 3 as shown in figure 6. Note that the edge that connects the disconnected cycles is the edge for 1 ≤ i ≤ m-1 when n is odd and the edge for 1 ≤ i ≤ m-1 when n is even. To prove the theorem, we will consider the following cases.

Case 1

n is odd, (n ≥ 3) and m ≥ 1.

Subcase 1

m = 1 and n ≥ 3.

If m = 1 and n ≥ 3 then the graph , n ≥ 3 which is a Super Root Square Mean Graph [9].

Subcase 2

m ≥ 2 and n ≥ 3.

Define the function f: V () → {1, 2, 3, . . ., p + q} by:

$$f(v_j^i) = \begin{cases} (2n+1)(i-1)+1 & 1 \le i \le m,\ j=1 \\ (2n+1)(i-1)+4j-5 & 1 \le i \le m,\ 2 \le j \le \frac{n+1}{2} \\ (2n+1)(i-1)+4n-4j+6 & 1 \le i \le m,\ \frac{n+3}{2} \le j \le n \end{cases}$$

The edge labels of are the following:

$$f^*(e_j^i) = \begin{cases} (2n+1)(i-1)+2 & 1 \le i \le m,\ j=1 \\ (2n+1)(i-1)+4j-3 & 1 \le i \le m,\ 2 \le j \le \frac{n+1}{2} \\ (2n+1)(i-1)+4(n-j+1) & 1 \le i \le m,\ \frac{n+3}{2} \le j \le n \end{cases}$$
$$f^*(\bar{e}^i) = (2n+1)i \qquad 1 \le i \le m-1$$

In view of the above labeling, f(V (M ())f(E(M ()) = {1, 2, 3, . . . ,p + q}.

Case 2

n is even, (n ≥ 4) and m ≥ 1.

Subcase 1

m = 1 and n ≥ 4.

If m = 1 and n ≥ 4 then the graph , n ≥ 4 which is a Super Root Square Mean Graph [9].

Subcase 2

m ≥ 2 and n ≥ 4.

Define the function f: V () → {1, 2, 3, . . ., p + q} by:

$$f(u_j^i) = \begin{cases} (2n+1)(i-1)+1 & 1 \le i \le m,\ j=1 \\ (2n+1)(i-1)+4j-4 & 1 \le i \le m,\ 2 \le j \le \frac{n+2}{2} \\ (2n+1)(i-1)+4n-4j+5 & 1 \le i \le m,\ \frac{n+4}{n} \le j \le n \end{cases}$$

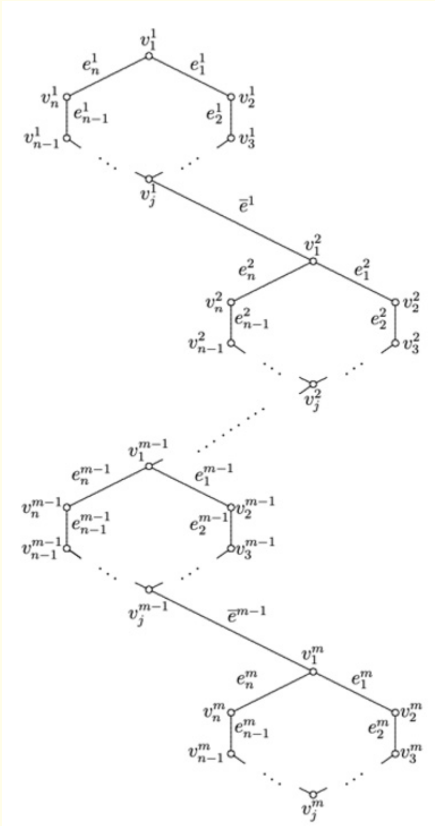

Figure 6: An alternate polygonal snake $\langle C_n : m\rangle$.

The edge labels of are the following:

$$f^*(e^i_j) = \begin{cases} (2n+1)(i-1)+4j-2 & 1 \le i \le m,\ 1 \le j \le \frac{n}{2} \\ (2n+1)(i-1)+4n-4j+3 & 1 \le i \le m,\ \frac{n+2}{2} \le j \le n \end{cases}$$
$$f^*(\bar{e}^i) = (2n+1)i \qquad 1 \le i \le m-1$$

In view of the above labeling,

$f(V(M(\langle C_n : m\rangle))) \cup f(E(M(\langle C_n : m\rangle))) = \{1, 2, 3, \ldots, p+q\}$.

Therefore, the Alternate Polygonal Chain Graph is a Super Root Square Mean Graph for all m ≥ 1 and n ≥ 3.

Theorem 3.7

The Kayak Paddle Graph KP(n,m,t) is Super Root Square Mean Graph for all n = m, n ≥ 3 and t ≥ 1.

Proof. Let $V(KP(n,n,t)) = \{v_1, v_2, \ldots, v_n, w_1, w_2, \ldots w_t, u_1, u_2, \ldots, u_n\}$ be the vertex set and $E(KP(n,n,t)) = \{e_i$, for $1 \le i \le n+t-1$ and $\bar{e}_i$ for $1 \le i \le n\}$ be the edge set of the Kayak Paddle Graph $KP(n,n,t)$ where $n \ge 3$ and $t \ge 1$ as shown in figure 7. Note that when n is odd, , when n is even, for all n, .

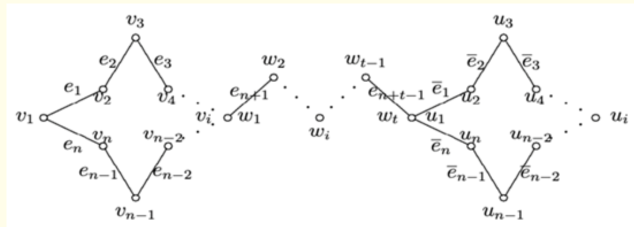

Figure 7: A kayak paddle graph KP (n, n, t),

To prove the theorem, we will consider the following cases.

Case 1

n is odd, (n ≥ 3) and t ≥ 1.

Subcase 1

n ≥ 3 and t = 1.

If n ≥ 3 and t = 1 then the graph KP (n, n, 1) , n ≥ 3 which is a Super Root Square Mean Graph by Theorem 3.5.

Subcase 2

n ≥ 3 and t = 2.

If n ≥ 3 and t = 2 then the graph KP (n, n, 2) , n ≥ 3 which is a Super Root Square Mean Graph by Theorem 3.6.

Subcase 3

n ≥ 3 and t ≥ 3.

Define the function f: V (KP (n, n, t)) → {1, 2, 3, . . ., p + q} by:

$$f(v_i) = \begin{cases} 1 & i = 1 \\ 4i-5 & 2 \le i \le \frac{n+1}{2} \\ 4n-4i+6 & \frac{n+3}{2} \le i \le n \end{cases}$$
$$f(w_i) = 2n+2i-2 \qquad 1 \le i \le t$$
$$f(u_i) = \begin{cases} 2n+2t-2 & i = 1 \\ 2n+2t+4i-8 & 2 \le i \le \frac{n+1}{2} \\ 6n+2t-4i+3 & \frac{n+3}{2} \le i \le n \end{cases}$$

The edge labels of KP (n, n, t) are the following:

$$f^*(e_i) = \begin{cases} 2 & i = 1 \\ 4i-3 & 2 \le i \le \frac{n+1}{2} \\ 4n-4i+4 & \frac{n+3}{2} \le i \le n \\ 2i-1 & n+1 \le i \le n+t-1 \end{cases}$$

$$f^*(\bar{e}_i) = \begin{cases} 2n+2t-1 & i = 1 \\ 2n+2t+4i-6 & 1 \le i \le \frac{n+1}{2} \\ 6n+2t-4i+1 & \frac{n+3}{2} \le i \le n \end{cases}$$

In view of the above labeling, f(V (M(KP(n,n,t)))f(E(M(KP (n,n,t))) = {1, 2, 3, . . . ,p + q}.

Case 2

n is even, (n ≥ 4) and t ≥ 1.

Subcase 1

n ≥ 4 and t = 1.

If n ≥ 4 and t = 1 then the graph KP (n, n, 1) , n ≥ 4 which is a Super Root Square Mean Graph by Theorem 3.5.

Subcase 2

n ≥ 4 and t = 2.

If n ≥ 3 and t = 2 then the graph KP (n, n, 2) , n ≥ 4 which is a Super Root Square Mean Graph by Theorem 3.6.

Subcase 3

n ≥ 4 and t ≥ 3.

Define the function f: V (KP (n, n, t)) → {1, 2, 3,…, p + q} by:

$$f(v_i) = \begin{cases} 1 & i = 1 \\ 4i-4 & 2 \le i \le \frac{n+2}{2} \\ 4n-4i+5 & \frac{n+4}{2} \le i \le n \end{cases}$$

$$f(w_i) = \quad 2n+2i-2 \qquad 1 \le i \le t$$

$$f(u_i) = \begin{cases} 2n+2t-2 & i = 1 \\ 2n+2t+4i-7 & 2 \le i \le \frac{n+2}{2} \\ 6n+2t-4i+2 & \frac{n+4}{2} \le i \le n \end{cases}$$

The edge labels of KP (n, n, t) are the following:

$$f^*(e_i) = \begin{cases} 4i-2 & 1 \le i \le \frac{n}{2} \\ 4n-4i+3 & \frac{n+2}{2} \le i \le n \\ 2i-1 & n+1 \le i \le n+t-1 \end{cases}$$

$$f^*(\bar{e}_i) = \begin{cases} 2n+2t-1 & i = 1 \\ 2n+2t+4i-5 & 1 \le i \le \frac{n}{2} \\ 6n+2t-4i & \frac{n+2}{2} \le i \le n \end{cases}$$

In view of the above labeling, f(V (M(KP(n,n,t)))f(E(M(KP (n,n,t))) = {1, 2, 3, . . . ,p + q}.

Therefore, the Kayak Paddle Graph KP (n, m, t) is a Super Root Square Mean Graph for all n = m, n ≥ 3 and t ≥ 1.

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