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Research Article

Validity Judgment of an EOQ Model Under Customers' Feedback

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Abstract

In this article, we have developed a classical backorder EOQ model. Constructing a cost function of the proposed model we optimize it with the help of Lingo 16.0 software. Taking sensitivity analysis of the proposed model we compute the mean, median and mode using those data sets and get their corresponding average inventory costs. Then we make a public judgment through the responses true or false over any of the cost values from model optimum, mean, median and mode values of the model itself. Incorporating phi-coefficient curves for the decision variables we have searched their point of intersections for validity testing.

Keywords: EOQ Model; Mean; Median; Mode; Phi-coefficient

Introduction

General overview

The main purpose of inventory management is helping a company for their unexpected fluctuations in consumption and supply. For this, a company can move a continuous production process. In this time modern researchers are handling inventory with the help of fuzzy environment. But nobody give attention to public judgment. Arrow., *et al*. [1] developed a method for deriving the optimal rules of inventory policy for finished goods. After that Arrow., *et al*. [2] proposed some mathematical theory of inventory and production. Kenkel [3] developed introductory statistics for management and economics which is specially designed for business. Kennedy and Bush [4] gave a proposed model for designing and analysis of experiments for behavioral research. Mehra., *et al*. [5] gave an introduction to the validity of the EOQ formula will be different under inflationary conditions. Nowadays in the finance industry, government and corporate sectors the main critical issue is the fraud. So past few years, many researchers are working on fraud inventory. Debreceny and Gray [6] gave an introduction on applying data mining techniques for journal entries. They also found the dataset with fraud indicators and using the data mining techniques compared it with other data. Hesse and Jr., [7] introduced seven practical recommendations for preventing and detecting fraud in this unique environment to fraud investigators. For fraud detection in financial statements, Kanapickiene and Grundiene [8] introduced a logistic regression model. They also illustrated financial ratios which help to find the fraud in financial statements. For detecting misstatements with fraud intention Kim., *et al*. [9] introduced a multi class financial misstatement detection model. The most ruinous risk lies in internal fraud and manufacturing firms face individual risk. Mu and Carroll, [10] proposed a fraud risk decision model in manufacturing firms for focus on fraud risk cases. West and Bhattacharya [11] gave a complete classification of existing fraud detection literature with the help of a detection algorithm. Burton [12] gave a numerous ways in which Leslie Fay ensured that quarterly sales met pre-established budgets.

Fahrmeir and Tutz [13] mainly introduced on a multivariate Statistical Model which may be estimated similarly to generalize linear models. Angulo., *et al*. [14] developed semi parametric

statistical approaches to estimate and predict space-time processes. Pruscha and Gottlein, [15] introduced the data of forest inventory which help to focusing on the space and time dependencies of the data. Besta., *et al*. [16] developed a model which utilized of some selected statistical tools in the area of inventory management. Ayloo., *et al*. [17] introduced a model which a gave relationship between bariatric surgical procedure and beck depression inventory. Iqbal., *et al*. [18] have developed a method which helps to verifying the statistical significance of multi-criteria inventory models with respect to customer order fill rates including statiscal methods and sensitivity analysis. Mutschler [19] designed a DSGE model for higher-order statistics.

De and Pal [20] proposed a decision-making model which may be helps book producer's conflict demand. Beyond the deterministic approach, researchers are shown their interested in fuzzy uncertainty modeling. De and Sana [21-24] developed a fuzzy backlogging model. De and Beg gave conceptions on dense fuzzy set and also its contemporary applications in inventory modeling are discussed by De and Mahata [25-27]. Maity., *et al*. [28] explained the heptagonal fuzzy set and its application in modeling, also for the pioneering works on intuitionistic dense fuzzy set by Maity., *et al*. [29] also analyzed its application through the utilization of score function. Many articles on inventory modeling have been developed after this invention by eminent researchers Karmakar., *et al*. [30,31]; Maity., *et al*. [32]; Maity., *et al*. [33]; Chakraborty., *et al*. [34]; Nobil., *et al*. [35]; Khan., *et al*. [36]; Rahman., *et al*. [37], Maity., *et al*. [38-41].

The main focus of interest of this article is customer judgment of an EOQ model. A statistical survey has taken over public agreement and disagreement by the retailer's view. Then we have solved the EOQ model and done the sensitivity analysis of this model. After calculating mean, median and mode of order quantity, backorder quantity and average cost from the sensitivity analysis table we have obtained the Phi-coefficient of order quantity, backorder quantity and average cost. The intersection of these three Phi-coefficient curves in the positive region of the coordinate axes indicates the actual range of the validity of our proposed model.

Specific study

Generally developing a model first is the main objectives of an inventory practitioner. Ten they introduced the model along with proposed constraints under some methodologies. Any kind of model can be fitted into any category of customers are assumed by the traditional inventory managers. Similar responses of the customers from different social setups might have in favors of a particular inventory. It is quite unnatural in practice and hence framing a model is much more difficult which is consistent with the customer's needs/attitude/behavior over the inventory of that particular place. For our proposed model common people play a vital role because the major parts of the customers from among them and are responsible because for the acceptance or rejection of an inventory set up socially in respective places. Though the managers have the supportive documents and survey/audit reports in favor of installing an inventory in a particular place but in many cases fraudulent is to be founded. So, in that case the truth must come from public choice. For this reason the truth of reality and fate of an inventory go side by side silently over the sphere of insights of the DM of any kind of inventory process. This is known by the inventory practitioner however the actual facts which have been ongoing whether right or wrong in the inventory process itself. To avoid minimum cost and sustain the real-world competition in many cases the DMs usually hide the actual facts from their customers. An another logic is that in some case it is shown that though some inventories have poor demand (fewer customers) but they continuously declare that they have huge number of customers to buy their products because the quality of the product is good and legal government transactions. But the facts don't agree in the actual sense by the claim of the customers. Day by day in this way the DM's are usually promoting their policy to motivate customers. There are few+ cases which may arise from the customers' side.

- Common people are shown their interested in knowing the maximum profit of an inventory practitioner rather than minimum cost.
- The main problem is that Common people usually believe the verbal message declared from the DM's desk or from his/her representatives during the item's delivery on demand.
- Whether it is true or false learned customers usually test or judge the forecasted message with real practice.
- So learned customers might agree with the DM's view or otherwise might have a disagreement with the DM's declaration based on the motivational study.

- We may say the inventory model is socially valid If the customers are satisfied through the agreement of positive motivational strategies otherwise the model is invalid. In other words we may call, the inventory model has social existence else without social existence the model is of virtual inventory or pseudo inventory or dummy inventory.

However, from the DM's behavior on inventory set up the following three views may arise:

- All transactions are of white paper (money) and government taxations are neat and clean and up to date. This can only be possible when profit is usually high and no other inventory set up is available in the market because the market is less competitive.
- Some transactions are of white papers and some others are of black papers. Here the govt. generates revenue partly. The inventory policy is a mixture of an actual transaction and dummy. That means it is a mixture of true and false. To qualify the models for the development of society in this situation it is critical. For this, it is necessary that the validation of the model should be judged under the social aspect.
- When the case arises such that it is not possible to earn a considerable amount (very high) of profit so that the inventory can last in crucial competition. For this case then all transactions might be performed in black papers (statement manipulation). This type of inventory is usually known as dummy inventories. In general this type of business under study may be called an underworld business. Government generates no revenue [the cases of parallel economy, terrorism]. For such type of business and has no social acceptance and hence the model is invalid and has no social existence by virtue of truth.

By the view of the DM's point it is impossible to run dummy inventory to sustain within the society because the real world is more competitive. The main focus of attention of the paper is to consider the inventory model having partly genuine and partly dummy (pseudo inventory) inventory.

Generally in practical case DM frames the policy such that some parts are true and the rest of them are false. As the profit mostly depend on the actual cost of the inventory the ultimate cost will not be declared by the DM in the paper work because of exemption from the income tax department of the concerned government. Generally public know the fact, that the inventory managers are making more cost or are not in genuine transactions.

Now we develop a backorder inventory model inspire by the case study. Generally, an inspection team set for a common questionnaire for the inventory practitioner between two public who is not a customer of that inventory as well as for the common public who is generally a customer of society first to know the facts of the concerned case. As we know that for any kind of statistical information central tendencies is the best popular measure so for the customer's feedback Mean, Median and Mode and the model optimum is taken here. Against each claim of cost the customer have a response like agreement and disagreement. To getting/earning cost value the claims of the inventory practitioner may be assumed by the response either of the model optimum, or mean, median and mode any one of these which may be true or false. Then we have a set of responses of the form (agree, disagree) against the claims which may be (true, false). We may check these responses as dichotomous variables and then here we use phi-coefficient to know correlation between DM's claims and public judgments. Generally the model is valid if the positive correlation indicates the DM's claim over public judgment is true and that of negative correlation indicates that the DM's claim over the public judgment is not true then the model is invalid. We compute phi-coefficient for order quantity, backorder quantity and the average cost itself over all possible cases for this paper. To check the actual validity of the model here we consider several validation hypotheses over phi coefficient tests. Then we gave graphical illustrations to the corresponding curves for each of them. At last we made a conclusion followed by the scope of future work based on our proposed model.

Case Study

For the case study of our proposed model we visited a cosmetic company 'Elco Cosmetics Pvt. Ltd' situated at Kolkata. After a long time discussion with the manager we collect some data regarding our model and we came to know that they are selling high-quality products and customers are fully satisfied with their products. The products basically are hair shampoos, face washes, body lotions, suns creams, and face creams. They strongly claim that customers are fully satisfied with their product as these products work nicely. We visited different selling counters of this company which are situated in Kolkata city and abroad. After visiting the different

counter of the company we collect the information regarding our proposed model then met with many customers at every counter. But they gave us different feedback regarding the quality of the products. The summary of the data so obtained is given below.

	Company's forecast			Customer's feedback		
	Order Quantity	Backorder Quantity	Cost	Order Quantity	Backorder Quantity	Expected Cost
Product A	465	130	535	445	147	449
Product B	425	175	555	400	155	520
Product C	441	150	519	423	140	470

Table 1: Data available the case study.

Holding cost $(H_c) = 2.5$, Shortage cost $(S_c) = 1.8$, Set up cost $(S_u) = 1200$, Demand $(d_1) = 100$.

Preliminaries

To get information from data statistics the best way. From a set of numbers we creating a tool for new understanding. For a statistics practitioner population is the group of all items of interest Then from the population we drawn a sample set of data. We calculate mean, median, and mode of the sample to get more information about the sample.

Mean

The sample mean is the average and is computed as the sum of all the observed outcomes from the sample divided by the total number of events. We use $\bar{x}$ as the symbol for the sample mean. The formula is given below $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, where n is the sample size. This is the most popular and useful measure of central location. It may be called the arithmetic mean to distinguish from the other mean. When each number x_i is to have weight w_i, the weighted mean can be defined ;$\bar{x} = \frac{w_1x_1+w_2x_2+\cdots+w_nx_n}{w_1+w_2+\cdots+w_n}$. In this paper to calculate mean we use the weighted mean formula.

Median

The median is the middle score. To find the median, first we have to rearrange the set of numerical data in ascending order. Then the median is the middle observation for an odd number of observations, and is the average of the two middle observations for an even number of observations. The formula which we have used in this paper to calculate median is given below. Median = $l + \left[\frac{\frac{n}{2}-cf}{f}\right] \times h$

Where, l = lower bound of median class, n = number of observations, f = frequency of median class. cf = Cumulative frequency of the class before the median class, h = length of median class.

Mode

The mode of a set of data is the number with the highest frequency, that one occurs the maximum number of times. The formula which we have used in this paper to calculate the mode is given below. Mode= $l + \left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right) \times h$. Where, l = lower bound of modal class frequency of the class before of modal class, f_1 =frequency of modal class, f_2 =frequency of the class after the modal class, h = length of the modal class.

Phi-(φ) coefficient

The phi coefficient is an important concept in statistic. It is also known "mean square contingency coefficient". It is generally denoted by r_φ or simply φ. Karl Pearson first discovered Phi coefficient. It measures the amount of association between two binary variables. The significance of the correlation coefficient and measure of phi coefficient are nearly the same. Actually, Pearson correlation coefficient for two binary variables is nothing but the phi coefficient. Also, Pearson product-moment correlation coefficient and the numerical result of phi correlation coefficient both are similar. Phi coefficient is based on chi-square coefficient which determines the statistical relationship between two variables. As, Phi coefficient has a known sampling distribution, it is very easy for computation of significance and standard deviation of the phi coefficient.

X / Y	True	False
Agree	p_1	p_2
Disagree	Q_1	Q_2

Table 2: Phi-coefficient matrix.

If P_1, P_2, Q_1 and Q_2 are the frequencies of the observations then φ is determined by the formula

$$\varphi = \frac{P_1Q_2 - P_2\,Q_1}{\sqrt{(P_1 + P)(Q_1 + Q)(P_2 + Q_2)(P_1 + Q_1)}}$$

The phi coefficient is very useful in many fields. Particularly, it is used for educational as well as psychological testing in which we frequently use a dichotomous continuous variable. When the variables having categories like pass or fail, success or failure, yes or no, agree or disagree, are observed, Phi coefficient can be utilized.

Considerations on model validation

We shall draw a logical decision for which we can conclude whether a model is practically valid or not. The following are the possible reasons of model validations.

- If φ assumes positive values then the model is valid. If φ assumes negative values then the model is invalid.
- If φ coefficient curves for several decision variables must intersect at least at one positive point then the model is valid.
- If φ coefficient curves for several decision variables do not intersect at all or all of them meet at a negative point in this case the model is invalid.
- If some of the φ coefficient curves for several decision variables intersect and few of them do not intersect at all then no decision can be made.

Crisp mathematical model

Notations and assumptions

Notations

H_c: Holding cost per quantity per unit time ($)

S_c: Shortage cost per unit quantity per unit time ($)

S_u: Set up cost per unit time period per cycle ($)

d_2: Demand in shortage period $(d_2 = d_1 e^{-t_2})$

I_1: Inventory level within time t_1 (days)

I_2: Shortage quantity during the time t_2 (days)

t_1: Inventory run time (days)

t_2: Shortage time (days)

T: Cycle time $(= t_1 + t_2)$ (days)

Z_c: Total average cost ($)

Assumptions

We have the following assumptions

- Demand rate is uniform and known
- Rate of replenishment is finite
- Lead time is zero/negligible
- Shortage are allowed and fully backlogged.

Model formulation

Let the inventory starts at time with order quantity and demand rate d. After time the inventory reaches zero level and the shortages starts and it continue up to time .

Let be the shortage quantity during that time period Also we assume that the shortage time demand rate is depending on the duration of shortage time . Therefore, the mathematical problem associated to the proposed model is shown in Figure 1 and the necessary calculation is given below.

Inventory holding cost = $\frac{1}{2}H_cI_1t_1$ ----------(1)

Shortage cost = $\frac{1}{2}S_cI_2t_2 = \frac{1}{2}S_cd_1(1 - e^{-t_2})\,t_2$ ---------(2)

Set up cost = S_u ---------(3)

Where $I_1 = d_1t_1$ ---------- (4)

And $I_2 = \int_0^{t_2} d_2\,dt_2 = d_1\int_0^{t_2} e^{-t_2}\,dt_2 = d_1(1 - e^{-t_2})$ ----------(5)

$T = t_1 + t_2$ ----------(6)

Therefore, the total average inventory cost is given by

$$Z_c = \frac{1}{2T}H_cI_1t_1 + \frac{1}{2T}S_cI_2t_2 + \frac{1}{T}S_u$$

$$= \frac{1}{2}H_c\,d_1\frac{t_1^2}{t_1+t_2} + \frac{1}{2}S_cd_1\frac{(1-e^{-t_2})\,t_2}{t_1+t_2} + \frac{S_u}{t_1+t_2} \quad \text{----------(7)}$$

$$= d_1\left\{\frac{1}{2}H_c\frac{t_1^2}{t_1+t_2} + \frac{1}{2}S_c\frac{(1-e^{-t_2})\,t_2}{t_1+t_2}\right\} + \frac{S_u}{t_1+t_2}$$

Therefore our problem is redefined as

Minimize $Z_c = d_1\Psi + \varphi$ ---------- (8)

Where $\begin{cases}\Psi = \frac{1}{2}H_c\frac{t_1^2}{t_1+t_2} + \frac{1}{2}S_c\frac{(1-e^{-t_2})\,t_2}{t_1+t_2} \\ \varphi = \frac{S_u}{t_1+t_2}\end{cases}$ ---------- (9)

Subject to the conditions (4-6).

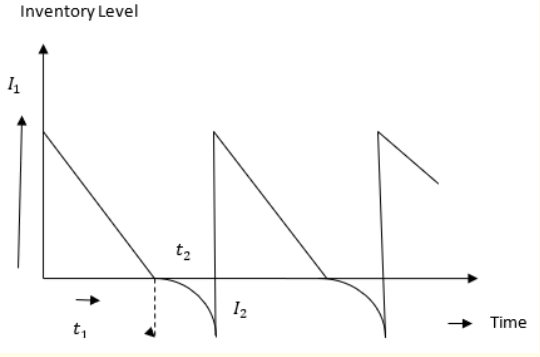

Figure 1: Logistic diagram of the EOQ model.

Numerical result

Let us consider H_c=2.5; S_c=1.8; S_u=1200; d_1=100; T=7; then we get the fallowing result Z_c=521.2791; t_1*=4.3500000; t_2*=2.650000; I_1*=265.0000; I_2*=92.93488.

Sensitivity analysis

Based on the numerical example considered above for the EOQ model, we now calculate the corresponding outputs for the changing inputs one by one. Again the sensitivity analysis by changing of each parameters, d_1, H_c, S_c and S_u by +40%, +20%, -20%,-40%, considering one at a time and keeping the remaining parameters unchanged.

Parameter	% Change	Time (t_1*)	Time (t_2*)	I_1*	I_2*	Minimum Cost (Z_c*)	(Z_c*- Z_c)/(Z_c) 100%
d_1	+40	4.36	2.64	369.60	130.00	663.38	0.273
	+20	4.305	2.695	323.40	111.89	582.95	0.118
	-20	4.295	2.705	216.40	74.65	444.55	-0.147
	-40	4.225	2.775	166.50	56.26	369.92	-0.290
H_c	+40	4.355	2.645	264.50	92.89	657.52	0.2613
	+20	4.36	2.64	264.00	92.86	590.71	0.133
	-20	4.37	2.63	263.50	92.83	455.55	-0.126
	-40	4.37	2.63	263.00	92.79	387.96	-0.256
S_c	+40	4.30	2.70	270.00	93.28	518.40	-0.0055
	+20	4.32	2.68	268.00	93.14	519.05	-0.0042
	-20	4.33	2.67	267.00	93.07	515.80	-0.0105
	-40	4.345	2.655	265.50	92.97	515.72	-0.0106
S_u	+40	4.345	2.655	265.50	92.97	589.08	0.130
	+20	4.325	2.675	267.50	93.11	551.71	0.058
	-20	4.31	2.69	269.00	93.21	480.84	-0.077
	-40	4.35	2.65	265.00	92.93	452.71	-0.131

Table 3: Sensitivity analysis of the backorder EOQ model.

Now from table 3, we have calculated the frequency distribution of the average inventory cost, average order quantity and backorder quantity which are shown in table 4, table 5 and table 6 respectively. Then we have calculated mean, median and mode of order quantity, backorder quantity and average inventory cost which is given in table 7.

Claim of decision maker under social judgment

Here we assume, the decision maker may have the following choices shown in table 8; i.e., the average cost be {Mean (M_n),

Class intervals	Frequency f_i	Cumulative frequency	Mid interval Value(x_i)	$f_i x_i$
364-414	2	2	389	778
414-464	3	5	439	1317
464-514	1	6	489	489
514-564	5	11	539	2695
564-614	3	14	589	1767
614-664	2	16	639	1278
Total	$\sum f_i$ =16			$\sum f_i x_i$ =8324

Table 4: Frequency distribution of average inventory Cost.

Class intervals	Frequency f_i	Cumulative frequency	Mid interval Value(x_i)	$f_i x_i$
160-195	1	1	177.5	177.5
195-230	1	2	212.5	212.5
230-265	5	7	247.5	1237.5
265-300	7	14	282.5	1977.5
300-335	1	15	317.5	317.5
335-370	1	16	352.5	352.5
Total	16			4275

Table 5: Frequency distribution of average order quantity.

Class intervals	Frequency f_i	Cumulative frequency	Mid interval Value(x_i)	$f_i x_i$
48-63	1	1	55.5	55.5
63-78	1	2	70.5	70.5
78-93	7	9	85.5	598.5
93-108	5	14	100.5	502.5
108-123	1	15	115.5	115.5
123-138	1	16	130.5	130.5
Total	16			1473

Table 6: Frequency distribution of backorder quantity

Median (M_d), Mode (M_o), Optimum (O_p). Each response of the decision maker might be judged by the public domain by responses agree and disagree only. Since there are four options in the response matrix made by the decision maker so, we must have at most 24 decision tests criteria and it can be minimized to 6 criteria by applying the similar terms taken once from the possible permutations. The following are the social judgments on the decision maker's claim.

Parameters	Expected Cost of the model	I_1^*	I_2^*
Mean (M_n)	520.25	267.19	92.06
Median (M_d)	534	270	90.86
Mode (M_o)	547.33	273.75	89.25

Table 7: Central measure of average inventory cost, order and backorder quantity.

Criteria	Decision maker's claim				Phi-coeff. Of I_1^*	Phi-coeff. Of I_2^*	Phi-coeff. Of Z_c^*
	Public Judgment Agree		Public Judgment Disagree				
	True	False	True	False			
1	O_p	M_n	M_o	M_d	-0.005506	-0.006820	-0.005669
2	O_p	M_n	M_d	M_o	0.001391	0.002118	0.006657
3	O_p	M_o	M_d	M_n	-0.010736	-0.013380	-0.018710
4	M_n	O_p	M_o	M_d	-0.001391	0.002118	-0.006657
5	M_n	O_p	M_d	M_o	0.005506	0.006820	0.005669
6	M_o	O_p	M_n	M_d	0.010736	-0.013380	0.018710

Table 8: Phi coefficient over several decision variables.

Secondly, DM claims: model optimum True, agreed; median value True, Disagreed by public. This is invalid for all (cost, ordering and backordering quantity) aspects under phi coefficients tests. Criteria 5 shows, DM claims: mean value True, agreed; median value True, Disagreed by public. This is valid for all (cost, ordering and backordering quantity) aspects under phi coefficients tests. Similar comments can be drawn from other criteria also

Graphical illustrations

From the above figure we see that the phi-coefficient curves for average cost function, order quantity and backorder quantity intersect at three points. Out of them, two points are negative and one is positive. Negative points are lying from the first and fourth criteria. These mean that the DM's 'True' claim on model optimum (actual cost) is rejected by the public judgments and hence the

model is invalid. That is the claim of the inventory cost $ 521.28 against the order quantity 265.00 units and the backorder quantity 92.93 units is false. However, the positive point occurs for criteria 5. This indicates that, the 'true' claim on mean cost (coming from sensitivity analysis) is accepted by the public judgments. Hence the model is valid and the actual cost of the DM is $ 520.25 against the order quantity 267.19 units and the backorder quantity 92.06 units exclusively.

Figure 2: Model validation curve under public judgment.

Main contributions

We have checked the validity of an inventory model under social aspect in this article. For this reason, we have collected public judgment over the declaration of inventory practitioner and used Phi-coefficient to find the actual range of validity of this model. As we shown that the decision maker's declaration is not true always. So, in this way we can check the validity of any other model to find the actual reality behind those models. The basic novelties are stated as follows:

- As per literature concerned the use of post sensitivity analysis is the first time application.
- Here we use a variable which is totally new concept in inventory management i.e. Dichotomous variable
- Use of Phi-coefficient in inventory modeling is new.
- Simultaneous work of "Modeling and Validating" is the new concept in inventory control problems.
- By this approach we can detect the fraud work.

Conclusion

We develop a classical backorder EOQ model in this article. We constructing a cost function of the proposed model which we optimize by the help of Lingo 16.0 Software. We compute the mean, median and mode using those data sets from the sensitivity analysis of our proposed model and get their corresponding average inventory cost. Then we make a public judgment through the responses true or false over any one of the cost values from model optimum, mean, median and mode values of the model itself. Incorporating phi-coefficient curves for the decision variables we have searched their point of intersections for validity testing. In this study we get positive and negative phi-coefficient values. Positive values indicate to model validation and negative values indicates to no validation of the model.

Again, by phi-coefficients we are intending to study the overall intention of the different views through the intersections of different curves. So, no matter how much the value of phi-coefficients gets closer to one another the results may vary if we consider the quartile points of the data set or the upper and lower bounds of the cost function instead. In addition, the model validity can be judged by another non-traditional method which we shall discuss in our future works.

Bibliography

1. Arrow K., *et al*. "Optimal inventory policy". *Econometrica* 19 (1951).
2. Arrow K., *et al*. "Studies in the mathematical theory of inventory and production". Stanford, Calif, Stanford University Press (1958).
3. Kenkel J L. "Introductory Statistics for Management and Economics". Boston: Prindil, Weber and Schmidt (1981).
4. Kennedy JJ and Bush AJ. "An introduction to the design and Analysis of Experiments of Behavioral Research". Newyork: University Press of America (1985).
5. Mehra S., *et al*. "Some comments on the validity of EOQ formula under inflationary conditions". *Decision Sciences* 22.1 (1991): 206-212.
6. Debreceny RS and Gray G L. "Data mining journal entries for fraud detection: An exploratory study". *International Journal of Accounting Information Systems* 11 (2010): 157-181.

7. Hesse MF and Jr JHC. "Fraud risk management: A small Business Perspective". *Business Horizons* 59 (2016): 13-18.

8. Kanapickiene R and Grundiene Z. "The model of fraud detection in financial statements by means of financial ratios". *Procedia-Social and Behavioral Sciences* 213 (2015): 321-327.

9. Kim Y J., *et al*. "Detecting financial misstatements with fraud intension using multi-class cost sensitive learning". *Expert Systems with Applications* 62 (2016): 32-43.

10. Mu E and Carroll J. "Development of a fraud risk decision model for prioritizing fraud risk cases in manufacturing firms". *International Journal of Production Economics* 173 (2016): 30-42.

11. West J and Bhattacharya M. "Intelligent Financial Fraud Detection: A Comprehensive Review". *Computers and Security* 57 (2016): 47-66.

12. Burton L. "Audit Report Details Fraud at Leslie Fay". *The Wall Street Journal*. March 28 (1995): B1.

13. Fahrmeir L and Tutz, G. "Multivariate Statistical Modeling Based on Generalized Linear Models". Springer-Verlag: New York.

14. Angulo JM., *et al*. "Semiparametric statistical approaches for space-time process prediction". *Environmental and Ecological Statistics* 5 (1998): 297-316.

15. Pruscha H and Gottlein A. "Regression analysis of forest inventory data with time and space dependencies". *Environmental and Ecological Statistics* 9.1 (2002): 43-56.

16. Besta P., *et al*. "The utilization of statistical methods in the area of inventory management". Jesenik, Czech Republic, EU (2012).

17. Ayloo, S., *et al*. "Correlation between the beck depression inventory and bariatric surgical procedures". *Surgery for Obesity and Related Disease* 3.11 (2015): 637-642.

18. Iqbal Q., *et al*. "Statistical analysis of multi-criteria inventory classification models in the presence of forecast upsides". *Production and Manufacturing Research* 5.1 (2017): 15-39.

19. Mutschler W. "Higher-order statistics for DSGE models". *Econometrics and Statistics* 6 (2018): 44-56.

20. De SK and Pal M. "An intelligent decision for a bi-objective inventory problem". *International Journal of System Science: Operation: Logistics* 3.1 (2016).

21. De SK and Sana SS. "Fuzzy order quantity inventory model with fuzzy shortage quantity and fuzzy promotional index". *Economic Modelling* 31 (2013a): 351-358.

22. De SK and Sana SS. "Backlogging EOQ model for promotional effort and selling price sensitive demand- an intuitionistic fuzzy approach". *Annals of Operations Research. Advance Online Publication* 233 (2015): 57-76.

23. De SK and Sana SS. "An EOQ model with backlogging". *International Journal of Management Sciences and Engineering Management* (2015).

24. De SK and Sana SS. "The (p, q, r, l) model for stochastic demand under intuitionistic fuzzy aggregation with Bonferroni mean". *Journal of Intelligent Manufacturing* 29 (2018): 1753-1771.

25. De SK and Mahata GC. "Decision of a fuzzy inventory with fuzzy backorder model under cloudy fuzzy demand rate". *International Journal of Applied and Computational Mathematics* 3 (2017): 2593-2609.

26. De S K and Mahata GC. "A cloudy fuzzy economic order quantity model for imperfect-quality items with allowable proportionate discounts". *Journal of Industrial Engineering International* (2019a).

27. De S K and Mahata G C. "A comprehensive study of an economic order quantity model under fuzzy monsoon demand". *Sadhana* (2019b).

28. Maity S., *et al*. "A comprehensive study of a backlogging EOQ model with nonlinear heptagonal dense fuzzy environment". *RAIRO-operations Research* (2018a).

29. Maity S., *et al*. "A study of a back order EOQ model for cloud type intuitionistic dense fuzzy demand rate". *International Journal of Fuzzy System* 22 (2019b): 201-211.

30. Karmakar S., *et al*. "A pollution sensitive dense fuzzy economic production quantity model with cycle time dependent production rate". *Journal of Cleaner Production* 154 (2017): 139-150.

31. Karmakar S., *et al*. "A pollution sensitive remanufacturing model with waste items: Triangular dense fuzzy lock set approach". *Journal of Cleaner Production* 187 (2018): 789-803.

32. Maity S., *et al*. "A Study of an EOQ Model under Lock Fuzzy Environment". *Mathematics* 7.1 (2019a).

33. Maity S., *et al*. "Two Decision Makers' Single Decision over a Back Order EOQ Model With Dense Fuzzy Demand Rate". *Finance and Market* 3 (2018b): 1-11.

34. Chakraborty A., *et al*. "Hexagonal fuzzy number and its distinctive representation, ranking, defuzzification technique and application in production inventory management problem". *Granular Computing* 6 (2020): 507-521.

35. Nobil A M., *et al*. "Reorder point for the EOQ inventory model with imperfect quality items". *Ain Shams Engineering Journal* 11.4 (2020): 1339-1343.

36. Khan M A., *et al*. "Inventory models for perishable items with advanced payment, linearly time-dependent holding cost and demand dependent on advertisement and selling price". *International Journal of Production Economics* 230 (2020): 107804.

37. Rahman MS., *et al*. "Necessary and Sufficient Optimality Conditions for non-linear Unconstrained and Constrained optimization problem with Interval valued objective function". *Computers and Industrial Engineering*147 (2020).

38. Maity S., *et al*. "A study of an EOQ model with public-screened discounted items under cloudy fuzzy demand rate". *Journal of Intelligent and Fuzzy System* 41.6 (2021): 6923-6934.

39. Maity S., *et al*. "A Study of an EOQ Model of Growing Items with Parabolic Dense Fuzzy Lock Demand Rate". *Applied System Innovation* 4 (2021): 81.

40. Maity S., *et al*. "Validity Judgement of an EOQ Model using Phi-coefficient, Optimal Decision Making in Operations Research and Statistics: Methodologies and Applications". CRC Press (2021): 378-387.

41. Maity S. "A Study of a Back Order EOQ Model Using Uncertain Demand Rate". *Acta Scientific Computer Sciences* (2020): 1-7.