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Research Article

Controller Identification for Control of Nonlinear Systems

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Abstract

Control of nonlinear systems has attracted attention of researchers for decades. In this work we applied the control identification technique to control nonlinear systems. This technique evaluates controllers even when they are not online. The knowledge of the plant parameters are not a priori required. We use exponentially weighted performance criteria. Simulation results are presented to analyze the effect of the discount factor for the control of the systems.

Keywords: Nonlinear Systems; Controller Identification Technique; Exponentially Weighted Performance; Discount Factor and Control; Algorithm

Introduction

Control of systems which exhibit chaotic behavior has been a great challenge for the past decades. To address this challenge we apply the controller identification technique to such systems. The controller identification technique tries to identify a good controller based on a given performance criterium and experimental data [1-3]. From a family of candidate controllers, the ones with better performance are chosen. Many controllers used in control of systems which exhibit chaotic behavior were of proportional type, as in [7-9] for the Rössler 3D system and in [10,11] for the Rössler 4D system, we propose proportional candidate controllers.

The performance criterion will be based on minimizing an error of following a reference model [3]. Key to these developments is the concept of fictitious model reference, which requires a left invertibility property of the control law.

On our previous work [4], it was applied controller identification technique to control nonlinear systems with chaotic behavior. In this work the controller identification technique will be applied to control the temporal trajectories of the following nonlinear systems: the Rössler 3D [5], and the Rössler 4D, with hyperchaos [6] models, using exponentially weighted performance criterium.

This work has two main differences from the previous work [4]. One is a more formal mathematical approach in relation to the use of the controller identification technique applied to nonlinear dynamic systems, and the other is the use of the exponentially weighted performance criterium, which made the control proposal more general and with better results.

We presented numerical simulation to illustrate the effectiveness of the proposed technique. These simulations are made to the previously cited systems with the requirement of following a given trajectory.

This paper is organized as follows. In Section 2, the mathematical modeling of the control problem is presented, which includes theoretical and practical aspects of modeling. In Section 3, the proposed technique is applied to control nonlinear systems, the 3D and 4D Rössler systems. Some simulations are presented. In Section 4, concluding remarks are given.

Mathematical modeling of the control problem

Nonlinear systems with control u, in the state space form:

$$\dot{x} = f(x) + Bu \quad ,\text{------- (1)}$$

Can be written using direct parameterization to bring the nonlinear dynamics to the state-dependent coefficient (SDC) form

$$\dot{x} = A(x)x + Bu \quad ,\text{-------- (2)}$$

Where A(x) is a state-dependent matrix, and B the control coefficient matrix. The design procedure consists of using direct parameterization of A(x) to bring the nonlinear system to a linear structure having state-dependent coefficients (SDC) f(x)=A(x)x. In general, A(x) is unique only if x is scalar. There are many possibilities for SDC parameterizations if x is not scalar. For the consistence of the control strategy, one has to guarantee that the pair [A(x), B] is a controllable parameterization of the nonlinear system (2). In this work we will show that [A(x), B] is pointwise controllable in the linear sense [12,13].

The first step to figure out solutions of the dynamical system (2) is obtaining the form of the control functions u. For this, it will be used the controller identification technique, which will be presented in the sequence.

Controller identification technique - theoretical aspects

A general overview of theoretical aspects for the controller identification technique can be found in [3]. According to this technique, the only plant information used is the plant experimental data. For the cases presented in this work, we need only the reference functions r and the data x, obtained from the dynamical system (2), to obtain the control u. Given a desired behavior, a class of candidate controllers is proposed. Then, a controller is selected through the use of a performance index and the fictitious reference concept. The work [4] presents applications to controlled systems with the use of controller identification technique. In this work we analyze the technique with exponentially weighted performance criterium, in comparison with the previous work of [4].

In this work, a class of proportional candidate controllers is proposed. The respective mathematical development is as follows.

First we introduce some definitions as follows [3].

Definition 1

With respect to data and measurements, we will assume that we make certain measurements which we will call the data. Assume that the data consists of observed realizations of the phenomenon itself. Thus, a dataset will be a nonempty set. The formal definition of a data set is, then, given by a pair (u, x). In our case x comes from the dynamical system.

Definition 2

The truncated L_2inner-products$<x, z>_t$ and norm $||x||_t$ is defined as

$$< x, z >_t = \int_0^t z^T x dt \quad \text{--------- (3)}$$

$$\|x\|_t = \sqrt{< x, z >_t} \quad \text{--------- (4)}$$

Definition 3

Given a constant >0, we define the exponentially weighted truncated L_2 inner-products $<x,z>_t$ and norm $||x||_t$ by

$$< x, z >_t = \int_0^t e^{-\sigma(t-\tau)} z^T(\tau) x(\tau) d\tau \quad \text{----------- (5)}$$

$$\|x\|_t = \sqrt{< x, z >_t} \quad \text{-----------(6)}$$

Now, the control law for a proportional controller is given by:

u=K(r-x), ---------- (7)

And, consequently, the fictitious reference is given by

$$r_K = \frac{u}{K} + x, \quad \text{------------ (8)}$$

Where K is a constant.

The performance index, in quadratic form, as norm (3), is given by

$$I_1 = \int_0^t (x - w_m * r_K)^2 dt \quad \text{---------- (9)}$$

Where w_m is the inverse Laplace transform of the transfer function W_m (s) of desired behavior

$$w_m = \mathcal{L}^{-1}[W_m(s)] \quad \text{------------(10)}$$

Note that w_m is the reference model of the desired behavior for the plant.

For convenience

$$u_m = w_m * u, \quad \text{----------(11)}$$

$$x_m = w_m * x. \quad \text{----------(12)}$$

With "*" denoting the convolution operator.

Theorem 1 [4]

Among the controllers of class (7), the one that minimizes the performance index (9) is given by

$$\widehat{K} = \frac{G}{F}, \quad \text{-----------(13)}$$

Where

$$F = \int_0^t (x - x_m) u_m d\tau, \quad \text{-----------(14)}$$

$$G = \int_0^t (u_m)^2 d\tau. \quad \text{-----------(15)}$$

Proof

From (7), the fictitious reference is given by (8), where K is a constant.

Using (11) and (12) we have that

$$x - w_m * r_K = x - w_m * \left(\frac{u}{K} + x\right) = x - \frac{u_m}{K} - x_m. \quad \text{----------(16)}$$

Substituting (16) the performance index (9), yields

$$I_1 = \int_0^t (x - w_m * r_K)^2 d\tau = \int_0^t (x - u_m/K - x_m)^2 d\tau$$

$$= \int_0^t (x^2 + (u_m/K)^2 + x_m^2 - 2xu_m/K - 2xx + 2u_m x_m/K) d\tau = E - 2\frac{F}{K} + \frac{G}{K^2}, \quad \text{----------(17)}$$

Where

$$E = \int_0^t (x - x)^2 d\tau, \quad \text{----------(18)}$$

$$F = \int_0^t (x - x_m) u_m d\tau, \quad \text{----------(19)}$$

$$G = \int_0^t (u_m)^2 d\tau. \quad \text{----------(20)}$$

The minimizationof theperformance indexmeans findingKthat satisfiesthe equation:

$$\frac{dI_1}{dK} = 2\frac{F}{K^2} - 2\frac{G}{K^3} = 0, \quad \text{----------(21)}$$

Which leads to the estimator for the proportional constant

$$\widehat{K} = \frac{G}{F}. \quad \text{----------(22)}$$

One can note that $\widehat{K} = \frac{G}{F}$s the multiplicative constant of control functions u. In order to obtain the simulation results, K will be periodically updated. The objective is to minimize the difference between the measured x and the desired x_m. We used a continuous time formulation for the dynamical systems.

From equations (7)-(12) and Theorem presented above, we obtained the constant K, that determines the proportionality constant of the optimal control function , given by (7). This development can be extended to any odd power of the control function .

Remark [4]

If n is an odd number, we replace (7) and (8):

$$u = (K(r - x))^n. \quad \text{----------(23)}$$

Isolating r produces

$$\sqrt[n]{u} = K(r - x), \quad \text{-----------(24)}$$

$$r_K = \sqrt[n]{u}/K + x, \quad \text{-----------(25)}$$

Then equations (11) and (16) are rewritten as

$$u_m = w_m * \sqrt[n]{u}, \quad \text{-----------(26)}$$

And

$$x - w_m * r_K = x - w_m * (\sqrt[n]{u}/K + x), \quad \text{---------- (27)}$$

Respectively. The other equations are not modified.

In the sequence we introduce in the performance index a exponentially weighted factor, , which produces a forgetting effect of the data as time evolves, also used as a discount factor in some applications.

Theorem 2

Among the controllers of class (7), the one that minimizes the performance index

$$I_2 = \int_0^t e^{-\sigma(t-\tau)} (x - w_m * r_K)^2 dt, \quad \text{--------- (28)}$$

Is , given by

$$\hat{K} = \frac{L}{J}, \qquad \text{----------(29)}$$

Where

$$J = \int_0^t e^{-\sigma(t-\tau)}(x - x_m)u_m d\tau, \qquad \text{----------(30)}$$

$$L = \int_0^t e^{-\sigma(t-\tau)}(u_m)^2 d\tau. \qquad \text{----------(31)}$$

Note that performance index (28) has the form of the norm (5).

Proof

First note that equation (16) will not be modified. Now substituting (16) the performance index (28), yields

$$I = \int_0^t e^{-\sigma(t-\tau)}(x - w_m * r_K)^2 d\tau = \int_0^t e^{-\sigma(t-\tau)}(x - u_m/K - x_m)^2 d\tau$$

$$= \int_0^t e^{-\sigma(t-\tau)}[x^2 + (u_m/K)^2 + x_m^2 - 2xu_m/K - 2xx + 2u_m x_m/K]d\tau = H - 2\frac{J}{K} + \frac{L}{K^2}, \qquad \text{----------(32)}$$

Where

$$H = \int_0^t e^{-\sigma(t-\tau)}(x - x)^2 d\tau, \qquad \text{----------(33)}$$

$$J = \int_0^t e^{-\sigma(t-\tau)}(x - x_m)u_m d\tau, \qquad \text{----------(34)}$$

$$L = \int_0^t e^{-\sigma(t-\tau)}(u_m)^2 d\tau. \qquad \text{----------(35)}$$

The minimization of the performance index (28) means finding K that satisfies the equation:

$$\frac{dI_2}{dK} = 2\frac{J}{K^2} - 2\frac{L}{K^3} = 0, \qquad \text{----------(36)}$$

Which leads to the estimator for the proportional constant

$$\hat{K} = \frac{L}{J}. \qquad \text{----------(37)}$$

Controller identification technique - practical aspects

It is stated that our goal is to select, based on the data, the constraints imposed by the control law (as proportionality and others) and the performance criterion, the best among the set of given controllers. In order to do that we introduce the cost functionals (9) and (28), and an optimality criterion (21) and (36).

Figure 1 shows a block diagram of the proposed control. Observe that y(t) are the system output trajectories.

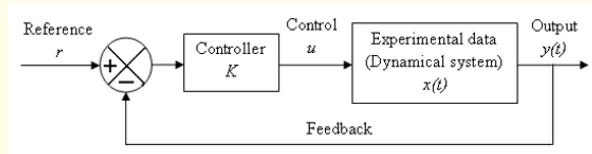

Figure 1: Block diagram for the proposed control system.

Among the candidate controllers, according to the performance index, the best one is inserted in the feedback loop. This is in essence the feedback scheme used in controller-oriented identification.

For the differential equation systems, the values obtained from equations (11)-(20), (30)-(35) and the state x were computed concomitantly. A vector Y was created to be used in the integration process of the numeric integration function (Runge-Kutta), defined as:

$$Y = [x_1 x_2 \dots x_n \mid x_{m_1} u_{m_1} E_1 F_1 G_1 \dots x_{m_n} u_{m_n} E_n F_n G_n]^T, \quad \text{otherwise}$$

$$Y = [x_1 x_2 \dots x_n \mid x_{m_1} u_{m_1} H_1 J_1 L_1 \dots x_{m_n} u_{m_n} H_n J_n L_n]^T, \qquad \text{--------------(38)}$$

Where n is the number of equations of the system.

The initial state is given by

$$Y(0) = Y_0, \qquad \text{-----------(39)}$$

And the initial values are

$$K_0 = [K_1 K_2 \dots K_n]^T, \qquad \text{----------(40)}$$

Where each K_i is associate with u_i, as shown in equation (7). The transfer function is

$$W_m(s) = \frac{1}{s+1}, \qquad \text{------------ (41)}$$

Which means that

$$U_m(s) = \frac{1}{s+1}U(s), \text{ and } X_m(s) = \frac{1}{s+1}X(s) \qquad \text{------------ (42)}$$

And

$$\dot{u}_m + u_m = u, \text{ and } \dot{x}_m + x_m = x, \qquad \text{-----------(43)}$$

Where the values u_m of x_m and are computed.

In the applications presented, the reference to the system, containing the desired trajectory, will be represented by a vector r, as

$$r = [r_1\, r_2 \ldots r_n]^T, \qquad \text{--------- (44)}$$

And the control functions by a vector u, as

$$u = [u_1 u_2 \ldots u_n]^T. \qquad \text{---------- (45)}$$

The controller identification technique can be translated to the following algorithm.

Input

Experimental data of the system x(t) (in our case are the plant data in state-space), sampling interval , reference r, number of intervals k.

Initialization

Set , set t=0, set ;

Procedure

for i=1:k;

> : step :(interval to compute K - step can be 0.05);

> integrate Y (which implies obtaining or - observe that the integrators are 1/S and 1/(S+), respectively);

> update reference r;

> calculate , otherwise ;

> calculate I_1 or I_2;

end;

OUPUT: Y, K=K(hat), I_1 or I_2

This algorithm returns for each time interval the value of K used in the control law. Also the pereformance index I_1 or I_2 and the controlled state x(t). By uniting the intervals, we can print the time trajectories of the controlled system.

Results and Discussion

To exemplify the use of this control technique, we present an application for two nonlinear dynamical systems: Rössler 3D [5] and Rössler 4D [6,14,15]. They have the peculiarity of, in addition to being nonlinear, presenting chaotic and hyperchaotic behavior, respectively. Both being taken as a dimensional.

We consider independent control laws for each equation of the system.

$$\begin{aligned} \dot{x}_1 &= -x_2 - x_3, \\ \dot{x}_2 &= x_1 + bx_2, \\ \dot{x}_3 &= b + x_3(x_1 - a). \end{aligned} \qquad \text{---------(46)}$$

It is well known that differential equations (46) define a continuous time system with chaotic behavior for b=0.2 and a=5.7. One can note that the third equation presents a nonlinearity . An interesting aspect of this system is its complex dynamical behavior in contrast with the simplicity of the description of its vector field.

Now, we introduce independent control functions in (46), and system with control is written as [7]:

$$\begin{aligned} \dot{x}_1 &= -x_2 - x_3 + u_1, \\ \dot{x}_2 &= x_1 + bx_2 + u_2, \\ \dot{x}_3 &= b + x_3(x_1 - a) + u_3. \end{aligned} \qquad \text{---------(47)}$$

The objective of the control strategy is to follow any specified trajectory r, independently of the initial state. The control signal u is used for this purpose. The desired trajectories chosen form a limit cycle in the phase diagram and are given by:

$$r = [5 + cos(t)\;\; 1 + sin(t)\;\; 1 + sin(t)]^T, \qquad \text{------------ (48)}$$

The initial values for K were

$$K_0 = [10\ 10\ 10]^T. \qquad \text{-------------(49)}$$

In order to obtain the simulation results, the values of K were periodically updated at each 0.5 from 0 to 50.

Figure 2 shows the desired trajectories r and the trajectories of the controlled system (47) by using the two performance indexes, equation (9) and with exponentially weighted factor, =0.01, equation (28), respectively. As the trajectories for both cases are almost the same, only one figure has been placed. The initial conditions are:

$$Y_0 = [0.1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T.- \qquad \text{------------(50)}$$

One can note the convergence with a residual error in the third component . However, when we reduce the time interval of the sim-

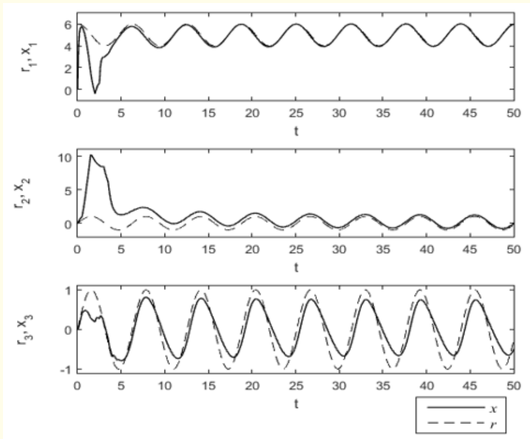

Figure 2: Temporal trajectories of the controlled system (47) and reference (48).

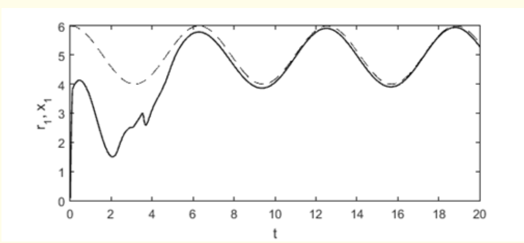

Figure 3: Temporal trajectory x_1 of the controlled system (47).

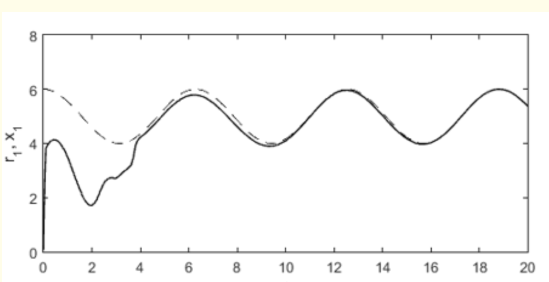

Figure 4: Temporal trajectory x_1 of the controlled system (47), considering exponentially weighted factor σ = 0.2.

ulations and increases the value of , such as =0.2, the trajectory of tends more quickly to the reference , as shown in figures 3 and 4. K were periodically updated every 0.1 second from 0 to 20.

The trajectories and have no noticeable difference.

The local controllability for the Rössler system (47) can be obtained by rewriting the system in the state-dependent coefficients form (2). One possible parameterization for (47) is:

$$\dot{x} = \begin{bmatrix} 0 & -1 & -1 \\ 1 & b & 0 \\ b/x_1 & 0 & x_1 - a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, \quad \text{---------- (51)}$$

The rank of the controllability matrix is computed for each update of K. It has value 3, what means that the nonlinear system (47) is locally completely state controllable. Note that because B is identity, controllability is always guaranteed.

Now consider the well-known Rösslerhyperchaos system [6], given by:

$$\begin{aligned} \dot{x}_1 &= -x_2 - x_3, \\ \dot{x}_2 &= x_1 + b_1 x_2 + x_4, \\ \dot{x}_3 &= b_2 + x_1 x_3, \\ \dot{x}_4 &= -b_3 x_3 + b_4 x_4. \end{aligned} \quad \text{------------ (52)}$$

The continuous time system (52) presents chaotic behavior for the set of parameter values b_1= 0.27857, b_2= 3.0, b_3= 0.3 and b_4= 0.05. As in the Rössler 3D system, the third equation presents a nonlinearity $x_1 x_3$. The phase portraits of the system (52) are shown in figure 5, in four different combinations of coordinate axes.

Introducing control functions in the 4D system (52), as in (47), yields the system with control, written as:

$$\begin{aligned} \dot{x}_1 &= -x_2 - x_3 + u_1, \\ \dot{x}_2 &= x_1 + b_1 x_2 + x_4 + u_2, \\ \dot{x}_3 &= b_2 + x_1 x_3 + u_3, \\ \dot{x}_4 &= -b_3 x_3 + b_4 x_4 + u_4. \end{aligned} \quad \text{-----------(53)}$$

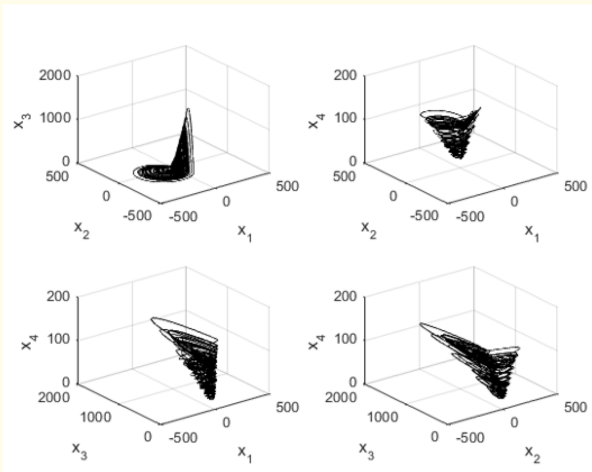

Figure 5: Phase portrait of Rösslerhyperchaos system (52).

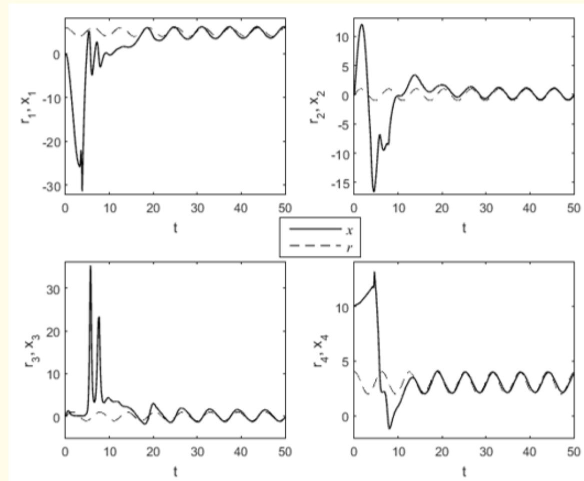

Figure 6: Temporal trajectories of the controlled system (53) and reference (54).

Analogously to the previous cases, the control strategy drives the system (53) from any initial state to desired trajectories r. The desired trajectories are given by:

$$r = [5 + cos(t)\ \ sin(t)\ \ sin(t)\ 3 + sin(t)]^T, \qquad \text{------------(54)}$$

The initial values for K were

$$K_0 = [0.0001\ 0.0001\ 0.0001\ 0.0001]^T, \qquad \text{------------(55)}$$

And 25 terms of Y_0, as

$$Y_0 = [0\ 0\ 0\ 10\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T \text{-------(56)}$$

In order to obtain the simulation results, the values of K, were periodically updated every 0.1 second from 0 to 50. Figure 6 presents the desired trajectories r_1, r_2, r_3, r_4 and the trajectories of the controlled system (53).

Considering the indexes (9) and with exponentially weighted factor (28), taking =0.01, the trajectories are almost the same, thus only simulations results without exponentially weighted factor are presented.

On the other hand, differently from the 3D Rössler system, for higher values, the 4D Rössler temporal trajectories tend more quickly to the desired trajectories. To show this, we repeat the simulations in a shorter period of time, as shown in figure 7, and simulate for this same time with sigma=0.06, as shown in figure 8.

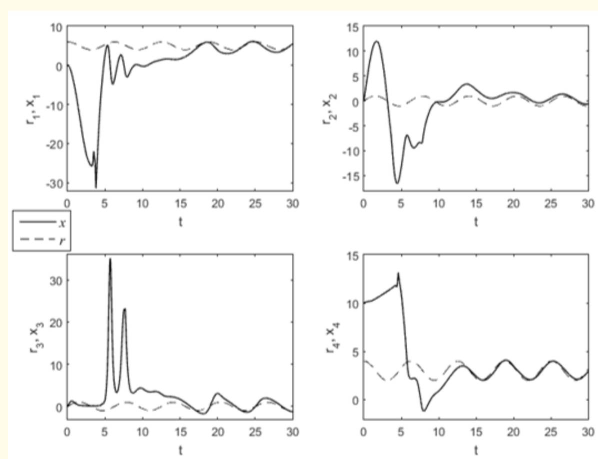

Figure 7: Temporal trajectories of the controlled system (53) and reference (54).

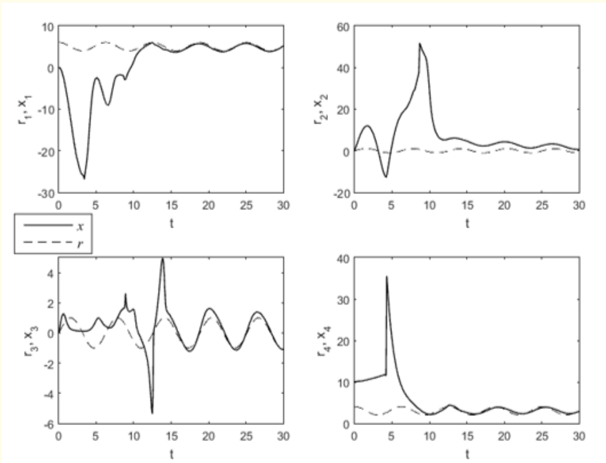

Figure 8: Temporal trajectories of the controlled system (53) and reference (54), considering exponentially weighted factor σ = 0.06.

One can observe by comparing figures 7 and 8 that, with the exception of , the trajectories tend more quickly to the reference.

As shown in [4], the local controllability for the 4D Rössler system (53) can be obtained by rewriting the system in the state dependent coefficients form (2). Also, the rank of the controllability matrix was computed for each update of K as having value 4, which means that the nonlinear system (53) is locally completely state controllable.

Using the controller identification technique in the systems with chaotic behavior requires particular care in the choices of the initial conditions and the initial values of K. Although not shown in this work, several tests were performed with different choices of initial conditions and initial K. Each change in choices generates different control performances. Therefore, it is up to the control designer to analyze these initial data in order to better control the system.

In addition, values also influence significantly the control of the system. Although each system has its particularity, the needs to be chosen carefully. For the examples presented the values should not exceed 0.2 for the 3D Rössler system, and 0.06 for the 4D Rössler system. Higher values can cause unexpected disturbances in the systems. Although not presented in this work, some tests with values higher than 0.2 and 0.06, respectively, were performed. These simulations showed that the control works well for the initial times of the system trajectories, but right away the system is then disturbed to the point of being no longer controllable.

Conclusions

In this work we used the controller identification technique to control nonlinear systems with chaotic behavior. Simulation results were presented. Convergence to the desired trajectories were observed.

We noticed that the results were dependent on the initial controller parameter K and on the periodicity of the updating of K.

It has also been shown that the use of the exponentially weighted factor can improve the performance of the control, although it is necessary to choose its value with care. Otherwise the system may not be controlled.

It was observed that including the exponentially weighted factor may be advantageous for controlling the trajectories, although we should be careful when choosing their values. For control the Rössler 3D system, the values should not exceed 0.2, and for control the Rössler 4D system, the values should not exceed 0.06.

The controller identification technique mains seems to be useful to control systems for its simplicity of implementation and for not having a priori requirement of knowledge of the plant parameters. Notice that, in this work, we used the state information for simulation purposes.

Conflict of Interest

The authors declare that they have no conflict of interest.

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