



Orthogonal Sequences

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Orthogonal sequences are used widely in the system communication channels as in the forward links for maxing the information and as in the backward links of these channels to sift this information which transmitted and the receivers get the information in a correct form.

Keywords: Orthogonal Sequences; Walsh Sequences; M-Sequences

Introduction

The works of Hocquenghem in 1959, Reed Solomon in 1960, Chaudhuri and Bose in 1960, BCH codes or Bose-Chaudhuri-Hocquenghem codes and others as Goppa, and Peterson 1961 were a new starting point for solving this issue.

Results and Discussion

Suppose G is a set of binary vectors of length n :
 $G = \{X; X = (x_0, x_1, \dots, x_{n-1}) \mid x_i \in F_2, i = 0, 1, \dots, n-1\}$

Let's $1^* = -1$ and $0^* = 1$, The set G is said to be orthogonal if the following two conditions are satisfied:

$$\forall X \in G, \left| \sum_{i=0}^{n-1} x_i \right| \leq 1; 2. \forall X, Y \in G, X \neq Y \Rightarrow \left| \sum_{i=0}^{n-1} x_i \cdot y_i^* \right| \leq 1, \text{ That is the absolute value}$$

of agreements minus the number of disagreements is less than or equals one.

The most important and easy orthogonal sequences

1. M-Sequences;
2. Walsh Sequences

M-Sequences

The recurring sequence $\{a_n\}$ which satisfies the following recurring formula:

$$a_{n+k} = \sum_{i=0}^{k-1} \lambda_i a_{n+i} = \lambda_{k-1} a_{n+k-1} + \lambda_{k-2} a_{n+k-2} + \dots + \lambda_0 a_n \quad \& \quad \lambda_i \in F_2, i = 0, 1, \dots, k-1$$

Where $(a_0, a_1, \dots, a_{k-1})$ is called the initial vector and $f(x) = x^k + \lambda_{k-1}x^{k-1} + \dots + \lambda_1x + \lambda_0$ is called the characteristic polynomial and if $f(x)$ is prime then the sequence $\{a_n\}$ is periodic with maximum length of the period equals $r = 2^k - 1$ and the all permutations of one period form an orthogonal set of linear and cyclic sequences. For the recurring sequence $a_{n+3} = a_{n+1} + a_n$

which has the characteristic prime polynomial $x^3 + x + 1$ and initial vector (101) the equivalent M-Sequence is: 1 0 1 1 1 0 0 1 0 1 1 1 0 0...

Walsh sequences

Walsh sequences has the length 2^k and we can generate them using Hadamard matrices as following: suppose $H_{2^0} = [1]$ $H = \begin{bmatrix} H & H \\ H & - \end{bmatrix}$ and we replace each 1 by 0 and each - by 1, thus we have

$$H_{2^1} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, H_{2^2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \text{ so and so, and the rows of the matrix except}$$

the zero row form a linear orthogonal set of sequences.

Generalization sequences not only for F_2 but for any finite field F_p where p is a prime number.

Conclusion

Through generalization and compose orthogonal sequences we can get other new orthogonal sequences with more large and safety but also for more cost of construction of these coders and decoders.

In the current time the interested problem is the following: given one orthogonal sequence then how we can find the coder which give as the orthogonal sequence.