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Review Article

Numerical Simulation of Interaction of Vortex Structures in Continuum

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Abstract

The results of numerical simulation of interaction of the vortex structures of various configuration, different symmetry orders and vorticities in the continuum are presented. In dependence on initial conditions the regimes of weak interaction with quasi-stationary evolution and active interaction with the "phase intermixing", when the evolution can lead to formation of complex forms of vorticity regions, are realized.

Keywords: Vortices; Interaction; Fluids; Plasma; Computer Simulation

Introduction

Basic equations

In this paper we study numerically the interaction of the vortex structures (so-called FAVRs [1]) in the continuum, and, specifically, in fluids and plasmas in two-dimensional approximation, when the Euler-type equations are applicable. In general case the set of the model equations describing a continuum (inviscid incompressible fluid) or the quasi-particles (charged filaments aligned with a uniform field **B**) with Coulomb interaction models is the following:

$$e_i d_t x_i = \partial_{y_i} H/B, \quad e_i d_t y_i = -\partial_{x_i} H/B, \quad \partial_v \equiv \partial/\partial v, \quad \text{---------- (1)}$$

$$\partial_t \rho + \mathbf{v}\cdot\nabla\rho = 0, \quad \mathbf{v} = -(\hat{\mathbf{z}}\times\nabla\psi)/B,$$

$$\Delta\psi - f = -\phi \quad \text{---------- (2)}$$

Where e_i is the strength (circulation) of discrete vortex or the charge per unit length of the filaments, ϕ is a *z*-component of vorticity ζ or charge density ρ, and ψ is a stream function or potential for the two-dimensional flow of inviscid fluid and guiding-centre plasma, respectively, and H is a Hamiltonian. Note, that in the continuum (fluid) model *B*=1 in the Hamiltonian Eqs. (1). Function *f* =0 for the continuum or quasi-particles (filaments) with Coulomb interaction models [2], and $f = k^2\psi$ for a screened Coulomb interaction model [3].

We will consider here only a case *f* = 0, and generalization of our approximation for $f = k^2\psi$ is rather trivial.

For numerical simulation we used the contour dynamics (CD) method [1], to some extent modified (see [4] for detail). This has yielded us a possibility not only to observe evolution of single vortex, but also to study the interaction between vortices having different sizes, vorticities and symmetry orders (different modes). A general idea of CD method is that the interaction between the boundaries of the regions with constant ϕ is considered, and due to this the dimension of the problem decreases on unit. Analytical solution of the Poisson equation (2) for current function y has form

$$\psi = -\frac{1}{2\pi}\iint d\xi\, d\eta\, [\ln r]\phi(\xi,\eta), \quad \text{------------- (3)}$$

Where $\ln r$ is the Green's function of Eq. (2), and $r = [(x-\xi)^2 + (y-\eta)^2]^{1/2}$. Then a value of velocity can be obtained by differentiation of integral (3), namely:

$$\mathbf{u}(x,y) = \phi_0 \oint_\Gamma [\ln r][\mathbf{e}_x\, d\xi + \mathbf{e}_y\, d\eta]. \quad \text{-----------(4)}$$

Further, obtain the change of the contour coordinates with time by solving differential equation $\mathbf{u}(x,y) = \dot{x}\,\mathbf{e}_x + \dot{y}\,\mathbf{e}_y$. For the computer simulation of the vortex structures the contour's boundary is divided into N lattice points (moreover, the point quantity should be rather great), and the temporal evolution is computed for each point. Thus Eq. (4) is written in discrete form, that allows us to found a value of velocity of each point of contour in dependence on influence to it of the points of both the same contour and the contour interacted with it. So, one can observe the time evolution of the vortex structure setting its initial form.

Numerical simulation and discussion

Let us consider the results of numerical simulation in terms of the vortex motion of the inviscid incompressible fluid, as more visual. In general, to study the evolution of vortex structures with different symmetry orders it is necessary to insert a small amplitude perturbation $r = R_0[1+\varepsilon\cos(m\alpha - \omega_m t)]$ (where R_0 is a conditional radius, ε is an eccentricity, m is symmetry order (mode), α is an angle and $\omega_m = \zeta_0(m-1)/2$) to the circle region with constant vorticity. But, taking into account that the results of evolution for one and two vortices with different m were described in detail in [5], let us stay on results on interaction of vortices and consider the most simple cases of circle vortices when m=1 and, therefore, $\omega_m = 0$. As it was found in [6] for such vortices the result of the evolution depends on sign of vorticity ζ ("polarity") and the distance δ between boundaries of vortices. We fulfilled a number of the series of numerical simulations for study of two-vortex interaction, the interaction in the N-vortex systems, including interaction between the vortex structures and the dust particles, and also interaction of two three-dimensional plane-rotating vortex structures within the framework of many-layer model of medium, in dependence on some parameters: initial distance between vortices, value and sign of their vorticities, and spatial configuration of the vortex system.

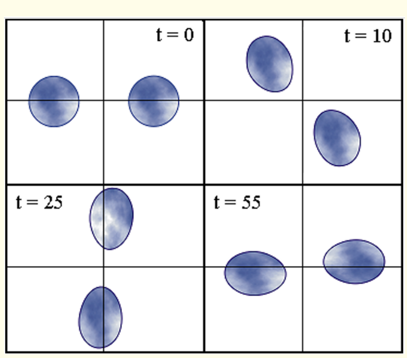

Figure 2: Evolution of two vortices with $\zeta_1 = \zeta_2 = -1$ at initial distance δ = 2d.

Two-vortex interaction

For two circle vortices having opposite polarities we observed that on the initial stage they approach and further move in the same direction, rotating in opposite directions (Figure 1). Thus, the vortices practically don't interact independently on value of δ.

For the circle vortices having the same polarities the result of evolution depends essentially on δ. So, for rather big δ they on a level with rotation about their own axes rotate around of common center (Figure 2), thus their interaction is weak and it is reduced to a cyclic change of their shape (so-called "quasi-recurrence" phenomenon [1] is observed). For rather small δ the vortices, on a level with rotation about their own axes and around of their common center, interact forming a common vortex region which consists of the vorticities of more small scales (Figure 3). Thus, in this case the regime of active interaction with the "phase intermixing" takes place. In our numerical experiments we have found that critical initial distance for two interacting vortices dividing these two types of interaction $\delta_e = 3d/4$, where d is the vortex diameter.

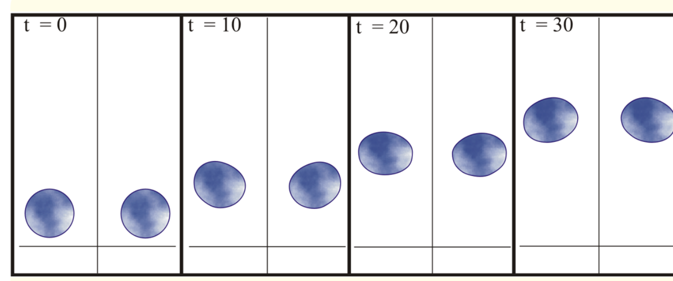

Figure 1: Evolution of two circle vortices with opposite polarities ($\zeta_2 = 1$ and $\zeta_2 = 1$).

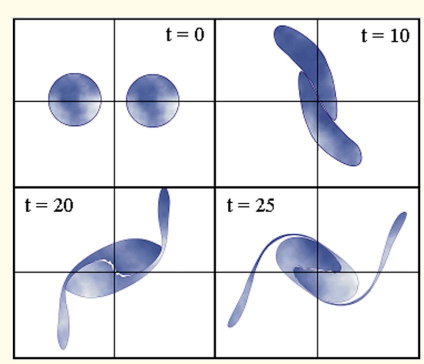

Figure 3: Interaction of two vortices with $\zeta_1 = \zeta_2 = -1$ at initial distance δ = d/2.

Note, that qualitative character of interaction of the vortices with different symmetry orders is, in general, the same, but in this case the vortex structures with more high symmetry order m liable to more high deformation (the vortex filaments appear) and have the greater tendency to destruction [7].

Interaction in N-vortex systems

To study the interaction in more complex *N*-vortex systems we have consider the problem with $N=3$ and $N=4$ in two variants: 1) for vortices linearly disposed at initial time, and 2) for vortices disposed at initial time in the corners of appropriate equilateral figures. Figure 4 shows an example of simulation of the interaction for initially linear disposition of four vortices. One can see that for rather big and equal initial distance between vortices the evolution leads to formation of two vorticity regions as a result of more strong interaction of each of the "outer" vortices with closest "inner" vortex. Thus, the interaction of forming pairs is similar to that of two vortex case. In case $\delta_i = d/2$ we observed the formation of a complex vortex structure which consists of many vorticities of more small scales (Figure 5). Further evolution of such structure leads to formation of complex turbulent field. Note that in last case we can also see that the interaction between outer vortices is stronger.

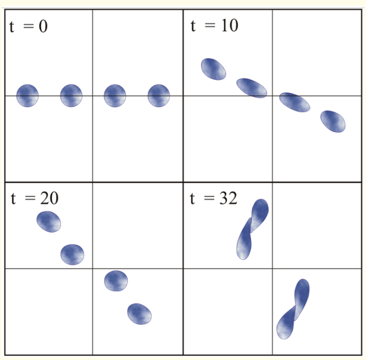

Figure 4: Interaction of four linearly disposed vortices with $\zeta_1 = \zeta_2 = -1$ and $\delta_i = d$.

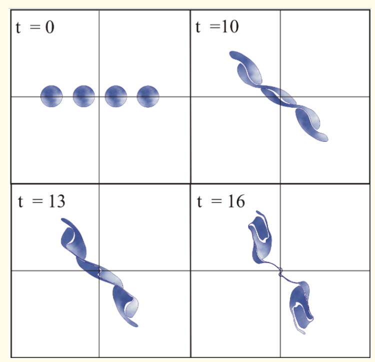

Figure 5: Interaction of four linearly disposed vortices with $\zeta_1 = \zeta_2 = -1$ and $\delta_i = d/2$.

This can be explained by the fact of more strong "attraction" of outer vortices to the "center of mass" of the vortex system because the outer vortex is attracted to the center by three other vortices, and the inner vortex is attracted to the center by two vortices and, to opposite side – by one outer vortex. To test this statement, in the next series of numerical experiments we have arranged outer and inner vortices on different initial distances. As a result, we observed the formation of common vortex structure from two inner vortices (see Figure 6). The results obtained for the 4-vortex system and the simulations for the 3-vortex system showed that in both cases, owing to effect noted above, the critical initial value δ_{cr} dividing quasi-stationary and active types of interaction is less than that for 2-vortex case.

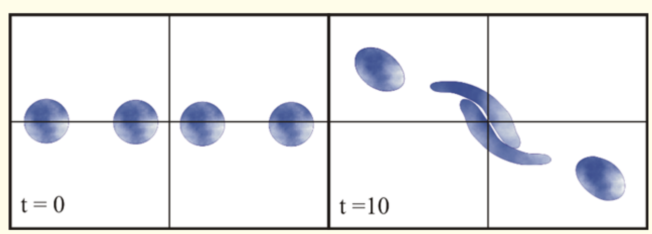

Figure 6: Interaction of four linearly disposed vortices with $\zeta_1 = \zeta_2 = -1$ for $\delta_{out} = d$ and $\delta_{inn} = d/2$.

In the next series of numerical experiments, we studied the interaction between the vortices disposed at initial time in the corners of appropriate equilateral figures. The following results were obtained. In case of evolution of three vortices with different signs of ζ being at initial time in the corners of triangle, we observed that a pair of them, having opposite polarities, behaves as well as pair of vortices with opposite polarities in 2-vortex case, and third vortex does not participate in interaction almost, practically independently on value of δ_i $(i = 1, 2, 3)$. The similar character of interaction is observed for four vortices with different signs of ζ being at $t = 0$ in the corners of square (see Figure 7 and 8, numbering of the vortices - clockwise, since the upper left corner).

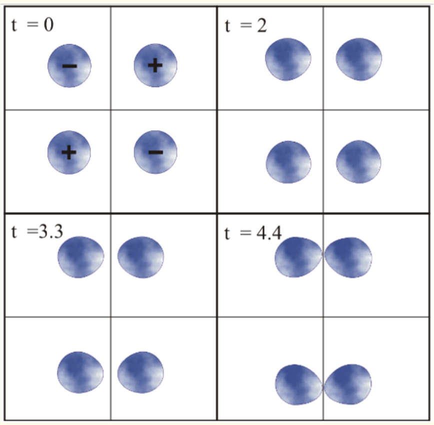

Figure 7: Interaction of four vortices with $\zeta_1 = \zeta_3 > 0, \zeta_2 = \zeta_4 < 0$ for $\delta_i = d$.

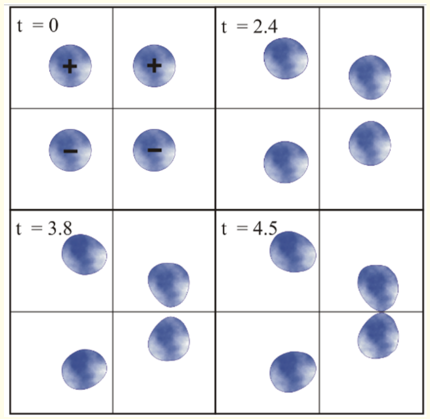

Figure 8: Interaction of four vortices with $\zeta_1 = \zeta_2 > 0, \zeta_3 = \zeta_4 < 0$ for $\delta_i = d$.

The character of interaction in the 3- and 4-vortex systems consisting of vortices having the same polarities depends essentially on the distances between them like that in the 2-vortex case. The examples of such interaction for $\delta = d/2 < \delta_{cr}$ and $\delta = d > \delta_{cr}$ are shown in figure 9 and 10. One can see that in the first case the four vortices are rotated forming one big vortex which consists of many vorticities of more small scales. In the second case we observed a "quasi-recurrence" phenomenon. Similar pictures take place in the 3-vortex system when at $t = 0$ the vortices are in the corners of triangle on the distances $\delta < \delta_{cr}$ or $\delta > \delta_{cr}$ one from another.

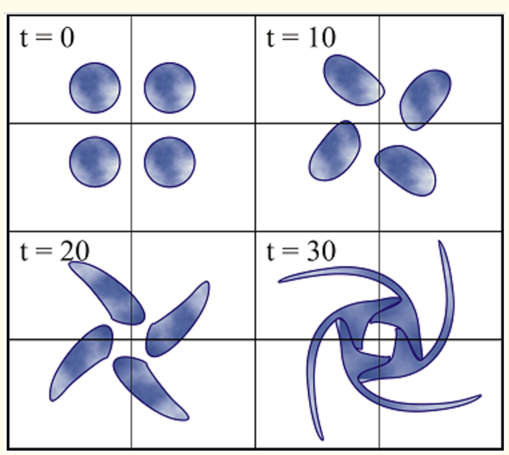

Figure 8: Interaction of four vortices with the same polarities for $\delta_i = d/2$.

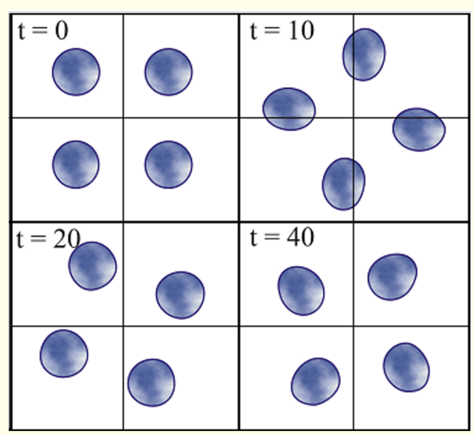

Figure 10: Evolution of four vortices with the same polarities for $\delta_i = d$.

Interaction in the vortex-dust particles system

The theoretical analysis and the experimental results [8] show that in plasma with gradient of dust charge the vorticity of dust particles can exist. This gives a possibility to study the interaction between the "hydrodynamic" vortex structures and dust particles by use of CD-method considering the dust particles as vortices of very small scales [9]. The results of our numerical simulations showed that the character of interaction in this case depends on value of particles' vorticity. If this value is very small that the interaction does not observed. When the vorticity of dust particles becomes like vorticity of the "hydrodynamic" vortex, the interaction becomes significant. An example of simulation is presented in figure 11, where one can see that the dust particles are involved by a vortex in large-scale rotation.

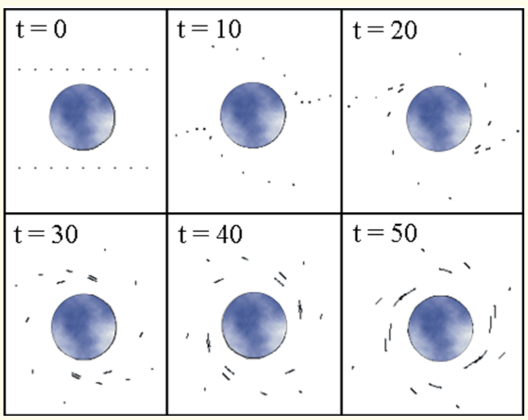

Figure 11: The case of dust particles with rather big values of vorticity.

Three-dimensional vortices interaction

In our investigations we also consider a possibility of simulation of the interaction dynamics of the three-dimensional plane-rotating vortex structures in the "two-dimensional approximation" within the framework of many-layer model of medium. Figure 12 shows the results of numerical simulation of interaction of two three-dimensional vortices with the exponential decreasing of their vorticity and (*x, y*)-sectional area with z-coordinate. One can see that, in the beginning, the vortices' central regions start to interact and only then other their areas are involved in interaction. Such behavior is explained by stronger interaction of central regions, which locate at the relatively short distance each other and their vorticities have relatively big values, so that the ratio ζ/δ is big in comparison with that for top and bottom of vortices. More strong analysis, however, requires more detail study of the regimes of two-vortex interaction.

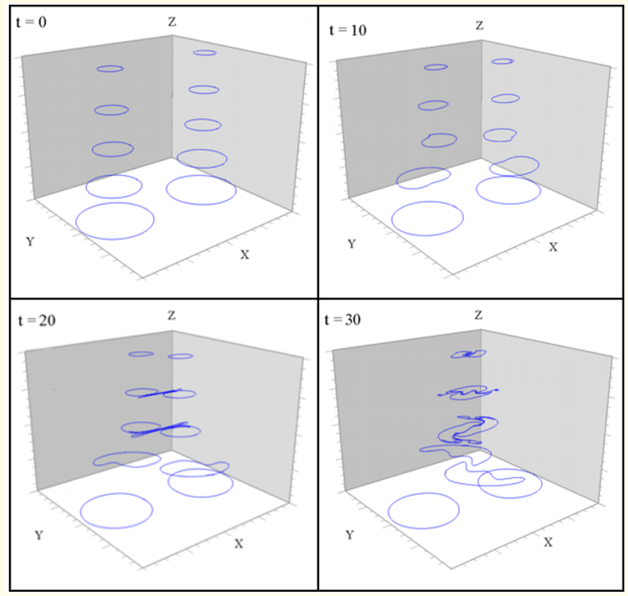

Figure 11: Interaction of three-dimensional plane-rotating vortex structures in the many-layer model.

Conclusion

So, we have presented, as more visual, some results of numerical simulation of Eqs. (1) and (2) in terms of the vortex motion of the inviscid incompressible fluid. But, as we noted in the beginning, the set (1) with $f=0$ can describe also the quasi-particles with Coulomb interaction model. At this, the results presented above can be easily extended on the two-dimensional simple system where the plasma is represented by charged filaments, aligned with a uniform field B, that move with the guiding-centre velocity $\mathbf{E}\times\mathbf{B}/B^2$. Moreover, the approach undertaken can be useful and also for other two-dimensional continuum models when $f \neq 0$ in the Poisson equation (2). They can describe the vortices or filaments with the non-Coulomb interaction. In the last case it is assumed that ions move with the guiding-centre velocity but electrons have a Boltzman distribution, thus the additional term $f = k^2\psi$ will describe the Debye screening [3].

Another plasma model that can be investigated using the described approach is the Hasegawa-Mima model [10], its equations can also be put in form (1) by way of inclusion of ion polarization drift through the ion equation of motion [3] $d_t\mathbf{v} = (e/M)(-\nabla\varphi + \mathbf{v}\times\mathbf{B})$. As to a hydrodynamic fluid model corresponding to a screened interaction that such model having form (1) has been proposed in [11] to describe the Earth's atmosphere with the equation of motion for the horizontal atmospheric flow $d_t\mathbf{v} = -g\nabla h + R\mathbf{v}\times\hat{\mathbf{z}}$, where *h* is the atmospheric depth and *R* is the Coriolis force. Another possible application can take a place in the study of the problems associated with the dynamics of the Alfven vortices in plasma of the ionosphere and magnetosphere of the Earth [12].

In conclusion, we studied numerically the problems of 2-vortex interaction, the interaction in the N-vortex systems (including interaction between the vortex structures and the dust particles), and also interaction of two three-dimensional plane-rotating vortex structures within the framework of many-layer model of medium, in dependence on the parameters: initial distance between vortices, value and sign of their vorticities, and spatial configuration of the vortex system. The results obtained showed that for all cases in dependence on initial conditions two regimes of the interaction can be observed, namely: weak interaction with quasi-stationary evolution and active interaction with the "phase intermixing", when the evolution can lead to formation of complex forms of vorticity regions. The theoretical explanation of the effects, which we observed, requires however more detail study of these regimes, including the transitions between two modes with a change of initial parameters of the vortex system.

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