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Research Article

Motions of the Gyroscope with one Side Support

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Abstract

The scientific innovations are the result of painstaking work accompanied by successes and omissions that is a normal process of human activity. The first edition of the theory of gyroscope effects for rotating objects is a typical example of such a statement. This theory contains incorrect processing of the integral equation for inertial torques generated by centrifugal forces. The torques of centrifugal forces act around two axes that give the slow and fast gyroscope motion around axes where the first one is impossible to measure. The validation of the theory was conducted by the measurement of the fast motion of the gyroscope around another axis whose value is not changed. The corrected mathematical model for the torques of centrifugal forces gives decreased angular velocity around one axis that is compensated by the increased dependency of the angular velocities about two axes. This manuscript shows the corrected mathematical models for motions of the gyroscope with one side support.

Keywords: Gyroscope Theory; Inertial Torque; Angular Velocity; Mathematical Model

Introduction

The gyroscopic effects did not have solutions for a long time and known publications had simplified mathematical models [1-4]. The separate solution of the gyroscope problems did not give an answer to their physics [5-8]. Researchers tried to solve gyroscope problems over two centuries [9-11]. The recent theory of gyroscope effects for rotating objects yields the expressions for the system of inertial torques generated by the centrifugal, Coriolis forces, and the change in the angular momentum of the spinning objects [12]. This theory is a breakthrough but contains incorrect processing of the integral equation for inertial torques generated by centrifugal forces. The system of inertial torques is interrelated by the dependency of the angular velocities of gyroscope motions around two axes. The motions of the gyroscope around the two axes are different. The precessed angular motion is measured by laboratory instruments and is bigger than the angular motion around another axis [13-16]. The slow rotation of the gyroscope does not measure because of the dynamical process drop of the revolutions of the gyroscope rotor, and technical problems.

The new theory of gyroscope effects for rotating objects contains incorrect processing of the integral equation for inertial torques generated by centrifugal forces. The torques of centrifugal forces act around two axes that give the slow and fast gyroscope motion around axes. The corrected mathematical model for the torques of centrifugal forces gives decreased angular velocity around one axis that is compensated by the increased dependency of the angular velocities about two axes. This is the reason for the validation of the theory that was conducted by the measurement of the fast motion of the gyroscope around another axis whose value is not changed. The corrected system of the inertial torques and the dependency of the angular velocities of the gyroscope is presented in Table 1 whose expressions are used for mathematical models of motions of the gyroscope with one side support.

Table 1 Inertial torques of the gyroscope of horizontal disposition and the dependency of the angular velocities.

Torque generated by	Acton	Equation
Centrifugal forces, T_{ct}	Resistance	$T_{ct} = \frac{4}{9}\pi^2 J\omega\omega_x$
	Precession	
Coriolis forces, T_{cr}	Resistance	$T_{cr} = (8/9)J\omega\omega_x$
Change in angular momentum, T_{am}	Precession	$T_{am} = J\omega\omega_x$
Dependency of the Angular Velocities		
$\omega_y = (8\pi^2 + 17)\omega_x$		

Table 1: Equations of the inertial torques acting on the spinning disc.

Table 1 parameters: ω_i is the angular velocity about axis i; ω is the angular velocity around axis oz; J is the moment of inertia of the gyroscope rotor.

Methodology

The mathematical models for the action of the torques and motions of the gyroscope with one side support were considered in detail in the publication with an error of expression of the centrifugal torque [16]. The corrected mathematical models for motions of the gyroscope of horizontal disposition are presented by Eqs. (1) - (3). The inertial forces generated by the rotating center mass of the gyroscope were removed because they have small values of a high order. Figure 1 shows the torques acting on the gyroscope with one side support in Cartesian 3D coordinate system Σoxyz.

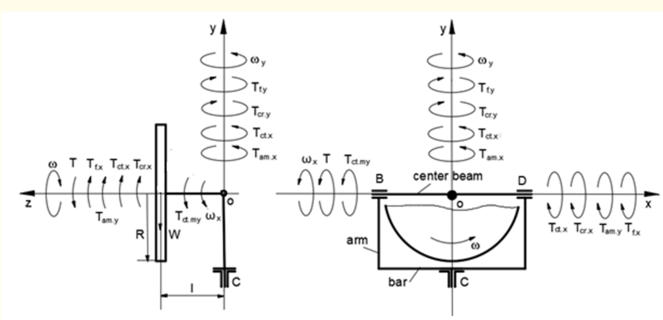

Figure 1: The torques acting on the gyroscope with one side support and motions.

The mathematical models of motions of the gyroscope are presented in Euler's differential equations based on acting torques around axes ox and oy.

$$J_x \frac{d\omega_x}{dt} = T - T_{f.x} - T_{ct.x} - T_{cr.x} - T_{am.y}\eta \quad \text{----------(1)}$$

$$J_y \frac{d\omega_y}{dt} = T_{ct.x} + T_{am.x} - T_{cr.y} - T_{y.cr.y} - T_{f.y} \quad \text{---------------(2)}$$

$$\omega_y = (8\pi^2 + 17)\omega_x \quad \text{---------------(3)}$$

Where $J_x = J_y$ is the moment of gyroscope inertia around axis ox and oy respectively that calculated by the parallel axis theorem; ω_x and ω_y is the angular velocity of around axis ox and oy respectively; T = Wl =Mgl is the torque generated by the gyroscope weight W about axis ox; M is the mass; g is the gravity acceleration; l is the overhang distance of the gyroscope center-mass to the one side support; t is the time; $T_{f.x.} = Egdf/2$ is the frictional torque acting on the supports B and D, E is the mass of the gyroscope with the center beam; $T_{f.y}$ is the frictional torque acting on the support C; $T_{f.y} = Agd_c f_c / 2$ is the frictional torque acting on the thrust sliding bearing of the pivot C produced by the weight of the gyroscope's stand axis ox: $T_{ct.x}$ is the resistance and precession torque of centrifugal forces around axis ox and oy respectively; $T_{cr.x}$ and $T_{cr.y}$ is the torque of Coriolis forces around axis ox and oy respectively; $T_{am.x}$ and $T_{am.y}$ is the precession and resistance torque of the angular momentum of the spinning rotor around axis ox and oy, respectively (Table 1); the correction coefficient η.

Substituting defined equations of the torques (Table 1) into Eqs. (1) and (2) and transformation yield the following:

$$J_x \frac{d\omega_x}{dt} = Mgl - Eg\frac{df}{2} - \left(\frac{4\pi^2+8}{9}\right)J\omega\omega_x - (8\pi^2+17)J\omega\omega_x \times \left[1 - \frac{Agd_c f_c/2}{\left(\frac{4\pi^2+9}{9}\right)J\omega\omega_x}\right] \text{-----(4)}$$

$$J_y \frac{d\omega_y}{dt} = \left(\frac{4\pi^2+9}{9}\right)J\omega\omega_x - \frac{8}{9}J\omega\omega_y - Ag\frac{d_c f_c}{2} \text{--------(5)}$$

Where parameters of the mass moment of inertia of the gyroscope are represented in table 2, and other components are as specified above.

Where R is the conventional radius of the disc-type rotor. Expression of ω_y of Eq. (3) is substituted into the last component of Eq. (4) with ω_y and transformation yield the equation of the gyroscope motion about axis ox.

Parameters	Mass, kg	Moment of inertia around axis ox and oy, J kg.m^2
Spinning rotor with shaft, m_r	0,1159	$J = m_r R^2/2 = 0,5726674\times10^{-4}$ (around axis oz)
Gyroscope, M	0,146	$J_{i\,M} = (2/3)m_f r_f^2 + m_f l^2 + (m_r R_c^2/4) + m_r l^2 = 2,284736*10^{-4}$
Centre beam with journals and screw, b	0,028	$J_{bx} = m_b r_b^2/2 = 0,00224\times10^{-4}$ (around axis ox) $J_{by} = m_b l_b^2/12 = 0,2105833\times10^{-4}$ (around axis oy)
Mass E = M + b	0,174	$J_x = J_{xM} + J_{bx} = 2,286976\times10^{-4}$ (around axis ox) $J_y = J_{xM} + J_{by} = 2,495319\times10^{-4}$ (around axis ox)
Bar with screws, b_s	0,067	$J_{bs} = m_{bs} l_{bs}^2/12 = 0,51456\times10^{-4}$ (around axis oy)
Arm, a	2×0,009 = 0,018	$J_a = m_a r_a^2/2 + m_a l_a^2 = 0,41553\times10^{-4}$ (around axis oy)
Total A = M + b + b_s + a	0,259	$J_y = J_{y\,M} + J_b + J_{bs} + J_a = 3,38437\times10^{-4}$ (around axis oy)
The coefficients of the sliding friction of supports and pivots are f = 0,1 and f_c = 0.3. The speed of the gyroscope's rotor is 10000 rpm. d = 4,24 mm, d_c = 9 mm.		

Table 2: Technical data of the test stand with Super Precision Gyroscope, "Brightfusion LTD" [16].

Solution of Eq. (4) and Eq. (3) gives the angular velocities of rotation ω_x and ω_y, respectively.

Working example

Substituting technical data of Table (2) into Eq. (4) for motion of the gyroscope with one side support yields:

$$2,86976\times10^{-4}\frac{d\omega_x}{dt} = 0,146\times9,81\times0,0355 - 0,174\times9,81\times\frac{0,00424\times0,1}{2} - \left(\frac{4\pi^2+8}{9}\right)\times0,5726674\times10^{-4}\times10000\times\frac{2\pi}{60}\omega_x - (8\pi^2+17)\times0,5726674\times10^{-4}\times10000\times\frac{2\pi}{60}\omega_x\times\left[1-\frac{0,259\times9,81\times\left(\frac{0,009}{2}\right)\times0,3}{\left(\frac{4\pi^2+9}{9}\right)\times0,5726674\times10^{-4}\times10000\times\frac{2\pi}{60}\omega_x}\right] \quad \text{------------(6)}$$

Where all parameters are as specified above.

Simplification and transformation of Eq. (6) yields:

$$4,72711055\times10^{-5}\frac{d\omega_x}{dt} = 0,018380899 - \omega_x \quad \text{------(7)}$$

Separating variables and transformation for this differential equation gives the following:

$$\frac{d\omega_x}{0,018380899 - \omega_x} = -21154,571dt \quad \text{----------(8)}$$

Transformation and presentation of the obtained equation by the integral form with defined limits yield the following:

$$\int_0^{\omega_x}\frac{d\omega_x}{0,018380899 - \omega_x} = -21154,571\int_0^t dt \quad \text{-------(9)}$$

The left integral of Eq. (9) is presented by the tabulated integral.

$\int\frac{dx}{x\mp a} = \ln|x\mp a| + C$ .The right integral is simple and integrals have the following solution.

$\ln(0,018380899 - \omega_x)\big|_0^{\omega_x} = -21154,571t\big|_0^t$ that give rise to the following

$$\ln\frac{0,018380899 - \omega_x}{0,018380899} = -21154,571t \quad \text{-----(10)}$$

Then, $\omega_x = 0,018380899 - 0,018380899e^{-21154,571t}$ --------(11)

The second component of the right side of Eq. (11) has a small value of the high order that can be neglected. Solving of Eq. (11) gives the equations of the angular velocity for the gyroscope about axis ox and oy as the result of the action of the gyroscope weight and the frictional torques.

$$\omega_x = 0,018381 \quad rad/s = 1,053153^\circ/s \quad \text{--------------- (12)}$$

$$\omega_y = (8\pi^2+17)\times0,018380899 = 1,763772896\ \text{rad/s} = 101,0567 \quad \text{-------(13)}$$

The time of one revolution around axis oy is

$t = 360^o/\omega_y = 360^o/101{,}056742^o\ /\ s = 3{,}562$ s --------(14)

Experimental and theoretical results of the gyroscope motions about two axes with the action of load and frictional torques on supports are represented in Table 3.

Gyroscope average parameters	Tests	Theoretical	Difference, %
Time of one revolution around axis *oy, s*	3,83	3,562	7.52
The angular velocity about axis *ox*, *s*		1,053°/s	

Table 3: Experimental and theoretical results of the gyroscope precessions.

Analysis of the results of the theoretical calculations and practical tests for the gyroscope precessions demonstrate acceptable discrepancies between them. Obtained data enable for stating the theoretical and practical results are well matched and mathematical model for the forces acting on the gyroscope and its motions are satisfied for engineering practice.

Results and Discussion

The test results are a validation of the analytic statement that the action of the frictional forces on supports and pivots of the gyroscope decreases the value of the precession torque generated around axis oy and acting around axis ox. This fact increases the value of the precession torques around axis oy that lead to the increase of the angular velocity of the gyroscope around axis oy. The mathematical model for the torques of the gyroscope explains the physics of its motions. The result of the theoretical calculation and practical test of the gyroscope (Table 3) are almost the same for the precessed motion about axis oy. The time of the gyroscope rotation on 20° about axis ox is almost twice less than in the first publications [13,16] that did not measure due to the very small value of the angular velocity. The discrepancy of theoretical and practical results does not exceed 10% which is the recommendation of the experts [17].

Conclusion

Mathematical modeling for gyroscopic effects is the result of painstaking work accompanied by successes and omissions that is a normal process. The first publications of the inertial torques generated by the rotating mass of the spinning disc contains incorrect processing of the integral equation for torques generated by centrifugal forces. The mathematical mistake in the expression of the one inertial torque is not fundamental but gives a wrong calculation of gyroscope motion around one axis. The gyroscope motion around another axis does not change because incorrectness of the expression at one torque is compensated by the increased dependency of the angular velocities about two axes. The renovated mathematical model for the centrifugal inertial torque was validated by practical tests of the gyroscope motions.

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