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Research Article

Extreme Value Charts and ANOM Based on Gumbel Distribution

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Abstract

The quality is assessable by its feature is implicit with a probability model which follow Gumbel distribution. From each subgroups extreme values are used to construct extreme value charts and variable control charts. Probability model of the extreme order statistics and the size of each sub group are used in control chart constants. To find the decision lines of Gumbel distribution we implement a method of analysis of means (ANOM). The proposed ANOM decision lines are constructed for given number of subgroup within means category and between means category given by Ott (1967). These decision lines are illustrated by giving few examples.

Keywords: Analysis of Means; Statistical Control; Quantile - Quantile Plot; Control Limits

Introduction

Analysis and prediction of the business conditions is the top most trend of research especially in marketing field. Every business has interested to know the future market value of their products. To analyze this type of conditions there are common statistical techniques are available like weekly averages, monthly averages and moving averages etc., Averages gives a summary about the product and business. Statistical methods are important for solving the economic problems of industry. To asses quality common tool is Shewart control charts. These charts are used to diagnose the assignable causes when there is any adjustments made in the process. Otherwise expected assignable cause is treated as to be a quality diverse in character of the subgroup. In this situation analysis of means (ANOM) technique is an effective and appropriate. For example if statistic is a sample mean, which control diversity of the process mean showing away from the expected mean. To analyse the subgroups those are categorized within identical group of means and between the mean of diverse category is possible through ANOM, which is the alternative method of Analysis of Variance (ANOVA). A further most characteristic of ANOM include its interpretation and graphical presentation. An ANOM charts and control charts are theoretically having the same pattern among decision lines, thus magnitude differences and Statistical significance of the treatments may be evaluated at the same time. The ANOM technique was first developed by Prof. Ott (1967) to know the association of means subgroup and to check if any one of them changes significantly from the overall mean. Statistical control charts are designed based on the normal distribution. Where as if the data follows skewed distribution these limits should be calculated according to the distribution based on the variable control limits.

In the present paper we consider Gumbel Distribution is one of the skewed distribution and developed control limits for the distribution.

The probability density function (pdf) of a Gumbel distribution (GD) with scale parameter σ is given by

$f(x) = \frac{1}{\sigma}\left[e^{-(z+e^{-z})}\right]$ where $z = \frac{x-\mu}{\sigma}$; $-\infty<x<\infty$, $-\infty<\mu<\infty$, $\sigma > 0$----------- (1.0.1)

Its cumulative distribution function (cdf) is

$F(x) = \exp[-e^{-(x-\mu)/\sigma}]$ ----------(1.0.2)

To construct control charts by extreme elements drawn from the production process of subgroup which follows Gumbel Distribution. Let us suppose that $x_1, x_2, \ldots, x_n$ are the select sample from the production process plotted on control chart. Individual samples are plotted into the control chart without estimating any statistic out of them. Based on the plotted sample using x_1 (sample minimum) and x_n(sample maximum) a right decision should be taken.

Let us suppose that $\bar{x}_1, \bar{x}_2 \ldots \bar{x}_n$ are arithmetic mean of k subgroups and n sample size drawn from the Gumbel distribution. To know whether the population from which subgroup means are drawn is working with allowable quality or not can be assessed with the help of subgroup means. Any sample mean lies outside these decision lines it is declared significantly different from the grand mean. Decision lines and significance of the samples can be assessed at a time by using the ANOM charts.

The main intention of developing ANOM using control limits of extreme value statistic for this we follow the concept and procedure only. We are not taken any developed ANOM tables or techniques. However, a detailed literature about ANOM is available in Rao (2005) [15], Ott (1967) and some related works in this direction are Enrick (1976) [4], Schilling (1979) [6], Ohta (1981) [11], Ramig (1983) [13], Bakir (1994) [1], Bernardand Wludyka., *et al.* (2001) [2], Montgomery (2000) [8], Nelson and Dudewiczand Nelson (2002) [10], Farnum (2004) [5], Guirguis and Tobias (2004) [6],Srinivasa Rao B., *et al.* (2012) [17], Srinivasa Rao. B and Pratapa Reddy. J., *et al.* (2012) [18], B.Srinivasa Rao and P. Sricharani (2018) [19], R. Subba Rao, A. Naga Durgamamba., *et al.* (2018) [20], Kalanka P Jayalatha and Honkeung Tony Ng. (2020) [21], Kalanka P Jayalatha and Jacob Turner (2021) [22], B.Srinivara Rao, K.N.V. R Lakshmi., *et al.* (2021) [23], Anil and B. Srinivasa Rao (2021) [24] all the references listed at the end of the article.

The rest of the paper is organized as follows. Extreme value control chart concepts and procedure of is explained in section 2. Using GD model in ANOM and extreme value control charts of GD is kept in section 3. Examples and illustrative are in section 4. Summary and conclusions are discussed in section 5.

Extreme value charts

If α is the level of significance and we should have the probability of all the subgroup means to fall within the control limits is (1-α). By considering subgroups the above probability statement be converted into n^{th} power of the probability of a subgroup mean to fall within the limits should be equal to (1- α). i.e., the confidence interval for meanof the sampling distribution of lie between two control limits should be equal to $(1-\alpha)^{1/n}$. using the same concept for developing the ANOM charts for Gumbel Distribution.

Subgroups and samples are drawn from the GD model. Using the extreme order statistics concepts control lines are determined based on GD. By selecting the x_i of $X = (x_1, x_2, \ldots, x_n)$ lies with probability $(1-\alpha)^{1/n}$ within the limits the control lines are to be determined. This is the probability inequality. $P(x_1 \le L) = \alpha/2$ and $P(x_n \ge U) = \alpha/2$. According to the concepts of order statistics, the cumulative distribution function of the least and highest order statistics in a sample of size n from any continuous population are $[F(x)]^n$ and $1-[1-F(x)]^n$ respectively. where F(x) is the cumulative distribution function of the population. If (1-α) is the desired at 0.9973 then α would be 0.0027. Taking F(x) as the CDF of a standard GD model (σ = 1), we can get solutions of the two equations$1-[1-F(x)]^n = 0.00135$ and $[F(x)]^n = 0.99865$, which in turn can be used to develop the control limits of extreme value chart. The resuls of the above two equations for n = 2 (1) 10 is given in table 1 denoted as $Z_{(1)\,0.00135}$ and $Z_{(n)\,0.99865}$.

n	$Z_{(1)0.00135}$	$Z_{(n)\,0.99865}$
2	0.03733	4.49294
3	0.09505	3.55494
4	0.13846	3.12189
5	0.2064	2.83479
6	0.2426	2.68648
7	0.2759	2.46582
8	0.29866	2.36053
9	0.30828	2.28933
10	0.32988	2.23825

Table 1: Control Limits of Extreme value charts.

The values of table 1 indicates the following probability statement:

$P(Z_{(1)0.00135} \leq {}_{i} \leq Z_{(n)\,0.99865,}\, \forall\, i = 1,2,.....n) = 0.9973$ ------- (2.0.3)

$P(\sigma\, Z_{(1)0.00135} \leq {}_{i} \leq \sigma\, Z_{(n)\,0.99865\,,}\, \forall\, i = 1,2,.....n) = 0.9973$ ------------- (2.0.4)

Taking $\overline{X}$ is 0.5772 as an unbiased estimate of σ, the above equation becomes

$\Rightarrow P(\, D_3^* < x_i < D_4^*\, ,"i = 1,2,.....n) = 0.9973$

Where $D_3^* = \frac{Z(1)0.00135}{0.5772}$ and $D_4^* = \frac{Z\,(n)\,0.99865}{0.5772}$ Thus D_3^* and D_4^*would constitute the control chart constants for the extreme values charts. Results are given in table 2 for n = 2(1) 10.

N	D_3^*	D_4^*
2	0.064674	7.784026
3	0.164674	6.15894
4	0.239882	5.40868
5	0.357588	4.911279
6	0.420305	4.654331
7	0.477997	4.272037
8	0.517429	4.089622
9	0.534096	3.966268
10	0.571518	3.877772

Table 2: Constants of Extreme value charts.

Analysis of Means (ANOM) - Gumbel distribution

Suppose , ,.....,are arithmetic means of k subgroups of size 'n' drawn from the GD model. To assess whether the population from which these subgroup means are drawn is working with allowable quality variations. Based on the population model, we generally, use the control chart constants developed by us or the well known Shewart constants available in any SQC text book. If all the subgroup means fall with in the limits then the process is in control. Otherwise we say the process is out of control. The above decisions can be converted into probability statements by assuming α is the level of significance.

P{LCL <, ∀i = 1 to k <UCL} = 1-α ------(3.0.6)

Using the notion of independent subgroups (3.0.6) becomes

P{LCL <, <UCL} = $(1\text{-}\alpha)^{1/k}$ -------(3.0.7)

With equi–tailed probability for each subgroup mean, we can find two constants say L* and U* such that

P{<L*} = P{ , >U*} = ------------(3.0.8)

In the case of normal population L* and U* satisfy U* = -L*.

If the distribution follows skewness like GD we have to calculate L*, U* separately from the sampling distribution of . Accordingly these depend on the subgroup size 'n' and the number ofsubgroups 'k'. We make us of the equations (3.07) and (3.08) for specified 'n' and 'k' toget L* and U* for α = 0.05 and α = 0.01. These are given in figure a and b.

n	LIMITS	k=1	2	3	4	5	6	7	8	9	10	15	20	30	40	50	60
2	L	0.1578	0.1115	0.0916	0.0768	0.0701	0.0602	0.0572	0.0544	0.0529	0.0504	0.0396	0.0369	0.0336	0.0327	0.0325	0.0275
	U	2.7799	3.1496	3.3567	3.5481	3.6613	3.7286	3.8671	3.9978	4.0332	4.0579	4.1905	4.5026	4.7724	4.8167	4.9507	4.9840
3	L	0.2426	0.1848	0.1649	0.1481	0.1427	0.1361	0.1325	0.1303	0.1248	0.1204	0.1021	0.0868	0.0809	0.0766	0.0758	0.0696
	U	2.4326	2.7079	2.8699	2.9975	3.0838	3.1147	3.1562	3.2480	3.3076	3.3109	3.4753	3.5798	3.6686	3.6903	3.7674	3.8161
4	L	0.3166	0.2634	0.2425	0.2247	0.2126	0.2035	0.1955	0.1877	0.1823	0.1776	0.1513	0.1383	0.1331	0.1056	0.0959	0.0941
	U	2.1684	2.4070	2.5534	2.6263	2.7249	2.7618	2.8250	2.8691	2.9139	2.9439	3.0517	3.1519	3.2006	3.2088	3.3905	3.5417
5	L	0.3645	0.3040	0.2731	0.2625	0.2529	0.2436	0.2370	0.2343	0.2337	0.2305	0.2170	0.2047	0.1904	0.1737	0.1652	0.1619
	U	2.0251	2.2184	2.3270	2.4111	2.4829	2.5311	2.5551	2.5911	2.6163	2.6752	2.8098	2.8413	2.9225	3.0219	3.0227	3.0310
6	L	0.4135	0.3563	0.3255	0.3156	0.2934	0.2877	0.2744	0.2620	0.2590	0.2565	0.2498	0.2417	0.2241	0.1941	0.1936	0.1927
	U	1.9667	2.1480	2.2335	2.3173	2.3781	2.4005	2.4406	2.4853	2.5065	2.5232	2.5591	2.6876	2.8087	2.8672	2.9782	2.9812
7	L	0.4381	0.3975	0.3669	0.3499	0.3420	0.3333	0.3219	0.3149	0.3097	0.3057	0.2880	0.2748	0.2552	0.2445	0.2372	0.2318
	U	1.8771	2.0356	2.1541	2.2093	2.2493	2.2859	2.3189	2.3406	2.3544	2.3778	2.4355	2.4884	2.5821	2.6140	2.6190	2.6195
8	L	0.4694	0.4069	0.3798	0.3676	0.3551	0.3476	0.3381	0.3322	0.3261	0.3141	0.3014	0.2952	0.2811	0.2659	0.2583	0.2582
	U	1.7852	1.9251	2.0019	2.0624	2.0987	2.1563	2.1787	2.2286	2.2396	2.2602	2.3061	2.3894	2.4105	2.5134	2.5274	2.5539
9	L	0.4998	0.4510	0.4234	0.4061	0.3858	0.3770	0.3685	0.3566	0.3473	0.3419	0.3228	0.3034	0.2888	0.2764	0.2635	0.2231
	U	1.7322	1.8783	1.9304	1.9801	2.0119	2.0482	2.0683	2.1213	2.1305	2.1581	2.1935	2.2962	2.3264	2.3354	2.3751	2.3771
10	L	0.5119	0.4548	0.4268	0.4134	0.4014	0.3966	0.3829	0.3728	0.3641	0.3576	0.3367	0.3286	0.3117	0.3079	0.2912	0.2846
	U	1.7092	1.8344	1.9156	1.9689	2.0252	2.0654	2.0885	2.1037	2.1073	2.1154	2.1961	2.2388	2.3016	2.3373	2.3603	2.3979

Figure a: Gumbel Distribution Constants for Analysis of Means (1- α = 0.95).

n	LIMITS	k=1	2	3	4	5	6	7	8	9	10	15	20	30	40	50	60
2	L	0.0685	0.0504	0.0391	0.0369	0.0359	0.0336	0.0334	0.0327	0.0325	0.0325	0.0223	0.0097	0.0097	0.0058	0.0058	0.0058
	U	3.6647	4.0579	4.1936	4.5026	4.5373	4.7724	4.8017	4.8167	4.9507	4.9840	5.0839	5.1209	5.3222	5.3222	5.6975	5.8811
3	L	0.1420	0.1204	0.1000	0.0868	0.0865	0.0809	0.0809	0.0766	0.0758	0.0758	0.0647	0.0635	0.0635	0.0545	0.0545	0.0545
	U	3.0910	3.3109	3.4848	3.5798	3.6131	3.6686	3.6759	3.6903	3.7674	3.8161	3.8275	3.8512	3.8555	3.8555	3.8945	3.9589
4	L	0.2122	0.1776	0.1469	0.1383	0.1343	0.1331	0.1211	0.1056	0.0959	0.0959	0.0940	0.0604	0.0604	0.0212	0.0212	0.0212
	U	2.7348	2.9439	3.0860	3.1519	3.1595	3.2006	3.2033	3.2088	3.3905	3.5417	3.5784	3.7032	4.0060	4.0060	4.1803	4.5836
5	L	0.2520	0.2305	0.2106	0.2047	0.1937	0.1904	0.1850	0.1737	0.1652	0.1652	0.1453	0.1448	0.1448	0.1293	0.1293	0.1293
	U	2.4932	2.6752	2.8212	2.8413	2.8670	2.9225	2.9663	3.0219	3.0227	3.0310	3.0328	3.0353	3.0693	3.0693	3.0797	3.2173
6	L	0.2924	0.2565	0.2474	0.2417	0.2316	0.2241	0.2219	0.1941	0.1936	0.1936	0.1902	0.1293	0.1293	0.1130	0.1130	0.1130
	U	2.3834	2.5232	2.5686	2.6876	2.7058	2.8087	2.8513	2.8672	2.9782	2.9812	3.0107	3.0111	3.1889	3.1889	3.2689	3.3749
7	L	0.3405	0.3057	0.2850	0.2748	0.2693	0.2552	0.2529	0.2445	0.2372	0.2372	0.2300	0.2055	0.2055	0.1938	0.1938	0.1938
	U	2.2549	2.3778	2.4395	2.4884	2.4940	2.5821	2.5873	2.6140	2.6190	2.6195	2.6728	2.7878	2.9143	2.9143	3.0613	3.6102
8	L	0.3519	0.3141	0.3007	0.2952	0.2837	0.2811	0.2765	0.2659	0.2583	0.2583	0.2344	0.1904	0.1904	0.1442	0.1442	0.1442
	U	2.1010	2.2602	2.3372	2.3894	2.4072	2.4105	2.4732	2.5134	2.5274	2.5539	2.5777	2.5924	2.7016	2.7016	2.7061	3.3559
9	L	0.3823	0.3419	0.3214	0.3034	0.2981	0.2888	0.2824	0.2764	0.2635	0.2635	0.1966	0.1845	0.1845	0.1775	0.1775	0.1775
	U	2.0138	2.1581	2.1989	2.2962	2.3021	2.3264	2.3299	2.3354	2.3751	2.3771	2.4031	2.4772	2.5265	2.5265	2.9165	2.9583
10	L	0.4010	0.3576	0.3346	0.3286	0.3191	0.3117	0.3090	0.3079	0.2912	0.2912	0.2716	0.2251	0.2251	0.1962	0.1962	0.1962
	U	2.0317	2.1154	2.2036	2.2388	2.2658	2.3016	2.3295	2.3373	2.3603	2.3979	2.3997	2.4648	2.5323	2.5323	2.5709	2.6815

Figure b: Gumbel Distribution Constants for Analysis of Means (1- α = 0.99).

All the subgroup means which are different themselves and homogeneous between groups fall with in indicates 'In Control'. Hence the constants of figure a and b can be used as Analysis of Mean technique. For a normal population one can use the tables of Ott (1967) [12]. For an GD our tables can be used. To evaluate the goodness of fit of Gumbel Distribution model we consider few examples with quantile-quantile plot technique (strength of linearity between observed and theoretical quantiles of a model) and check the homogeneity of means in each group.

Illustrative examples

Example 1

Wardsworth (1986): Consider the following data of 25 observations on "A manufactures of metal products that suspected variations in iron content of raw material supplied by five suppliers. Five ingots were randomly selected from each of the five suppliers. The following table contains the data for the iron determinations on each ingots in percent by weight.

Example-1				
Suppliers				
1	2	3	4	5
3.46	3.59	3.51	3.38	3.29
3.48	3.46	3.64	3.4	3.46
3.56	3.42	3.46	3.37	3.37
3.39	3.49	3.52	3.46	3.32
3.4	3.5	3.49	3.39	3.38

Table a

Example 2

Three brands of batteries are under study. It is suspected that the life (in weeks) of the three brands is different. Five batteries of each brand are tested with the following results. Test whether the lives of these brands of batteries are different at 5% level of significance.

Example 3

Four catalysts that may effect the concentration of one component in a three component liquid mixture are being

Example-2		
Weeks of life		
Brand-1	Brand-2	Brand-3
100	76	108
96	80	100
92	75	96
96	84	98
92	82	100

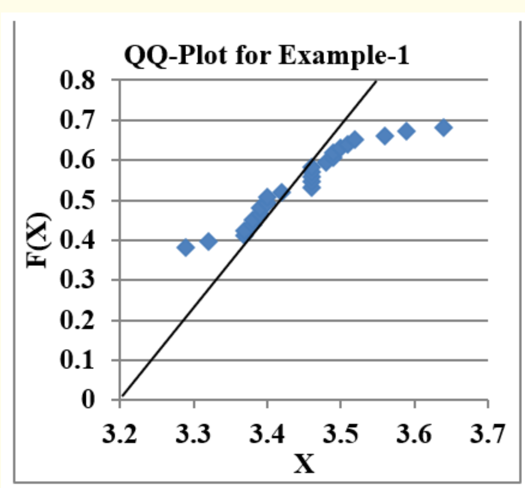
Table b

investigated. The following concentrations are obtained. Test whether the four catalysts have the same affect on the concentration at 5% level of significance.

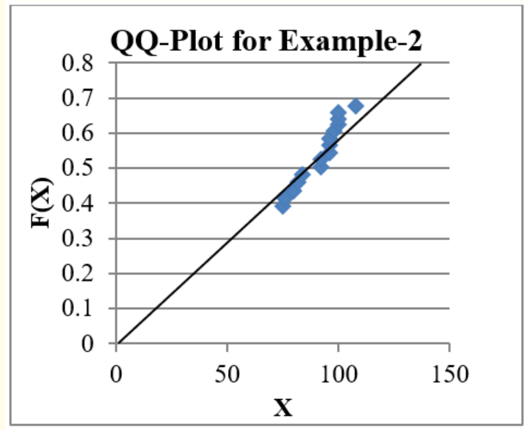
Example-3			
Catalyst			
1	2	3	4
58.2	56.3	50.1	52.9
57.2	54.5	54.2	49.9
58.4	57	55.4	50
55.8	55.3	54.9	51.7

Table c

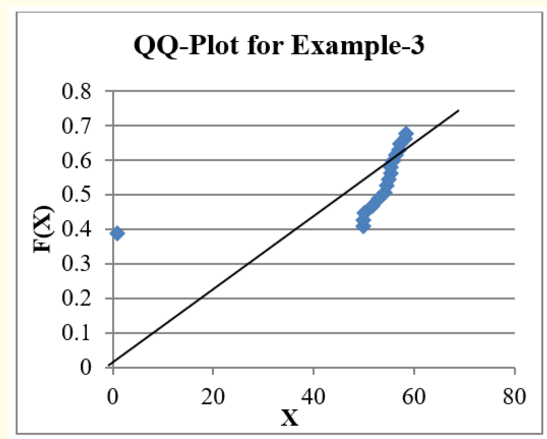
Graphical presentation of QQ-Plot for the data in the illustrated examples are shown below.



Graph 1



Graph 2



Graph 3

From the above graphs shows the goodness of fit of data given in three examples are represented by qunatile–quantile plot (correlation coefficient) are summarized in the following table, that indicates Gumbel Distribution is a better model, revealing significance linear relationship between sample and population quantiles.

	GD	Normal
Example 1	0.9595	0.2067
Example 2	0.9716	0.4149
Example 3	0.9815	0.4447

Table d

In the given data observations are considered as a single sample, we have determined the decision limits for the Normal population as well as Gumbel population and have given them in the table 3 respectively.

	(LDL, UDL)	No. of. Subgroups fall			
		With in the decision lines	Coverage probability	Outside decision limits	Coverage probability
Example-1 n = 5, k = 5, α = 0.05	[3.379,3.517] [0.8719, 8.5602]	3 5	0.6 1.0	2 0	0.4 0.0
Example-2 n = 5, k = 3, α = 0.05	[87.82,95.52] [25.0305,213.3121]	2 3	0.7 1.0	1 0	0.3 0.0
Example-3 n = 4, k = 4, α = 0.05	[26.14, 82.84] [12.2455, 143.0999]	2 4	0.5 1.0	2 0	0.5 0.0

Table 3

In each cell the first row values represents the Normal distribution and second row values represents the Gumbel distribution.

Summary and Conclusions

From the data when we use ANOM tables of Ott (1967) [11] the number of homogeneous means for each data set are 3,2,2 respectively and those away from heterogeneity are 2,1,2 respectively. Where as when we use ANOM tables of Gumbel Distribution we get the number of homogeneous means to be 5,3,4 respectively without showing deviation of any mean from homogeneity i.e., out side the control limits. Therefore there is a possibility of rejection when we use normal model resulted in homogeneity for within means and deviation from between means. Through quantile-quantile plot technique we made a conclusion that Gumbel Distribution is a better model than Normal. Therefore, Gumbel Distribution is a better decision if all the means to be homogeneous. We recommend the environmentalist hydrologist and experimenters those who are to adopt this techniques. Also suggest for predictive analysis occurrence of natural extreme events such as flood water level and high winds.

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