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Research Article

Estimation of Lyapunov Exponents for Systems with Periodic Coefficients On-base Structure Approach

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Abstract

We propose an approach to Lyapunov exponents (LE) identification. It bases on the analysis of geometric frameworks (GF) describing the dynamics of the LE change. We obtain the upper bound for the smallest LE and mobility limit for the large LE and the indicator set for the system with periodic coefficients. Graphical criteria (GC) propose to assess of adequacy the obtained LE. GC is based on the proposed histograms method and is applied to assess LE adequacy. We show that the dynamic system has the LE set. Identifiability conditions of LE are obtained.

Keywords: Dynamic Systems with Periodic Coefficients; Lyapunov Exponents; Framework; Histogram; Almost Periodic Function; Identifiability

Introduction

Lyapunov exponents apply to the analysis of dynamic system qualitative behavior. Introduction should reflect the background, purpose and significant of the study that is carried out. They allow estimating trajectory behavior of various objects in physics [1], medicine [2-4], economy [5], astronomy [6]. LE determine based on time series analysis. Many researchers assume that a priori information on the structure of the system is known. [7], contains the calculation review of large LE for various classes of systems. Lyapunov exponents estimation algorithm for an unknown dynamic system is proposed in [8]. The algorithm calculates all LE based on the application of networks and the multidimensional prediction. Various algorithms are used for the calculation of the large LE for time-varying systems on experimental data. Takens theorem (TT) [9] is the base for the application of these algorithms. Wolf [10] and Rosenstein [11] methods apply to the definition of the large LE. Many authors generalize and develop these methods [12]. In [12], the approach proposes large (first) LE estimation and the method for logarithm allocation. It bases on the interpolation of a time series. The application of the interpolation algorithm gives the best results for time-varying systems. Notice that implementation of the Rosenstein method is a labour-consuming procedure and is associated with the choice of system parameters. The neural network algorithm [13] is proposed for the estimation of the largest LE. It is based on the application of multilayer perceptron.

Two main methods exist for Lyapunov exponents estimation on the time series [14]. The application of these methods is based on an attractor and the TT use. In [10], the method estimates LE by two close trajectories in the recovered phase space, and it traces trajectories behaviour at some time interval (Benettin's algorithm) [15]. The LE spectrum estimation coincides with LE calculation on equations of an initial system and equations in variations. The relative simplicity is the advantage of this method. The shortcoming of this method is the difficulty of all spectrum LE identification as the estimate of the large LE effects on the analysis of two close trajectories. The second method [16,17] is based on Jacobin. There the LE estimate is based on the calculation of Jacobi matrix eigenvalues for a system generating the time series. The advantage of this method is the possibility of non-negative Lyapunov exponents spectrum estimation on short implementation, and the shortcoming is high sensitivity to noise and errors.

Takens theorem application depends on features of the time series [18]. The implementation complexity of LE identification methods is explained with features of the time series. These approaches are applied to LE estimation of time-varying systems.

Therefore, various modifications of Rosenstein, Benetton, Wolf and Takens theorem widely applies to Lyapunov exponents identification of stationary systems. Properties of the time series affect the accuracy of obtained LE estimations. The simplification of specified methods is based on consideration of a priori information. As a rule, the considered methods allow finding the high (first, large) Lyapunov exponent. The overwhelming number of publications is devoted to the analysis of chaos systems. Time-varying systems have specific [19]. In particular, they may contain the Lyapunov exponents set. A further modification is required for the approaches and methods considered above. Criteria and procedures for verification of obtained decisions are proposed not always. In [20], the approach to the LE identification based on the analysis of frameworks is proposed. Frameworks describe LE change dynamics of stationary systems under uncertainty. They do not demand the application of procedures and methods considered above. We gave a generalization of this approach on a class of periodic systems.

Problem statement

Consider the linear dynamic system [19]

$$\begin{aligned} \dot{X} &= A(t)X + BU, \\ Y &= CX + W_U U, \end{aligned} \qquad \text{--------------(1)}$$

Where $X \in R^m$ is state vector, $Y \in R^n Y \in R^n$ are input and output, $W_U \in R^{n\times k}$, $C \in R^{n\times m}$, $A(t) \in R^{m\times m}$, , $B \in R^{m\times k}$.

Let the matrix $A(t)$ satisfy conditions.

A1. $A(t)$ is a continuous Frobenius matrix and limited

$$\|A(t)\| \le \alpha_A \qquad \text{--------------(2)}$$

Where $\alpha_A > 0$, $\|\cdot\|$ is norm of a matrix.

A2. $A(t)$ is almost periodic [17].

$$A_i(t) = A_i(t - \tau_i) \qquad \text{--------------(3)} .$$

A3. $A(t)$ is Hurwitz matrix for almost all $t \ge 0$.

Experimental information for (1) has the form

$$\mathrm{I}_o = \left\{Y(t), U(t), t \in J = \left[t_0, t_1\right]\right\}. \qquad \text{--------------(4)}$$

Write the solution of the system (1) as

$$X(t) = \mathcal{X}(t_0, U, t) \qquad \text{--------------(5)} ,$$

Where $\mathcal{X}$ is the operator who is determined by matrixes A, B.

Obtain from (5) the solution of system (1) at $X_0 = X(t_0)$

$$X(t) = X_g(t) + X_q(t) \qquad \text{--------------(6)} ,$$

Where $X_q(t)$ is a particular solution (1) with $U \in \mathrm{I}_o$, $X_g(t)$ is a general solution (1) with $U(t) = 0$ at the unknown $X_0 \in \mathrm{I}_o$.

Let $X \ (X_0, t)$ isageneralsolutionofsystem(1)with $X_0 = X_0(Y_0) \in \mathrm{I}_o$

Problem: determine by solution estimations

$$X_g(t) = X_g(X_q, X_0, t)$$

on the set I_o determine the eigenvalues spectrum and the order of system (1).

Estimation $X_g(t)$

Apply operation $\left\{X(t)\right\} \setminus \left\{X_q(t)\right\}$ and create the set $\left\{X_g(t)\right\}$ for LE estimation. We will state the task solution method [20] on the example of a second-order system (1) with one input and an output. Introduce designations: $y = Y$, $u = U$.

Let $W_U = 0$, $D_u(\omega) D_u(\omega)$ are frequency spectrum u, y, $|y(t)| < \infty$, $|u(t)| < \infty$. As the matrix $A(t)$ satisfies the A3 condition obtain $D_y(\omega) \ne D_u(\omega)$, i.e. the system (1) is time-varying. The proposed approach to the design of a model eliminates this delay.

Present I_o as

$$\mathrm{I}_o = \mathrm{I}_o^q(J_q) \cup \mathrm{I}_o^g(J_g) ,$$

Where $J_q \cup J_g = J \subseteq R$; I_o^q, I_o^g are sets containing the information about X_q and X_g.

Determine by particular solution estimation of the system (1) on the set $\mathrm{I}_o^q(J_q)$. As $x_1 = y, y \in R$ apply variable y differentiation operation to obtaining the component $x_2 = \dot{x}_1$ vector $X \in R^2$. Designate $x_2 = \dot{y}$.

Statement 1 [20]. Model

$$\hat{X}_q(t) = \hat{A}_q W(t) \quad \forall t \in J_q \qquad \text{--------------(7)}$$

is applied for the identification $X_q(t)$ on the set I_o^q, where $\hat{A}_q \in R^{2\times 2}$ is the parameter matrix of the model, $W = [u\; u']^T$.

Model (7) properties depend on the choice of the interval $J_q \subset J$. Model (7) is applied also on $m > 2$.

Determine by the estimation of the particular solution $\hat{X}_q(t)$ of the system (1) using the model (7) on I_o^g. Next obtain the estimation of general solution

$$\hat{X}_g(t) = X(t) - \hat{X}_q(t) \quad \forall t \in J_g,$$

where $\hat{X}_g(t) = [\hat{y}_g(t)\; \dot{\hat{y}}_g(t)]^T$.

Further, we consider the system (1) with one input u and one output y and suppose that the system (1) is identifiable. Perform this condition check based on the approach proposed in [20].

Structural properties coefficient of system. LE

Apply Lyapunov exponents [19] to the estimation of system (1) properties. LE for a real function $h(t)$ determine as

$$\chi[h] = \overline{\lim_{t\to\infty}} \frac{\ln|h(t)|}{t}, \qquad \text{--------------(8)}$$

where $\overline{\lim\limits_{t\to\infty}}$ is upper limit.

LE $\chi_i[h]$ $\left(i = \overline{1,m}\right)$ of the stationary system (1) coincides with real parts of eigenvalues λ_i for matrix A.

Let the estimation of general solution $\forall t \in J_g$ be known for the system (1) and condition A3 is satisfied.

Apply (8) to $\hat{y}_g(t)$

$$\chi\left[\hat{y}_g\right] = \overline{\lim_{t\to\bar{t}}} \frac{\ln\left|\hat{y}_g(t)\right|}{t} \qquad \text{--------------(9)},$$

where $\bar{t} \in J_g$ is the upper bound t on an interval $J_g \subset J$.

(9) is large LE. If the limit (8) exists, that $\chi\left[\hat{y}_g\right]$ is matrix A maximum eigenvalue estimation. Therefore, $\chi\left[\hat{y}_g\right]$ is the stability degree of the system (1). If $m = 2$, then obtain for $\dot{\hat{y}}_g$

$$\chi\left[\dot{\hat{y}}_g\right] = \overline{\lim_{t\to\bar{t}}} \frac{\ln\left|\dot{\hat{y}}_g\right|}{t}. \qquad \text{--------------(10)}$$

Also, the indicator

$$\eta[h] = \underline{\lim_{t\to\infty}} \frac{\ln|h(t)|}{t} \qquad \text{--------------(11)}$$

is applied, where $\underline{\lim\limits_{t\to\infty}}$ is the bottom limit. It is the Perron bottom index [19,20].

The application idea of Lyapunov exponents in identification problems is presented in [20]. The proposed approach is based on the analysis of structural properties coefficient (SPC) [20]. Hereinafter development of this approach is given. At first, considers an association between CSP and LE.

Let

$$\rho\left(\hat{y}_g\right) = \rho_g = \ln\left|\hat{y}_g(t)\right| \quad \forall t \in \bar{J}_g \subset J_g, \qquad \text{--------------(12)}$$

where $\bar{J}_g = \left[t_0, \bar{t}\right]$ determine based on (9).

Then CSP for the estimation of system structural properties has the form

$$k_s(t,\rho) = \frac{\rho\left(\hat{y}_g(t)\right)}{t} \qquad \text{--------------(13).}$$

$k_s(t,\rho)$ is the main variable for indicator $\chi[\hat{y}_g]$ calculation on the interval $\bar{J}_g$.

So, interdependence between LE $\chi\left[\hat{y}_g\right]$ and the SPC coefficient of structural properties $k_s(t,\rho)$ is showed on the informational set $\mathrm{I}_\rho = \left\{\rho\left(\hat{y}_g(t)\right), t \in \bar{J}_g\right\}$.

Consider the set

$$\mathrm{I}_g\left(\hat{y}_g, t\right) = \left\{\hat{y}_g(t), t \in J_g\right\} = \mathrm{I}_{\hat{X}_g} \setminus \left\{\dot{\hat{y}}_g(t)\; t \in J_g\right\} \qquad \text{-------------(14),}$$

which data contains about the change of the variable $\hat{y}_g$ on the interval J_g.

We suppose that the system (1) is stable, i.e. $\mathrm{Re}(\lambda_i(t)) \le 0$, $i=\overline{1,m}$ for $\forall t>0$, where $\lambda_i(t)\in\sigma(A)$ is i-th matrix eigenvalue.

Problem: estimate Lyapunov exponents and the of the system (1) order based on sets I_ρ, I_g analysis.

As the set (14) is formed based on the model (7), the estimation $\hat{y}_g$ will contain error ε. Therefore, the function $\hat{y}_g(t)$ is almost "periodic".

Almost periodic functions to bohr

Consider a class of almost periodic functions to Bohr.

Definition 1 [21]

The numerical set $\Xi=\{\xi\}$ is called relatively dense on the real axis $-\infty<x<\infty$ if number $l>0$ exists such that each segment $a\le x\le a+l$ of length l contains at least one element of our set, i.e. at any a we have

$$[a,a+l]\cap\Xi\ne 0.$$

Definition 2 [21]

The number $T=T_f(\delta)$ is almost the function $f(x)$ period with an accuracy of δ if inequality

$$\left|f(x+T_f)-f(x)\right|<\delta,\quad \delta>0 \qquad \text{-------------(15)}$$

fairly for any $x\in(-\infty,\infty)$.

Definition 3 [21]

A function $f(x)\in(-\infty,\infty)$ is almost periodic in Bohr sense ($\mathcal{BF}$-function) if a relatively dense set of almost T_f-period for the function $f(x)$ exists within accurate δ; i.e. such positive number $l=l(\delta)$ exists that any segment $[a,a+l]$ contains, at least, one number T_f for which it is fair

$$\left|f(x+T_f)-f(x)\right|<\delta \text{ for } x\in(-\infty,\infty).$$

where δ is any positive number.

Function $\hat{y}_g(t)$ belongs to exponential-sinusoidal function class. Therefore, condition (15) cannot be satisfied. Provide the belonging $\hat{y}_g(t)$ to $\mathcal{BF}$-functions [22]. Perform the following operations.

Consider some point $t\in R$ and its neighbourhood O_t. Determine the average value $\hat{y}_g(t)$ for $\forall t\in O_t$

$$\alpha=\bar{\hat{y}}_{g,O_t}=\frac{1}{N_t}\sum_i \hat{y}_i,$$

where N_t is the point quantity on O_t, $t_i\in O_t$ is the current coverage of the interval O_t with a step τ.

Obtain for $t\in R$ belonging to the neighbourhood $O_{t+T_{\hat{y}_g}}$

$$\pi=\bar{\hat{y}}_{g,O_{t+T_{\hat{y}_g}}}=\frac{1}{N_{t+T_{\hat{y}_g}}}\sum_i \hat{y}_i,$$

Definition 4 [22]

A function $\hat{y}_g(t)\in(-\infty,\infty)$ is almost periodic in Bohr sense ($\mathcal{BF}_{\alpha\pi}$-function) if a relatively dense set of almost T_f-period for the function $\hat{y}_g(t)$ exists within accurate δ; i.e. such positive number $l=l(\delta)$ exists that any segment $[a,a+l]$ contains, at least, one number T_f for which it is fair

$$\left|\frac{\hat{y}_g(t+T_f)}{\pi}-\frac{\hat{y}_g(t)}{\alpha}\right|<\delta \text{ for } t\in[0,\infty).$$

Frameworks for LE estimation

Consider sets

$$\mathrm{I}_{k_s}=\left\{k_s\left(t,\rho\left(\hat{y}_g(t)\right)\right),t\in\bar{J}_g\right\},\ \mathrm{I}_{k_s'}=\left\{k_s\left(t,\rho\left(\hat{y}_g(t)\right)\right),t\in\bar{J}_g\right\}.$$

Determine mapping $\mathcal{S}_{k_{s,\rho}}\subset\mathrm{I}_{k_s}\times\mathrm{I}_{k_s'}$ on I_{k_s}, $\mathrm{I}_{k_s'}$. Consider on the set $\mathrm{I}_{k_s'}$ the function

$$\Delta k_s'(t)=k_s\left(t,\rho\left(\hat{y}_g(t+\tau)\right)\right)-k_s\left(t,\rho\left(\hat{y}_g(t)\right)\right) \qquad \text{-------------(16)}$$

describing the change of the first difference $k_s(t,\rho(\dot{\hat{y}}_g(t))$, where $\tau>0$.

Form the set $\mathrm{I}_{\Delta k_s'}=\left\{\Delta k_s\left(t,\rho\left(\dot{\hat{y}}_g(t)\right)\right),t\in\bar{J}_g\right\}$ and consider the framework $\mathcal{SK}_{\Delta k_{s,\rho}'}$ defined on $\mathrm{I}_{k_{s,\rho}}\times\mathrm{I}_{\Delta k_{s,\rho}'}$.

Consider the mapping

$$\mathcal{LSK}_{\Delta k_{s,\rho}'}\to\mathrm{I}_{k_{s,\rho}}\times B\left(\mathrm{I}_{\Delta k_{s,\rho}'}\right), \qquad \text{-------------(17)}$$

where $B\left(\mathrm{I}_{\Delta k_{s,\rho}'}\right)\subset\{-1;1\}$. Define by elements of the binary set $B\left(\mathrm{I}_{\Delta k_{s,\rho}'}\right)$ as

$$b(t)=\begin{cases}1\ if\ \ \Delta k_s'(t)\ge 0,\\ -1\ if\ \ \Delta k_s'(t)<0,\end{cases}\quad t\in\bar{J}_g.$$

Remark 1. The border chosen for limit superior to (9) is based on the change $\mathcal{S}_{k_{s,\rho}}$.

Remark 2. Values range choice of the function $b(t)$ is defined by the convenience of its graphic analysis. $b(t)$ is possible to define on the binary set {0; 1}.

The stated approach is proposed for stationary systems. Some modification has required this approach for $\alpha\pi$ -almost periodic systems. Some modification has required this approach for $\alpha\pi$ -almost periodic systems. In particular, the framework $\mathcal{LSK}_{\Delta k'_{s,\rho}}$ effectively works in the analysis of stationary systems. $\mathcal{LSK}_{\Delta k'_{s,\rho}}$ is inefficient for periodic systems as the function $b(t)$ reflects all changes in the framework $\mathcal{SK}_{\Delta k'_{s,\rho}}$. $\mathcal{LSK}_{\Delta k'_{s,\rho}}$ is inefficient for periodic systems as the function $b(t)$ reflects all changes in the framework $\mathcal{SK}_{\Delta k'_{s,\rho}}$.

System order estimation

All results presented further belong to the system (1) with Frobenius matrix A, $y \in R$, $W_U = 0$, $u \in R$. We consider that the matrix A satisfies A1-A3 conditions. We consider that the matrix A satisfies A1-A3 conditions.

Consider the framework defined on $\mathrm{I}_{k_{s,\rho}} \times \mathrm{I}_{\Delta k'_{s,\rho}}$ and described by function $f_{sk}(t): k_s \to \Delta k_s$. The function $f_{sk}(t)$ is $\mathcal{BF}_{\alpha\pi}$-function. $f_{sk}(t)$ contains areas $\mathcal{D}_{sk}$ which have sharply changing amplitude.

Definition 5 [22]

The area $\mathcal{D}_{sk}$ of the function f_{sk} is $\alpha\pi$ -area on the interval $J_{sk} = [t, t+T]$ if it corresponds to the change $\mathcal{BF}_{\alpha\pi}$-function $k_s(t)$ on this interval.

Theorem 1

Let the system (1) satisfy conditions A1-A3. Then the system (1) have an order m, if the function $f_{sk}(t)$ contains not fewer m areas $\mathcal{D}_{sk}$ on the interval $[t_0, t^*] \subset \bar{J}_g$ $(t^* \le \bar{t})$.

Remark 3. As matrix A eigenvalues $\lambda_i(t)$ are periodic functions of time, then a lineals $\mathcal{L}^i(t)$ and $\mathcal{L}^{i+1}(t)$ can be intersected. This case can lead to an infinite LE spectrum. This feature is noted in [19].

Remark 4. If frequency spectra of lineal $\mathcal{L}^i(t)$ and $\mathcal{L}^{i+1}(t)$ are intersected, then, following remark 3, we have a pyramid with almost smooth faces. Such representation influences on LE spectrum. The pyramid considered in [19] corresponds to a stepped set of lineals

$$0 \equiv \mathcal{L}^{l_0}(t) \subset \mathcal{L}^{l_1}(t) \subset \ldots \subset \mathcal{L}^{l_n}(t) \equiv \mathcal{L}^m(t).$$

Structural approach to LE estimation

We introduce the framework $\mathcal{SK}_{k^i_{s,\rho}}$ which is defined on the set $\mathrm{I}_{k_s} \times \mathrm{I}_{k^i_s}$ and $\mathcal{SK}_{\Delta k'_{s,\rho}}$ where i designates i -th derivative $\hat{y}_g(t)$,

$$\mathrm{I}_{k^i_s} = \left\{ k_s\left(t, \rho\left(\hat{y}^{(i)}_g(t)\right)\right), t \in \bar{J}_g \right\} \quad \text{--------------(18)}$$

The framework $\mathcal{SK}_{k^i_{s,\rho}}$ [20] reflects LE change. Indicators $\chi_i[\hat{y}_g]$ correspond to local minima $\mathcal{SK}_{k^i_{s,\rho}}$. χ_m correspond to a global minimum, and χ_1 correspond to a maximum of the function describing the change $\mathcal{SK}_{k^i_{s,\rho}}$.

Theorem 2 [22]

If the system (1) is stable and contains simple eigenvalues, then frameworks $\mathcal{SK}_{k^i_{s,\rho}}$, $i = \overline{1,m}$ reflect the information on Lyapunov exponents.

The local minima location on $\mathcal{SK}_{k^i_{s,\rho}}$ coincides with areas $\mathcal{D}^i_{sk}$ of the framework $\mathcal{SK}$. The analysis $\mathcal{D}^i_{sk}$ gives the set $\mathcal{M}_{LE}$ containing LE estimations of the system (1). The cardinal number $\mathcal{M}_{LE}$ cannot coincide with LE number of the system. $\mathcal{M}_{LE}$ characterizes an available set of system (1) lineales.

Perform the choice of time $\bar{t}$ in (10) based on framework $\mathcal{SK}_{\Delta k'_{s,\rho}}$ analysis. The described approach generalization is given in [22], where the procedure is described for the LE spectrum estimation.

On LE structural identifiability

The system structural identifiability (SI) estimation guarantees the computability of LE at a given input. The SI estimation method is presented in [23]. It is based on the analysis of properties of a specific class of geometric frameworks $\mathcal{S}_{ey}$. Input requirements guaranteeing SI and obtaining of LE are presented in [23]. They endow $\mathcal{S}_{ey}$ with the required properties and the application of the above approach.

Simulation results

Consider the system (1) which phase portrait is showed in Figure 1. The information set (4) is known. Input is $u(t)=5+2\sin(0.2\pi t)$. Figure 1 shows that the system has oscillations. The input has only one frequency. Therefore, existence in the framework of oscillations with other frequencies indicates what the system is possible belongs to the non-stationary system class.

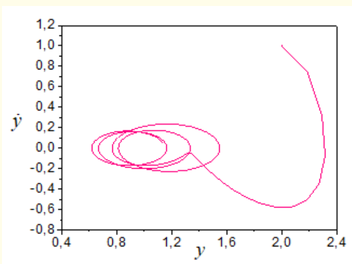

Figure 1: Phase portrait of system.

Find the model (7) for obtaining of the general solution for $y, \dot{y}$. Apply operation of numerical differentiation to calculation $\dot{y}$. Models (7) have the form

$$\hat{y}_q(t)=[0.75;0.07;-0.22][1\,u(t)\,\dot{u}(t)]^T,$$

$$\hat{\dot{y}}_q(t)=[-0.394;-0.059;0,078][1\,u(t)\,\dot{u}(t)]^T.$$

The determination coefficient for these models is 0.99. Next, find estimations for system free motion.

Consider frameworks $\mathcal{SK}_{\Delta k'_{s,\rho}}$, $\mathcal{S}_{k^1_{s,\rho}}$ presented in Figure 2, and apply them to order estimation of the system.

The $\mathcal{SK}_{\Delta k^1_{s,\rho}}$ analysis will show that the system has a third order. We obtain Lyapunov exponent set

$$\mathcal{M}_{LE}=\{-2.04;-1.842;-1.77;-1.167;-0.878\}$$

based on the analysis $\mathcal{SK}_{\Delta k^1_{s,\rho}}$ and $\mathcal{SK}_{\Delta k^2_{s,\rho}}$.

The upper bound for the smallest LE is $\kappa_m=-2.04$. The mobility admissible limit of the higher indicator χ_1 is –0.8. χ -adequacy

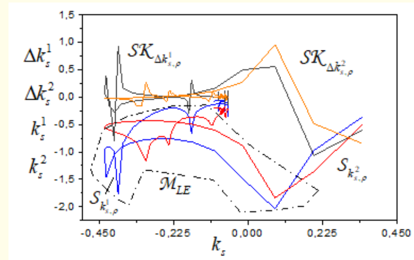

Figure 2: LE set.

[22] check results of Lyapunov exponents are presented in figure 3. We see that estimations are χ -adequate. Figure 4 represents Lyapunov exponent distribution.

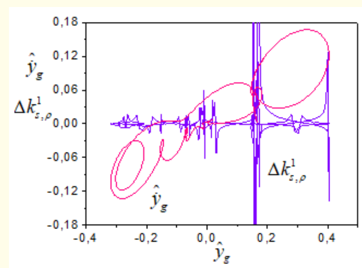

Figure 3: χ - adequacy estimation results in spaces $\mathcal{R}_y=(\hat{y}_g,\hat{\dot{y}}_g)$ and $\mathcal{R}_y=(\Delta k^1_{s,\rho},\hat{\dot{y}}_g)$

The initial system has following eigenvalues of the state matrix $\lambda_1(t)=-1+0.2\sin(0.02\pi t)$, $\lambda_2(t)=-2+0.3\sin(0.04\pi t)$, $\lambda_3(t)=-3+0.2\sin(0.06\pi t)$.

So, modeling results show that the proposed approach allows obtaining LE estimations. It gives a complex assessment of the Lyapunov exponent spectrum. This is the main advantage of this method. The basis of this method is frameworks describing the LE dynamics change. Proposed frameworks are to criteria for LE set selection.

Conclusion

We propose frameworks reflecting dynamics of Lyapunov exponent change. We show that the dynamic system can have a Lyapunov exponent set. The LE structural identifiability problem is considered. The estimation procedure of system order is presented. Modeling results show that the proposed approach allows obtaining LE estimation.

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