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Review Article

Shrinkage Estimation of Strength Reliability for Geometric Distribution Using Record Values

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Abstract

In this paper we obtain the shrinkage estimate of R= P(X≤Y) when X the stress and Y the strength are independent geometric variable and the sample on Y the strength is upper records.

Keywords: Shrinkage; Geometric Distribution; Strength

Introduction

The geometric distribution, the discrete counter part of the exponential distribution is widely used to model discrete reliability system owing to its lack of memory property. The estimation of stress strength reliability is an important problem of interest in the recent times. When X, the stress (or demand) and Y, the strength (or supply) has geometric distribution, then R = P(X≤Y) the system reliability (or economic equilibrium). Matti [15] obtained the UMVUE, MLE and Bayes estimate of R based on independently and identically distributed observation on X and Y under geometric setup. Jeevanand [12] obtain the Bayes estimate of R when outliers are present in the sample. Mattachan and Jeevanand [18] discuss the estimation of R by the method of least squares. Mohamed [24] obtained the bootstrap and Bayes confidence Interval for R. Matti and Murmu [16] and Matti, Murmu and Chattppadhaya [17] extended the result to two parameter geometric distribution.

Record values and associated statistics are of interest and importance in several branches of studies such as psychology, medicine, pharmaceutical science, engineering and pedagogy. All of us constantly hear of new records being created in events such as stock market prices, rainfall, temperature, flood level, sales of goods, in sport items etc. In any field, whenever a new high or a new low value is observed, in connection with the phenomena under study, it becomes a part of history and will be called as a record.

In his pioneering paper, Chandler [3] defined records and laid the groundwork for a mathematical study of records. Record values, record value times and inter record times were first discussed in this article. Glick [7], Nevzorov [27] Nagaraja [26], Galambose [6], Ahsanullah [1], Arnold., *et al*. [2] and Gulati and Padgett [10] have highlighted many of the advances in the theory of records and have mentioned several of its applications. The estimation of the geometric parameters based on record was discussed by Mahdi and Ahmadi [14], Mathachan Pathiyil, Johny Scaria and E.S.Jeevanand [20], Mathachan and E.S.Jeevanand [19], Gouet, Raúl., *et al*. [9], Wenbo and Jinyun [30]. The estimation of stress-strength reliability R of geometric distribution using lower record was discussed in Mohamed [25] and the upper records in Glifin and Anjana [8].

Preliminaries

Let X and Y have probability mass functions

$$f(x:\theta) = \theta^{x}(1-\theta), \qquad x = 0,1,2,\dots, 0<\theta<1 \quad \text{-------------} (2.1)$$

And

$$f(x:\mu) = \mu^y(1-\mu), \quad y = 0,1,2,\dots, 0 < \mu < 1 \text{ ------------(2.2)}$$

Then

$$R = P(X \le Y) = \frac{1-\theta}{1-\theta\mu}, \quad 0 < R \le 1 \text{ ---------------- (2.3)}$$

In the present paper we obtain the Shrinkage estimate of R when the data on the strength (Y) are records and a normal sample for the stress (X). We present the finding in the following manner. After a formal derivation of the stress strength reliability in section 2, we obtain the shrinkage estimate of R in section 3. Finally the last section is devoted to the simulation study.

Estimates of R

Let be a random sample of n observation from (2.1), then its likelihood function is giv by

$$L\,(\underline{x}|\theta) = \theta^{n\bar{x}}(1-\theta)^n \text{ -------------- (3.1)}$$

Now let $\{Y_i\}$ i = 1, 2, ...N be a sequence of observations from (2.2). Let be the first (n + 1) upper record obtained from the above sequence. Then the likelihood function of is given by

$$L\,(r_0, r_1, \dots, r_n|\mu) = \mu^{r_n}(1-\mu)^{n+1} \text{ ---------- (3.2)}$$

Using (3.1) and (3.2) the joint likelihood function can be written as

$$L\,(\underline{x}, \underline{r}|\theta,\mu) = \theta^{n\bar{x}}(1-\theta)^n\mu^{r_n}(1-\mu)^{n+1} \text{ -------------- (3.3)}$$

Taking Logarithm on both side of (3.3) and differentiating partially with respect to and equating to zero we get the MLE of as

$$\hat{\theta}_{mle} = \frac{\bar{x}}{\bar{x}+1} \text{ ------------- (3.4)}$$

And

$$\hat{\mu}_{mle} = \frac{r_n}{r_n+n+1} \text{ -------------- (3.5)}$$

Substituting (3.3) and (3.5) in (2.3) we obtained the MLE of R as

$$\hat{R}_{mle} = \frac{1-\hat{\theta}}{1-\hat{\theta}\hat{\mu}} = \frac{r_n+n+1}{r_n+(n+1)(\bar{x}+1)} \text{ ------ (3.6)}$$

From the above expression, it is very difficult to find the exact variance and distribution of . So we use the multivariate delta method (See Wasserman [29], Soliman., *et al*. [28], Dhanya, M. and Jeevavand, E. S. [4], Khan, M.J.S. and Khatoon, B. [13]) to find the approximate estimate of the asymptotic variance of which is given as follows.

Let the Fisher Information matrix

$$\emptyset\,(\theta,\mu) = \begin{bmatrix} E\left(\frac{-\partial^2 \ln L}{\partial\theta^2}\right) & E\left(\frac{-\partial^2 \ln L}{\partial\theta\,\partial\mu}\right) \\ E\left(\frac{-\partial^2 \ln L}{\partial\theta\,\partial\mu}\right) & E\left(\frac{-\partial^2 \ln L}{\partial\mu^2}\right) \end{bmatrix} = \begin{bmatrix} \frac{n}{\theta(1-\theta)^2} & 0 \\ 0 & \frac{n+1}{\mu(1-\mu)^2} \end{bmatrix} \text{ ------- (3.7)}$$

And

$$B' = \begin{bmatrix} \frac{\partial R}{\partial\theta} & \frac{\partial R}{\partial\mu} \end{bmatrix} = \begin{bmatrix} \frac{\mu-1}{(1-\theta\mu)^2} & \frac{\theta(1-\theta)}{(1-\theta\mu)^2} \end{bmatrix} \text{ --- (3.8)}$$

Then

$$\sigma_R^2 = V(R) = B'\emptyset^{-1}B = \frac{\theta(n\mu\theta+n+1)(1-\theta)^2(1-\mu)^2}{(n^2+n)(1-\mu\theta)^4} \text{ .--------------------- (3.9)}$$

Substituting the estimated value , in (3.9) we get the estimate of V(R) as

$$\hat{\sigma}_R^2 = V(\hat{R}_{mle}) = \frac{\hat{\theta}_{mle}(n\hat{\mu}_{mle}\hat{\theta}_{mle}+n+1)(1-\hat{\theta}_{mle})^2(1-\hat{\mu}_{mle})^2}{(n^2+n)(1-\hat{\mu}_{mle}\hat{\theta}_{mle})^4} \text{ .---------- (3.10)}$$

Shrinkage estimation with constant shrinkage factor

In this case we use the Thompson shrinkage estimator of say as

$$\hat{\beta}_{sh} = \psi(\hat{\beta})\hat{\beta}_{ub} + (1-\psi(\hat{\beta}))\hat{\beta}_0 \text{ ----------- (3.11)}$$

With $\psi(\hat{\beta}) = 0.01$ the constant shrinkage weight factor suggested by Hameed., *et al*. (2020) this leads the Shrinkage estimates of and as

$$\hat{\theta}_{sh} = 0.01\hat{\theta}_{ub} + 0.99\hat{\theta}_0 \text{ -------- (3.12)}$$

And

$$\hat{\mu}_{sh} = 0.01\hat{\mu}_{mle} + 0.99\hat{\mu}_0 \text{ -------------- (3.13)}$$

Where $\hat{\theta}_0$ and $\hat{\mu}_0$ is taken as the boot strap estimate of and .

This leads to the constant shrinkage estimate of R as

$$\hat{R}_{sh} = \frac{1-\hat{\theta}_{sh}}{1-\hat{\theta}_{sh}\hat{\mu}_{sh}} \text{ --------------- (3.14)}$$

To obtain the boot strap estimate we use the following boot strap algorithm (Efron [5]).

Step-1

Generate the sample of size n using Monte Carlo method for geometric distribution with parameter . To carry out the simulation study using record values from geometric distribution, first we simulate samples of size 1,000 each from the distribution with specified parameter values. Then, from these, samples having 6 or more record values (ie. having at least are sorted out. Using these short listed sets of records, thus obtained, the estimators of the respective entropy measures are computed.

Compute the MLE of θ and μ say $\hat{\theta}_{mle}, \hat{\mu}_{mle}$, given (3.4) and (3.5).

Step-2

Generate an independent parametric bootstrap sample using say , instead of and .

Step-3

Calculate the maximum likelihood estimate of , using the sample obtained in step 2.

Step-4

Repeat the step-2 and step-3 N times to obtained the parametric bootstrap estimates, , and .

The modified Thompson type shrinkage estimator

Here we use two type of shrinkage estimate first one the modified Thompson type shrinkage weight factor suggested by Hameed., *et al*. [11] and Shrinkage weight factor suggested by Mehta and Srinivasan [23]) to find out the shrinkage estimator.

Weight factor suggested by Hameed., *et al*. here we take the weight factor as

$$\phi(\hat{R}) = \frac{\hat{R}_{mle}-\hat{R}_0}{(\hat{R}_{mle}-\hat{R}_0)^2+var(\hat{R}_{mle})}(0.001) \text{ ----------- } (3.15)$$

Where $var(\hat{R}_{mle})$ is as defined in (3.10). So the modified Thomason type shrinkage estimator will be

$$\hat{R}_{Th} = \phi(\hat{R})\hat{R}_{mle} + (1-\phi(\hat{R}))\hat{R}_0 \text{ -------------- } (3.16)$$

Shrinkage weight factor suggested by Mehta and Srinivasan (1971) here we take the weight factor as

$$\varphi(\hat{R}) = a \exp\left\{-\frac{b(\hat{R}_{mle}-\hat{R}_0)^2}{var(\hat{R}_{mle})}\right\} \text{ ------------------- } (3.17)$$

Where 0<a<1and b>0 So the modified Thomason type shrinkage estimator will be

$$\hat{R}_{MS} = \varphi(\hat{R})\hat{R}_{mle} + (1-\varphi(\hat{R}))\hat{R}_0, \text{---------------} (3.18)$$

Empirical Study

Here, we first obtain a bootstrap estimate for R by using a parametric percentile bootstrap method (Efron [5]) given in section 3. Then we calculate the Shrinkage estimated suggested above. The result are exhibited in table 1.

No of records			sh		
r1	2,3	Bias	0.3449	0.269	0.1156
		MSE	0.1741	0.1047	0.0789
	2,5	Bias	0.0818	0.1155	0.0545
		MSE	0.0172	0.0294	0.0068
	5,10	Bias	0.1410	0.1046	0.0848
		MSE	0.0308	0.0719	0.0467
r3	2,3	Bias	0.1290	0.1277	0.0498
		MSE	0.0584	0.0434	0.0090
	2,5	Bias	0.1174	0.095	0.0746
		MSE	0.0533	0.0485	0.0266
	5,10	Bias	0.1665	0.0354	0.0009
		MSE	0.0519	0.0024	0.0003
r5	2,3	Bias	0.1355	0.1013	0.0791
		MSE	0.0119	0.0235	0.0059
	2,5	Bias	0.0301	0.0773	0.0495
		MSE	0.0119	0.0235	0.0207
	5,10	Bias	0.2124	0.0453	0.0016
		MSE	0.0671	0.0025	0.0001

Table 1: Estimators of the reliability measures.

Conclusion

The conclusions from the empirical study are

- For all values of and , the estimators of R are close to their respective actual values and have low mean square errors.
- The biases and mean square errors reduce considerably when the value of and increase or when higher records are used in the estimation procedure.
- Among the three estimates $_{sh}$, and , the estimator has lesser bias and have low mean square errors then the other two.
- The estimate performs better than $_{sh}$, in all cases.

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