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Review Article

Model of Transforming Individual Preferences into Collective Preference Using the Quadratic Mean

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Abstract

An election is to designate one candidate among many others: it is a social choice. The object of this social choice is the selection of options by an individual or a group of individuals. It is not without cost. Indeed, the organization of elections in developing countries is extremely expensive. Consequently, in the event of a tie between the candidates to decide between them, the decision makers are obliged to take over the elections for a second time. This, generates additional efforts to mobilize resources. To put an end to such a concern, the choice of an appropriate and adequate procedure would be suitable. this study of the se- lection procedure can be done in a relatively practical way (electoral procedures). This approach draws its historical source from the theory of social choice, including the analysis of voting procedures by Borda and Condorcet. The major problem of social choice is that of the passage from data of individual preference to that of social or collective preference.

Therefore we develop a social aggregation function based on the quadratic means. We do a digital application and get some interesting results.

Keywords: Quadratic Mean; Individual Preference; Social Preference; Aggregation Function

Introduction

Quantitative and qualitative mathematical models of group decision support are often used by decision makers for a better group choice decision at the economic, social and political level etc. In this work we propose to build a model of social choice making it possible to choose a candidate for an election, which amounts to making a selection among many others. In literary reviews with the theory of social choice, there is little of choice model of this type. uppercase to this end we are going to focus more on the issue of elections. There are one-round elections (English system), two-round elections (French system) and ranked list elections (Australian system). The need to make an election credible and competitive with a favorable outcome in the first round will spare us conflicts and multiple challenges during the electoral process. According to [1], the organization of elections is often complex due to the need to take into account the points of view of all the actors involved in the electoral decisionmaking process, who sometimes have conflicting opinions. The question of aggregating individual preferences in a coherent fashion into a social or collective preference can serve as a basis for drawing up a recommendation taking into account the choice of voters. According to [7], it has long been discussed within the framework of election theory. According to [2,3,6] in the literature relating to social choice theory, many methods exist such as the Borda method and others. It consists in seeking a mechanism allowing to aggregate in a reasonable way the opinions expressed during an election by several voters concerning various candidates so as to determine a winner (the elected candidate) or to rank in order of preference the various candidates. According to [8], although this problem has a very ancient origin, it is customary to trace its modern analysis back to the work of Borda and Condorcet. In this work precisely we will propose a model of transformation of individual preferences into a collective preference using the root mean square to solve social choice problems. We will do a digital application, and do some comparison.

Review of the literature on the aggregation function in social choice theory

Borda method

This part comes from [4].

A ‘candidate a’ is preferred to a ‘candidate b ’if the sum of the ranks of a in the lists of voters is strictly less than that of b (it is assumed here that the lists are complete orders, without a link, and rank 1 is assigned to the first of the list, 2 to the second and so on; the method can be generalized without difficulty to deal with cases of link).

Denote the following sets:

- Let $C = \{c_1,..c_j,...c_m\}$ the set of candidates for an election;
- Let P the set of profiles of a candidate;
- Let n_j be the number of voters for candidate j with > 1;
- Let n be the total number of voters in the election with ;
- Let k_j the rank occupied by candidate j in the ranking of voters’ preferences; $\mathbf{j}$ ranging from 1: 2;...., m and k_j ranging from 1^{st} up to n^{th} rank under the order of preference of voters .

Let be the function of social aggregation assigning the score to the candidate

$$S_j = \sum_{j=1}^{m} n_j \times k_j \text{ ,avec } \mathbf{j} = 1,2,...m \text{ ------------(1)}$$

Presentation of the method

The quadratic mean

Definition

The quadratic mean of n values is obtained as follows

$$q = \sqrt{\frac{\sum x_i}{n}} \text{ --------(2)}$$

Presentation of the quadratic mean applied to the theory of social choice

- Let $C = \{C_1,...C_j..., C_m\}$ the set of candidates for an election;
- Let P the set of profiles of a candidate;
- Let n_j be the number of voters for candidate j with ;
- Let n be the total number of voters in the election with ;
- Let k_j the rank occupied by candidate j in the ranking of voters’ preferences; $\mathbf{j}$ ranging from 1, 2,...., m and ranging from 1^{st} up to n^{th} rank under the order of preference of voters .
- Let the social aggregation function assigning the score to the candidate j

Properties

This subsection is inspired by [8]. The new aggregation function has the following properties.

Unanimity

If all decision-makers prefer candidate a over candidate b, then social preference shows that candidate a is preferred over candidate b. The result of the method should not contradict a unanimous opinion of decision-makers.

Monotony

If a decision-maker changes his judgment in favor of the candidate a, then a cannot be ranked lower than before.

Neutrality

The relative preferences between candidate a and b do not depend on the other candidates present in the corpus of candidates.

Transitiviy

The method must provide a classification in the form of a full preorder.

Universality

Every list configuration is admissible.

Demonstration

Unanimity

Indeed, if all decision-makers prefer the candidate a over the candidate b, then the social preference shows that candidate a is preferred to candidate b because $S_a > S_b$.

Monotony

If a decision-maker changes his judgment in favor of the candidate a, then 'a' cannot be ranked lower than before. Indeed, If at the beginning $S_a > S_b$ and after a decision-maker who votes in favor of a, then $S_a > S_b$.

Neutrality

The relative preferences between candidate a and b do not depend on the other candidates present in the corpus of candidates. Indeed, it is just enough to consider only S_a and S_b.

Transitiviy

The method must provide a classification in the form of a full pre-order, i.e., if $S_a > S_b$ and $S_b > S_c$, then $S_a > S_c$.

Universality

Each of the list configurations is eligible. Indeed, whatever the list considered, we can always classify the candidates.

Digital experiences

Note that these experiences will allow us to make comparisons.

Example 1

This example comes to [4]. It is used to make an illustration and a comparison of the Borda and Condorcet methods. The results is satisfactory, hence the need to apply it with the new method is advisable.

The subject

Let {a, b, c, d} the set candidate for an election. Consider three voters whose preferences are as follows

- 2 voters have preferences bP aP c
- 1 voter has a preference aP cP dP b

Resolution by Borda method

Using Borda method,

Let's calculate the social preference of each candidate.

Let S_j be the function of social aggregation assigning the score to the candidate j

$S_j = \sum_{j=1}^{m} n_j \times k_j$ avec $j = 1,2, \dots .m.$

Digital application

$$S_a = \sum n_a \times k_a = 2 \times 2 + 1 \times 1 = 5$$
$$S_b = \sum n_b \times k_b = 2 \times 1 + 1 \times 4 = 6$$
$$S_c = \sum n_c \times k_c = 2 \times 3 + 1 \times 2 = 8$$
$$S_d = \sum n_d \times k_d = 2 \times 4 + 1 \times 3 = 11$$

Candidate	Social score
a	5
b	6
c	8
d	11

Table 1: Social score by candidate.

The 'candidate a 'wins the election with a social score of 5.

Resolution by the new method (that we proposed)

Using the new method

$$S_j = \sqrt{\frac{\sum_{j=1}^{m}(n_j k_j)^2}{n}}$$

Digital application

$$S_a = \sqrt{\frac{\sum(n_a k_a)^2}{n}} = \sqrt{\frac{4^2+1^2}{3}} = \sqrt{\frac{17}{3}} = 2.38$$

$$S_b = \sqrt{\frac{\sum(n_b k_b)^2}{n}} = \sqrt{\frac{2^2+4^2}{3}} = \sqrt{\frac{20}{3}} = 2.58$$

$$S_c = \sqrt{\frac{\sum(n_c k_c)^2}{n}} = \sqrt{\frac{6^2+2^2}{3}} = \sqrt{\frac{40}{3}} = 3.65$$

$$S_d = \sqrt{\frac{\sum(n_d k_d)^2}{n}} = \sqrt{\frac{8^2+3^2}{3}} = \sqrt{\frac{73}{3}} = 4.93$$

Candidate	Social score
a	2.38
b	2.58
c	3.65
d	4.93

Table 2: Social score by candidate.

The ‘candidate a ’wins the election with a social score of 2.38.

Example 2 [5]

This example comes from the famous work of Jean-Charles, Chevalier de Borda (1733-1799) Marie Jean Antoine Nicolas de Caritat, Marquis de Condorcet (1743-1793), Kenneth Arrow (1921-). It was applied to three voting systems for the election of deputies by the French system (two-round majority); by the English system (one-round majority); and by the Australian system (ranked list voting). The results obtained were satisfactory. This example motivated us to apply it to the new method that we proposed which subsequently, we will compare the different results.

The subject

Number of voters	Preferences
1	pineapple > banana > cherry
7	pineapple > banana > cherry
7	pineapple > banana > cherry
6	pineapple > banana > cherry

Table 3: Which fruit wins?

Resolution by Borda method

Using the method

Digital application

$$S_{pineapple} = \sum n_{pineapple} \times k_{pineapple} = 1 \times 1 + 7 \times 1 + 7 \times 3 + 6 \times 3 = 1 + 7 + 21 + 18 = 47$$

$$S_{banana} = \sum n_{banana} \times k_{banana} = 1 \times 2 + 7 \times 3 + 7 \times 1 + 6 \times 2 = 2 + 21 + 7 + 12 = 42$$

$$S_{cherry} = \sum n_{cherry} \times k_{cherry} = 1 \times 3 + 7 \times 2 + 7 \times 2 + 6 \times 1 = 3 + 14 + 14 + 6 = 37$$

Type of fruit	Social score
Pineapple	47
Banana	42
Cherry	37

Table 4: Overall Score.

C herry is the fruit to choose with an overall score of 37.

Resolution by the new method (that we proposed)

Using the news method

Digital application

$$S_{pineapple} = \sqrt{\frac{\Sigma(n_{pineapple} k_{pineapple})^2}{n}} = \sqrt{\frac{1^2+7^2+21^2+18^2}{21}} = \sqrt{\frac{815}{21}} = 6.22$$

$$S_{banana} = \sqrt{\frac{\Sigma(n_{banana} k_{banana})^2}{n}} = \sqrt{\frac{2^2+21^2+7^2+12^2}{21}} = \sqrt{\frac{638}{21}} = 5.51$$

$$S_{cherry} = \sqrt{\frac{\Sigma(n_{cherry} k_{cherry})^2}{n}} = \sqrt{\frac{3^2+14^2+14^2+6^2}{21}} = \sqrt{\frac{437}{21}} = 4.56$$

Type of fruit	Collective score
Pineapple	6,22
Banana	5,51
Cherry	4,56

Table 5: Overall Score.

C herry is the fruit to choose with an overall score of 4,56.

Example 3

This example is an initiative to illustrate cases of situation where there is difficulty in deciding among the candidates on the list with the Borda method.

The subject

Let {a, b, c, d} the set of candidates in an election for which there are 10 voters whose preferences are:

- 3 bP cP dP a
- 2 dP bP aP c
- 3 cP dP aP b

Resolution by Borda method

Using the method

Digital application

$$S_a = \sum n_a \times k_a = 3 \times 4 + 2 \times 3 + 3 \times 3 = 12 + 6 + 9 = 27$$

$$S_c = \sum n_c \times k_c = 3 \times 2 + 2 \times 4 + 3 \times 1 = 6 + 8 + 3 = 17$$

$$S_d = \sum n_d \times k_d = 3 \times 3 + 2 \times 1 + 3 \times 2 = 9 + 2 + 6 = 17 \; S_b = \sum n_b \times k_b = 3 \times 1 + 2 \times 2 + 3 \times 4 = 3 + 6 + 12 = 19$$

Candidate	Collective score
a	27
c	17
d	17
b	19

Table 6: Collective Score.

Candidates c and d are tied in score (17 each).

Resolution by the new method (that we proposed)

Using the news method

Digital application

$$S_a = \sqrt{\frac{\Sigma(n_a k_a)^2}{n}} = \sqrt{\frac{12^2+6^2+9^2}{8}} = \sqrt{\frac{144+36+81}{8}} = \sqrt{\frac{261}{8}} = 5.71$$

$$S_b = \sqrt{\frac{\Sigma(n_b k_b)^2}{n}} = \sqrt{\frac{3^2+4^2+12^2}{8}} = \sqrt{\frac{9+16+144}{8}} = \sqrt{\frac{169}{8}} = 4.59$$

$$S_c = \sqrt{\frac{\Sigma(n_c k_c)^2}{n}} = \sqrt{\frac{6^2+8^2+3^2}{8}} = \sqrt{\frac{36+64+9}{8}} = \sqrt{\frac{109}{8}} = 3.69$$

$$S_d = \sqrt{\frac{\Sigma(n_d k_d)^2}{n}} = \sqrt{\frac{9^2+2^2+6^2}{8}} = \sqrt{\frac{81+4+36}{8}} = \sqrt{\frac{121}{8}} = 3.88$$

Candidate	Collective score
a	5.71
c	3.69
d	3.88
b	4.59

Table 7: Social score.

The candidate c is elected at the election with a collective score of 3.69.

Comparison

In the first example, Borda method and the new method give the same best candidate : candidate a is elected. In addition, candidates are ranked in the same order according to each of these methods. The same results are obtained with Borda's method in the famous work by which the example is drawn. As for the second example, we notice that the new method and the Borda method give the best fruit: the cherry fruit is chosen. The same result is obtained with the method of Borda in the illustration and comparison of the methods of Borda and Condorcet to the election of the deputies. Also the fruits are arranged in the same order by each of these methods. In the third example, the candidates c and d are linked in the election by Borda method: a case of indifference which does not make it possible to decide between the candidates on the list. Yet a winner is needed. The new method makes it possible to decide among the candidates c and d who are in ballot in the election by the method of Borda.

Conclusion

Borda method does not allow us to elect a candidate. However, the objective of democratic elections is to be able to choose among the candidates. In a word, the method based on the quadratic means makes it possible to elect a candidate thanks to his or her discriminating power.

We also note that there are more calculations with the new method compared to the weighted average (the Borda method). Descriptively, the Borda's method and the quadratic mean method are also easy to use. The question we can ask ourselves is the following: if the values are very disturbed, can we expect the method to still retain its discriminating power?

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