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Research Article

Bifuzzy Commutative Ideals in BCK-algebras

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Abstract

The tricky point when studying bifuzzy mathematics lies in how to carry out the ordinary concept to bifuzzy case. In other words, how to pick out the ordered pair of the rational generalization from the large ordered pairs available in [0,1]x[0,1]. The elements of the bifuzzy sets are featured by an additional degree which is called the degree of uncertainty. This paper aims to introduce a new notion of bifuzzy commutative idealsin BCK-algebras, and various properties of these notions in the context of bifuzzy sets are established.

Introduction

The notion of BCK/BCI-algebras was introduced by Isèki [2,3]. Since then, a great deal of literature has been produced on the theory of BCK/BCI-algebras. The notion of fuzzy sets was introduced by Zadeh [11]. In 1991, Xi [10] applied the concept to BCK-algebras. In 2020Muhiuddin and Jun [8] give further Results of Neutrosophic subalgebras in BCK/BCI -algebras based on Neutrosophic points.

In 1983, Atanassov introduced the notion of intuitionistic fuzzy sets as a generalization of fuzzy sets. In 1995, Gerstenkorn and Mańko [4] re-named the intuitionistic fuzzy sets as bifuzzy sets. The elements of the bifuzzy sets are featured by an additional degree which is called the degree of uncertainty. Bifuzzy sets have also been defined by Takeuti and Titans in [9]. Takeuti and Titanti considered bifuzzy logic in the narrow sense and derived a set theory from logic which they called bifuzzy set theory. The bifuzzy sets have drawn the attention of many researchers in the last decades. This is mainly because bifuzzy sets are consistent with human behavior, by reflecting and modeling the hesitancy present in real-life situations. In 2019Muhiuddin, Kim and Jun [7] discussed Implicative N – ideals of BCK – algebras based on Neutrosophic N – structures.Bifuzzy sets give kind of fuzzy set extensions in the fuzzy set theory whose membership degree range in thus the elements of bifuzzy sets are featured by an additional degree which is called the degree of uncertainty which is an essential tool for giving applications in mathematics and computer science. Bifuzzy sets takes the advantage of fuzzy sets to handle information with various facts of uncertainty such as fuzziness and randomness the bifuzzy set has become a formal and useful tool for computer science to deal with bifuzzy information and uncertain information. In this paper, we introduce the notions of bifuzzy commutative ideal of BCK-algebras and investigate their properties.

Preliminaries

An algebra (X;*,0) of type (2, 0) is called a BCI-algebra if it satisfies the following axioms:

(BCI -1), $((x*y)*(x*z))*(z*y)=0$

(BCI -2) , $(x*(x*y))*y=0$

(BCI -3) , $x*x=0$

(BCI -4) $x*y=0$ and $y*x=0$ impl $x=y$ y for all $x,y,z \in X$.

A BCI-algebras is said to be a BCK-algebra if it satisfies:

BCK-5. $0*x=0$

In a BCI-algebra*X*, a partially ordered relation ≤ can be defined by $x \le y$ if and only if $x*y=0$,then $(X,\le)$ is a partially ordered setwith least element 0.

In any BCI-algebra*X*, the following hold:

$(x*y)*z=(x*z)*y$,

$x*0=x$,

$0*(x*y)=(0*x)*(0*y)$,

$0*(0*(x*y))=0*(x*y)$,

$(x*z)*(y*z) \le x*y$,

$x*y=0$ implies $x*z \le y*z z*y \le z*x$ and

For all x, y, $z \in X$.

Definition 2.1 [5]

A nonempty subsetI of a BCK-algebraX is called an ideal of X if it

$0 \in I$,

$x*y \in I$ and $y \in I$ imply $x \in I$.

Definition 2.2 [2]

A nonempty subset *I* of a BCK-algebra X, is called a commutative ideal X if it satisfies $0 \in I$;

$\forall x,y,z \in X,(x*y)*z \in I$ and $z \in I$ imply $x*(y*(y*x)) \in I$

Definition 2.3 ([5])

A partially ordered set $\langle X,\le\rangle$is said to be a lower semi lattice if every pair of elements X has a greatest lower bound (meet (∧)); it is called to be an upper semi lattice if every pair of elements in X has the least upper bound (Join (∨))$\langle X,\le\rangle$. It is both an upper and a lower semi lattice, then it is called a lattice.

Bifuzzy commutative ideals of BCK-algebras

Definition 3.1 [1]

A mapping $A=(\mu_A,\lambda_A):X\to[0,1]\times[0,1]$ is called bifuzzy set in X if $\mu_A(x)+\lambda_A (x)\le 1$ for all $x \in X$, where the mappings $\lambda_A:X\to[0,1]$ and denote the degree of membership (namely μ_A (x) and the degree of non-membership (namely λ_A (x) of each element $x \in X$ to A respectively.

Definition 3.2

A bifuzzy set $A=(\mu_A,\lambda_A):X\to[0,1]\times[0,1]$ is called a bifuzzy ideal of BCK - algebra X if $\forall x,y,z \in X$ the following conditions hold:

$\mu_A (0)\ge\mu_A (x),\lambda_A (0)\le\lambda_A (x)$,

$\mu_A (x)\ge\min\{ \mu_A (x*y),\mu_A (y)\}$,

$\lambda_A (x)\le\max\{ \lambda_A (x*y),\lambda_A (y)\}$.

Example 3.3

Let X={0,a,b,c} in which * is defined by

*	0	a	b	c
0	0	0	0	0
a	a	0	a	c
b	b	b	0	a
c	c	c	b	0

Figure a

Then (X;*,0) is BCK-algebra.

We define a bifuzzy set $\mu_A (0)=0.7$by, $\mu_A (x)=0.01$ and $\lambda_A (0)=0.02$, $\lambda_A (x)=0.6$ for all $x\ne 0X$ in. By routine computations, we can easily verify that the bifuzzy set A is a bifuzzy ideal of a BCK-algebra X.

Definition 3.4

A bifuzzy set $A=(\mu_A,\lambda_A):X\to[0,1]\times[0,1]$ is called a bifuzzy commutative ideal of BCK - algebra X if $\forall x,y,z\in X$ the following conditions hold:

$\mu_A(0)\ge\mu_A(x),\lambda_A(0)\le\lambda_A(x),\mu_A(x*(y*(y*x)))\ge\min\{\mu_A((x*y)*z),\mu_A(z)\}$,

$\lambda_A(x*(y*(y*x)))\le\max\{\lambda_A((x*y)*z),\lambda_A(z)\}$.

If we put y=0, then follows that $A=(\mu_A,\lambda_A):X\to[0,1]\times[0,1]$ is abifuzzy ideal of. X Thus every bifuzzy commutative ideal of BCK – algebra X is a bifuzzy ideal X.

Example 3.5

Let X={0,1,2,3,4} be a BCK-algebra with Cayley table as follows:

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	0
3	3	3	3	0	0
4	4	4	4	3	0

Figure b

We define a bifuzzy set $\mu_A(0)=0.67$ by, $\mu_A(x)=0.03$ and $\lambda_A(0)=0.06$, $\lambda_A(x)=0.73$ for all $x\neq 0$Xin. By routine computations, we can easily verify that the bifuzzy set Ais a bifuzzy commutative ideal of a BCK-algebra X.

Definition 3.6 [1]

Let $A=(\mu_A,\lambda_A)$ be a bifuzzy set on Xand let$(\alpha,\beta)\in[0,1]\times[0,1]$ with$\alpha+\beta\le1$. Then the set

$X_{(\alpha,\beta)A}=\{x\in X/\alpha\le\mu_A(x),\lambda_A(x)\le\beta\}$ is called an (α,β) -level subset A. The set of all $(\alpha,\beta)\in\mathrm{Im}(\mu_A)\times\mathrm{Im}(\lambda_A)$ such that $\alpha+\beta\le1$is called the image of$A=(\mu_A,\lambda_A)$.

We notice that, $X_{(\alpha,\beta)A}=\{x\in X/\mu_A(x)\ge\alpha,\lambda_A(x)\le\beta\}$

$$=\{x\in X/\mu_A(x)\ge\alpha\}\cap\{x\in X/\lambda_A(x)\le\beta\}$$

$$=\mu_\alpha A\cap\lambda_{\beta A}.$$

Where $\mu_\alpha A=\{x\in X/\mu_A(x)\ge\alpha\}and\lambda_{\alpha A}=\{x\in X/\lambda_A(x)\le\alpha\}$are level subsets of μ_A,λ_Arespectively.

Theorem 3.7

Let $A=(\mu_A,\lambda_A)$ be a bifuzy commutative ideal of BCK – algebraX, then the following are satisfies:

$\mu_A(x*(y*(y*x)))\ge\mu_A(x*y)$ and $\lambda_A(x*(y*(y*x)))\le\lambda_A(x*y)$.

(ii)If $\mu_{\alpha1},\mu_{\alpha2}$ two-level commutative ideals (with $\alpha_1<\alpha_2$)of Aare equal if and only if there is no $x\in X$ such that$\alpha_1<\mu_A(x)<\alpha_2$. And if $\lambda_{\alpha1}$, $\lambda_{\alpha2}$ two-level commutative ideals (with $\beta_1<\beta_2$) of Aare equal if and only if there is no $x\in X$such that $\beta_1<\lambda_A(x)<\beta_2$.

Proof.(i)That$A=(\mu_A,\lambda_A)$ is a bifuzzy commutative ideal of BCK – algebraX, then $=\mu_A(x*(y*(y*x)))\ge\min\{\mu_A(x*y),\mu_A(0)\}=\mu_A(x*y)$, thus$\mu_A(x*(y*(y*x)))\ge\mu_A(x*y)$.

$\lambda_A(x*(y*(y*x)))\le\max\{\lambda_A(x*y),\lambda_A(0)\}=\lambda_A(x*y)$, thus $\lambda_A(x*(y*(y*x)))\le\lambda_A(x*y)$.

(ii)Assume that $\mu_{\alpha1}=\mu_{\alpha2}$ for $\alpha_1<\alpha_2$and that there exists $x\in X$ such that $\alpha_1<\mu_A<\alpha_2$then $\mu_{\alpha1}$ is a proper subset$\mu_{\alpha2}$, which contradicts the hypothesis.

Conversely, suppose that there is no $x\in X$such that$\alpha_1<\mu_A(x)<\alpha_2$. Since $\alpha_1<\alpha_2$we have $\mu_{\alpha1}\subseteq\mu_{\alpha2}$. Let $x\in\mu_{\alpha1}$ then $\mu_A(x)\ge\alpha_1$ and hence$\mu_A(x)\ge\alpha_2$, because $\mu_A(x)$does not lie between α_2. Hence$x\in\mu_\alpha$ which implies $\mu_{\alpha1}\subseteq\mu_{\alpha2}$, this completes the proof in case $\mu\alpha_1,\mu_{\alpha2}$ By the same method we can provein case $\lambda_{\alpha1}$, $\lambda_{\alpha2}$.

Theorem 3.8

A bifuzzy set $A=(\mu_A,\lambda_A)$ of BCK – algebra Xis a bifuzycommutative ideal of Xif, for all $(\alpha,\beta)\in[0,1]\times[0,1]$, $X_{(\alpha,\beta)A}$is either empty or acommutative ideal of X.

Proof. Suppose that $A=(\mu_A,\lambda_A)$ is a bifuzzy commutative ideal of Xand $X\neq\phi$for any $(\alpha,\beta)\in[0,1]\times[0,1]$. it is clear that $0\in\mu_{\alpha A}$ since $\mu_A(0)\ge\alpha$. Let $(x*y)*z\in\mu_{\alpha A}$ and $z\in\mu_{\alpha A}$, then $\mu_A((x*y)*z)\ge\alpha$ and $\mu_A(z)\ge\alpha$, since $A=(\mu_A,\lambda_A)$ is a bifuzzy commutative ideal we have $\mu_A(x*(y*(y*x)))\ge\min\{\mu_A((x*y)*z),\mu_A(z)\}\ge\alpha$

Thus $x*(y*(y*x))\in\mu_{\alpha A}$, this shows that $\mu_{\alpha A}$ is a commutative ideal X.

It is clear that $0\in\lambda_{\alpha A}$since$\lambda_A(0)\le\beta$. Let $(x*y)*z\in\lambda_{\alpha A}andz\in\lambda_{\alpha A}$, then$\lambda_A((x*y)*z)\le\beta$ and $\lambda_A(z)\le\beta$, since $A=(\mu_A,\lambda_A)$ is a bifuzzy is a commutative ideal we have $\lambda_A(x*(y*(y*x)))\le\max\{\lambda_A((x*y)*z),\lambda_A(z)\}\le\beta$, thus $x*(y*(y*x))\in\lambda_{\beta A}$, this shows $\lambda_{\beta A}$is a positive implicative ideal ofX. Since that$X_{(\alpha,\beta)A}=\{x\in X/\mu_A(x)\geq\alpha,\lambda_A(x)\leq\beta\}=\mu_\alpha A\cap\lambda_{\beta A}$, thus $X_{(\alpha,\beta)A}$ is a commutativeideal of X.

Conversely, suppose that for each $(\alpha,\beta)\in[0,1]\times[0,1]$, $X_{(\alpha,\beta)A}$ is either empty or acommutative ideal of. X For any $x\in X$ let $\mu_A(x)=\alpha$, then $x\in\mu_{\alpha A}$. Since $\mu_{\alpha A}(\neq\phi)$is acommutative ideal of X, therefore $0\in\mu_{\alpha A}$ and hence $\mu_A(0)\geq\alpha=\mu_A(x)$, thus $\mu_A(0)\geq\mu_A(x)$ for all $x\in X$. Now we only need to show that $\mu_A(x*(y*(y*x)))\geq\min\{\mu_A((x*y)*z),\mu_A(z)\}$. If not, then there exists $x',y',z'\in X$ such that $\mu_A(x'*(y'*(y'*x')))<\min\{\mu_A((x'*y')*z'),\mu_A(z')\}$

Taking $\alpha_0=1/2\{\mu_A(x'*(y'*(y'*x')))+\{\mu_A((x'*y')*z'),\mu_A(z')\}\}$, then we have. $\mu_A(x'*(y'*(y'*x')))<\alpha_0<\min\{\mu_A((x'*y')*z'),\mu_A(z')\}$ Hence

$x'*(y'*(y'*x'))\notin\mu_{\alpha 0A}$,$(x'*y')*z'\in\mu_{(\alpha_0 A)}$,$z'\in\mu_{\alpha 0A}$ i.e. $\mu_{\alpha 0A}$is not a commutative ideal of X, which is a contradiction therefore

$\mu_A(x*(y*(y*x)))\geq\min\{\mu_A((x*y)*z),\mu_A(z)\}$.

For$(\alpha,\beta)\in[0,1]\times[0,1]$. For any $x\in X$ let $\lambda_A(x)=\beta$, then $x\in\lambda_{\beta A}$. Since $\lambda_{\beta A}(\neq\phi)$ is a positive implicative ideal ofX, therefore $0\in\lambda_{\beta A}$and hence $\lambda_A(0)\leq\beta=\lambda_A(x)$, thus $\lambda_A(0)\leq\lambda_A(x)$for all $x\in X$. Now we also only need to show that $\lambda_A(x*z)\leq\max\{\lambda_A((x*y)*z),\lambda_A(y*z)\}$. If not, then there exists $x',y',z'\in X$such that $\lambda_A(x'*(y'*(y'*x')))>\min\{\lambda_A((x'*y')*z'),\lambda_A(z')\}$.

Taking$\beta_0=1/2\{\lambda_A(x'*(y'*(y'*x')))+\max\{\lambda_A((x'*y')*z'),\lambda_A(z')\}\}$, then we have$\lambda_A(x'*(y'*(y'*x')))>\beta_0>\max\{\lambda_A((x'*y')*z'),\lambda_A(z')\}$. Hence $x'*(y'*(y'*x'))\notin\lambda\beta_{0A}$,$((x'*y')*z')\in\lambda\lambda_{0A}$,$z'\in\lambda_{\beta 0A}$) i.e.$\lambda_{\beta 0A}$ is not a The commutative ideal of X, which is a contradiction therefore $\lambda_A(x*(y*(y*x)))\leq\max\{\lambda_A((x*y)*z),\lambda_A(z)\}$, thus $A=(\mu_A,\lambda_A)$ a bifuzzy commutative ideal.

Theorem 3.9

Let $\{A_i/i\in I\}$be a family of bifuzzy commutative ideals of BCK-algebra Xthen $\{A_i,\subset\}$ is ordering set under the ordering of bifuzzy set inclusion$\subset$.

Proof. Let$\{A_i/i\in I\}$ is a family of bifuzzy commutative idealsX. Since [0,1] is a completely distributive lattice for the usual ordering[0,1], it is sufficient to show that $\cap A_i=(\wedge\mu_{Ai},\vee\lambda_{Ai})$ is a bifuzzy commutative idealX.

For any $x\in X$, where $\vee$and $\wedge$denoted the operation join and meet respectively.

$$(\bigwedge_{i\in I}\mu_{A_i})(0)=\inf_{i\in I}\mu_{A_i}(0)\geq\inf_{i\in I}\mu_{A_i}(x)=(\bigwedge_{i\in I}\mu_{A_i})(x)$$

and

$$(\bigvee_{i\in I}\lambda_{A_i})(0)=\sup_{i\in I}\lambda_{A_i}(0)\leq\sup_{i\in I}\lambda_{A_i}(x)=(\bigvee_{i\in I}\lambda_{A_i})(x)$$

Let $x,y,z\in X$ then

$$(\wedge\mu_{A_i})(x*(y*(y*x)))==\inf_{i\in I}\mu_{A_i}(x*(y*(y*x)))=inf_{i\in I}\{\mu_{A_i}(x*(y*(y*x)))/i\in I\}$$

$$\geq inf_{i\in I}\{min\{\mu_{A_i}((x*y)*z),\mu_{A_i}(z))/i\in I\}\}$$

$=min\{inf_{i\in I}\{\mu_{A_i}((x*y)*z),\mu_{A_i}(z))/i\in I\}\}$,

$=min\{inf_{i\in I}\mu_{A_i}((x*y)*z),inf_{i\in I}\mu_{A_i}(z))/i\in I\},=min\{\wedge_{i\in I}\mu_{A_i}((x*y)*z),\wedge_{i\in I}\mu_{A_i}(z))/i\in I\}$

$$(\vee\lambda_{A_i})(x*(y*(y\times x)))=\sup_{i\in I}\lambda_{A_i}(x*(y*(y*x)))=sup_{i\in I}\{\lambda_{A_i}(x*(y*(y*x)))/i\in I\}$$

$$\geq sup_{i\in I}\{max\{\lambda_{A_i}((x\times y)*z),\lambda_{A_i}(z))/i\in I\}\}$$

$=max\{sup_{i\in I}\{\lambda_{A_i}((x*y)*z),\lambda_{A_i}(z))/i\in I\}\}$,

$=max\{sup_{i\in I}\lambda_{A_i}((x*y)*z),sup_{i\in I}\lambda_{A_i}(z))/i\in I\},=max\{\vee_{i\in I}\lambda_{A_i}((x*y)*z),\vee_{i\in I}\lambda_{A_i}(z))/i\in I\}$

Hence $\cap A_i=(\vee\mu_{A_i},\vee\lambda_{A_i})$ is a bifuzzy commutative ideal of X Υ

Theorem 3.10

Let Ibe a commutative ideal of a BCK-algebraX. Then there exists a bifuzzy commutative ideal$A=(\mu_A,\lambda_A)$ of Xsuch that $X_{\alpha,\beta A}=I(\alpha,\beta)\in[0,1]\times[0,1]$ was with$\alpha+\beta\leq1$.

Proof. Define $\mu_A:X\to[0,1]$ by

$$\mu_A(x)=\begin{cases}\alpha & ; \quad x\in I\\ 0 & ; \quad x\notin I\end{cases}$$

Whereα is a fixed number (0,1] and define

$\lambda_A:X\to[0,1]$ by

$$\lambda_A(x)=\begin{cases}0 & ; \quad x\in I\\ \beta & ; \quad x\notin I,\end{cases}$$

Where β is a fixed number(0,1]. Since $0\in I$, $\mu_A(0)=\alpha\geq\mu_A(x)$ for all $x\in X$, also since $0\in I$, $\lambda_A(0)=0\leq\lambda_A(x)$ for all $x\in X$. Forμ_A. SinceI is a commutative ideal of X, we have $(x*y)*z\in I$ and $z\in I$ then $x*(y*(y*x))\in I$. Hence$\mu_A((x*y)*z)=\mu_A(z)=\mu_A(x*(y*(y*x)))=\alpha$, thus

$\mu_A(x*(y*(y*x)))=\min\{\mu_A((x*y)*z),\mu_A(z)\}=\alpha$.

If at least one $(x*y)*z$ and z is not in I, then at least one $\mu_A((x*y)*z)$and $\mu_A(z)$is 0.

Therefore

$\mu_A(x*(y*(y*x)))\geq\min\{\mu_A((x*y)*z),\mu_A(z)\}$.

For λ_A. Since I is commutative of X, if $(x*y)*z\in I$ and $z\in I$ then $x*(y*(y*x))\in I$.

Hence $\lambda_A(((x*y)*z))=\lambda_A(z)=\lambda_A(x*(y*(y*x)))=\beta$, thus

$\lambda_A(x*(y*(y*x)))=\max\{\lambda_A((x*y)*z),\lambda_A(z)\}=\beta$.

If at least one $(x*y)*z$ and z is not in I, then at least one $\lambda_A((x*y)*z)$ and $\lambda_A(z)$ is β.

Therefore

$\lambda_A(x*(y*(y*x)))\leq\max\{\lambda_A((x*y)*z),\lambda_A(z)\}$.

Thus $A=(\mu_A,\lambda_A)$ is a commutative ideal of X. It is clear that

$I=\mu_\alpha A\cap\lambda_{\beta A}=X_{(\alpha,\beta)A}$ and so the result follows □

Definition 3.11 [1]

Let X be a nonempty set. Then we call mapping

$A=(\mu_A,\lambda_A):X\times X\to[0,1]\times[0,1]$

a bifuzzy relation on X if $\mu_A(x,y)+\lambda_A(x,y)\leq 1\ \forall(x,y)\in X\times X$.

Definition 3.12 [1]

Let $A=(\mu_A,\lambda_A)$ and $B=(\mu_B,\lambda_B)$ be a bifuzzy sets on a set X, then $A=(\mu_A,\lambda_A)$ is a bifuzzy relation on $B=(\mu_B,\lambda_B)$ if

$\mu_A(x,y)\leq\min\{\mu_B(x),\mu_B(y)\}$ and

$\lambda_A(x,y)\geq\max\{\lambda_B(x),\lambda_B(y)\}$.

Definition 3.13 [1]

Let $A=(\mu_A,\lambda_A)B=(\mu_B,\lambda_B)$ and be two bifuzzy sets on a set X. Then the Cartesian product A×B is defined as follows

$A\times B=(\mu_A,\lambda_A)\times(\mu_B,\lambda_B)=(\mu_A\times\mu_B,\lambda_A\times\lambda_B)$, where

$\lambda_A\times\lambda_B(x,y)=\max\{\lambda_A(x),\lambda_B(y)\}$

We note that the Cartesian product A×B is always a bifuzzy set in X×X, if

$0\leq\min\{\mu_A(x),\mu_B(y)\}+\max\{\lambda_A(x),\lambda_B(y)\}\leq 1$.

Definition 3.14 [1]

Let $A=(\mu_A,\lambda_A)B=(\mu_B,\lambda_B)$ and be a bifuzzy set on X. then the strongest bifuzzy relation on $B=(\mu_B,\lambda_B)$ is A_B, defined by $A_B=(\mu_{A\mu B},\lambda_{A\lambda B})$ where $\lambda_{A\mu B}(x,y)=\min\{\lambda_B(x),\lambda_B(y)\}$ and.

Theorem 3.15

If $A=(\mu_A,\lambda_A)$ and $B=(\mu_B,\lambda_B)$ are two bifuzzy commutative ideal of BCK – algebra X, Then A×B is a bifuzzy commutative idea of X×X.

Proof. Let

$0=(0,0),x=(x_1,x_2),(x*y)*z=((x_1*y_1)*z_1,(x_2*y_2)*z_2),z=(z_1,z_2)$
$x*(y*(y*x))=(x_1*(y_1*(y_1*x_1)),x_2*(y_2*(y_2*x_2)))$ and, any elements of.

X×X Then we have $(\mu_A\times\mu_B)(0)=(\mu_A\times\mu_B)((0,0))$

$=\min\{\mu_A(0),\mu_B(0)\}\geq\min\{\mu_A(x_1),\mu_B(x_2)\}$

$=(\mu_A\times\mu_B)((x_1,x_2))=(\mu_A\times\mu_B)(x)$.

$(\lambda_A\times\lambda_B)(0)=(\lambda_A\times\lambda_B)((0,0))$

$=\max\{\lambda_A(0),\lambda_B(0)\}\leq\max\{\lambda_A(x_1),\lambda_B(x_2)\}$

$=(\lambda_A\times\lambda_B)((x_1,x_2))=(\lambda_A\times\lambda_B)(x)$.

$(\mu_A\times\mu_B)(x*(y*(y*x)))=(\mu_A\times\mu_B)((x_1*(y_1*(y_1*x_1)),x_2*(y_2*(y_2*x_2))))=\min\{\mu_A(x_1*(y_1*(y_1*x_1))),\mu_B(x_2*(y_2*(y_2*x_2)))\}$

$\geq\min\{\min\{\mu_A((x_1*y_1)*z_1),\mu_A(z_1)\},\min\{\mu_B((x_2*y_2)*z_2),\mu_B(z_2)\}\}=\min\{\min\{\mu_A((x_1*y_1)*z_1),\mu_B((x_2*y_2)*z_2)\},\min\{\mu_A(z_1),\mu_B(z_2)\}\}$

$=\min\{(\mu_A\times\mu_B)(((x_1*y_1)*z_1,((x_2*y_2))*z_2)),(\mu_A\times\mu_B)((z_1,z_2))\}$

$=\min\{(\mu_A\times\mu_B)((x*y)*z),(\mu_A\times\mu_B)(z)\}$.

$(\lambda_A\times\lambda_B)(x*(y*(y*x)))=(\lambda_A\times\lambda_B)((x_1*(y_1*(y_1*x_1)),x_2*(y_2*(y_2*x_2))))$

$=\max\{\lambda_A(x_1*(y_1*(y_1*x_1))),\lambda_B(x_2*(y_2*(y_2*x_2)))\}$

$\leq\max\{\max\{\lambda_A((x_1*y_1)*z_1),\lambda_A(z_1)\},\max\{\lambda_B((x_2*y_2)*z_2),\lambda_B(z_2)\}\}=\max\{\max\{\lambda_A((x_1*y_1)*z_1),\lambda_B((x_2*y_2)*z_2)\},\max\{\lambda_A(z_1),\lambda_B(z_2)\}\}$

$=\max\{(\lambda_A\times\lambda_B)(((x_1*y_1)*z_1,((x_2*y_2))*z_2)),(\lambda_A\times\lambda_B)((z_1,z_2))\}$

$=\max\{(\lambda_A\times\lambda_B)((x*y)*z),(\lambda_A\times\lambda_B)(z)\}$.

Hence A×B is a bifuzzy commutative ideal of X×X □

Theorem 3.16

Let $A=(\mu_A,\lambda_A)$ and $B=(\mu_B,\lambda_B)$ be a bifuzzy sets in BCK – algebra X and A_B the strongest bifuzzy relation on X. Then $B=(\mu_B,\lambda_B)$ is a bifuzzy commutative ideal of X if and only if A_B is a bifuzzy commutative ideal of X×X.

Proof. Let

$0=(0,0),x=(x_1,x_2),z=(z_1,z_2),(x*y)*z=((x_1*y_1)*z_1,(x_2*y_2)*z_2)$ and

$x*(y*(y*x))=(x_1*(y_1*(y_1*x_1)),x_2*(y_2*(y_2*x_2)))$

, any elements of X×Xand let$B=(\mu_B,\lambda_B)$ be a bifuzzy positive implicative ideal ofX.

$\mu_{A\mu B}(0)=\mu_{A\mu B}((0,0))$

$=\min\{\mu_B(0),\mu_B(0)\}$

$\geq\min\{\mu_B(x),\mu_B(x)\}$

$=\mu_{A\mu B}((x_1,x_2))$

$=\mu_{A\mu B}(x)$.

$\lambda_{A\mu B}(0)=\lambda_{A\mu B}((0,0))$

$=\max\{\lambda_B(0),\lambda_B(0)\}$

$\leq\max\{\lambda_B(x),\lambda_B(x)\}$

$=\lambda_{A\mu B}((x_1,x_2))$

$=\lambda_{A\mu B}(x)$.

$\mu_{A\mu B}(x*(y*(y*x)))=\mu_{A\mu B}((x_1*(y_1*(y_1*x_1)),x_2*(y_2*(y_2*x_2))))$

$=\min\{\mu_B(x_1*(y_1*(y_1*x_1))),\mu_B(x_2*(y_2*(y_2*x_2)))\}$

$\geq\min\{\min\{\mu_B((x_1*y_1)*z_1),\mu_B(z_1)\},\min\{\mu_B((x_2*(y_2*x_2))*z_2),\mu_B(z_2)\}\}=\min\{\min\{\mu_A((x_1*y_1)*z_1),\mu_B((x_2*y_2)*z_2)\},\min\{\mu_A(z_1),\mu_B(z_2)\}\}$

$=\min\{\mu_{A\mu B}(((x_1*y_1)*z_1,(x_2*y_2)*z_2)),\mu_{A\mu B}((z_1,z_2))\}$

$=\min\{\mu_{A\mu B}((x*y)*z),\mu_{A\mu B}(z)\}$.

$\lambda_{A\mu B}(x*(y*(y*x)))=$λ_(A_μB) $((x_1*(y_1*(y_1*x_1)),x_2*(y_2*(y_2*x_2))))$

$=\max\{\lambda_B(x_1*(y_1*(y_1*x_1))),\lambda_B(x_2*(y_2*(y_2*x_2)))\}$

$\leq\max\{\max\{\lambda_B((x_1*y_1)*z_1),\lambda_B(z_1)\},\max\{\lambda_B((x_2*(y_2*x_2))*z_2),\lambda_B(z_2)\}\}=\max\{\max\{\lambda_A((x_1*y_1)*z_1),\lambda_B((x_2*y_2)*z_2)\},\max\{\lambda_A(z_1),\lambda_B(z_2)\}\}$

=max {λ_(A_μB) $(((x_1*y_1)*z_1,(x_2*y_2)*z_2))$,λ_(A_μA) $((z_1,z_2))\}$

$=\max\{\lambda_{A\mu B}((x*y)*z),\lambda_{A\mu B}(z)\}$

This shows that $A_B$is a bifuzzy commutative ideal X×X.

Conversely, suppose that$A_B=(\mu_{A\mu B},\lambda_{A\mu B})$ is a bifuzzy commutative ideal X×X. Then $\min\{\mu_B(0),\mu_b(0)\}=\mu_{A\mu B}((0,0))\geq\mu_{A\mu B}((x,y))=\min\{\mu_B(x),\mu_B(y)\}$

$\max\{\lambda_B(0),\lambda_B(0)\}=\mu_{A\mu B}((0,0))\leq\lambda_{A\mu B}((x,y))=\max\{\mu_B(x),\mu_B(y)\}$.

For all, it follows that$\mu_B(x)\leq\mu_B(0)$ and$\lambda_B(x)\geq\lambda_B(0)$.

For any $x*(y*(y*x))=(x_1*(y_1*(y_1*x_1)), x_2*(y_2*(y_2*x_2))),\in$ X×X and $(x*y)*z=((x_1*y_1)*z_1,(x_2*y_2)*z_2)$, $z=(z_1,z_2)\in X\times X$

$\min\{\mu_B(x_1*(y_1*(y_1*x_1))),\mu_B(x_2*(y_2*(y_2*x_2)))\}=\mu_{A\mu B}((x_1*(y_1*(y_1*x_1)), x_2*(y_2*(y_2*x_2))))=\mu_{A\mu B}(x*(y*(y*x)))=\geq\min\{\mu_{A\mu B}((x*y)*z),\mu_{A\mu B}(z)\}=$

$=\min\{\mu_{A\mu B}(((x_1*y_1)*z_1,(x_2*y_2)*z_2)),\mu_{A\mu B}((z_1,z_2))\}=\min\{\min\{\mu_B((x_1*y_1)*z_1),\mu_B((x_2*y_2)*z_2)\},\min\{\mu_B(z_1),\mu_B(z_2)\}\}$ Putting $x_2=z_2=0$ gives

$\mu_B(x_1*(y_1*(y_1*x_1)))\geq\min\{\mu_B((x_1*y_1)*z_1),\mu_B(z_1)\}$.

$\max\{\lambda_B(x_1*(y_1*(y_1*x_1))),\lambda_B(x_2*(y_2*(y_2*x_2)))\}=\lambda_{A\mu B}((x_1*(y_1*(y_1*x_1)),x_2*(y_2*(y_2*x_2))))=\lambda_{A\mu B}(x*(y*(y*x)))=\leq\max\{\lambda_{A\mu B}((x*y)*z),\lambda_{A\mu B}(z)\}=$

$=\max\{\lambda_{A\mu B}(((x_1*y_1)*z_1,(x_2*y_2)*z_2)),\lambda_{A\mu B}((z_1,z_2))\}=\max\{\max\{\lambda_B((x_1*y_1)*z_1),\lambda_B((x_2*y_2)*z_2)\},\max\{\lambda_B(z_1),\lambda_B(z_2)\}\}$Putting $x_2=z_2=0$ gives

$\lambda_B(x_1*(y_1*(y_1*x_1)))\geq\max\{\lambda_B((x_1*y_1)*z_1),\lambda_B(z_1)\}$.

Hence $B=(\mu_B,\lambda_B)$ is a bifuzzy commutative ideal of X □

Definition 3.17 [1]

Let a mapping f: $X_1\to X_2$ from BCK - algebra X_1 into BCK - algebra $X_2$and let $A=(\mu_A,\lambda_A)$ be a bifuzzy set X_2. The map$A=(\mu_A,\lambda_A)$ is called the pre-image of $A=(\mu_A,\lambda_A)$ underf:$X_1\to X_2$, if

$\mu_A^{f(x)}=\mu_A(f(x))$ and $\lambda_A^{f(x)}=\lambda_A(f(x))$ $\forall x\in X_1$.

Theorem 3.18

Let a mapping f:$X_1\to X_2$be a homomorphism from BCK - algebra X_1 into BCK - algebra X_2 such that f(0)=0, then$A=(\mu_A,\lambda_A)$ is a bifuzzy commutative ideal X_2 X_1

Proof. Let

$A=(\mu_A,\lambda_A)$ is a bifuzzy commutative ideal X_2. Then for any $x\in X_1$, we have $\mu_A^f(0)=\mu_A(f(0))=\mu_A(0)\geq\mu_A(f(x))=\mu_A^f(x)$and

$\lambda_A^f(0)=\lambda_A(f(0))=\lambda_A(0)\geq\lambda_A(f(x))=\lambda_A^f(x)$.

For any$x,y,z\in X_1$, since $A=(\mu_A,\lambda_A)$ is a bifuzzy commutative ideal of $X_2\mu_A^f(x*(y*(y*x)))=\mu_A(f(x*(y*(y*x))))\geq\min\{\mu((f(x)*f(y))*f(z))*\mu(f(z))\}$

$=\min\{\mu_A((f(x*y)*z)),\mu_A(f(z))\}$

$=\min\{\mu_A^f((x*y)*z),\mu_A^f(z)\}$.

$\lambda_A^f(x*(y*(y*x)))=\lambda_A(f(x*(y*(y*x))))\leq\max\{\lambda((f(x)*f(y))*f(z))*\lambda(f(z))\}$

$=\max\{\lambda_A((f(x*y)*z)),\lambda_A(f(z))\}$

$=\max\{\lambda_A^f((x*y)*z),\lambda_A^f(z)\}$

Proving that $A^f=(\mu^f{}_A,\lambda^f{}_A)$ is a bifuzzy commutative ideal X_1.

Conversely Let $A^f=(\mu^f{}_A,\lambda^f{}_A)$ is a bifuzzy commutative idealX_1 $x,y,z\in X_2$. For any, there exists $a,b,c\in X_1$ such that $x=f(a)$, $y=f(b)$, $z=f(c)$ follow that

$\mu_A(x)=\mu_A(f(a))=\mu_A^f(a)\leq\mu_A^f(0)=\mu_A(f(0))=\mu_A(0)$

$\lambda_A(x)=\lambda_A(f(a))=\lambda_A^f(a)\geq\lambda_A^f(0)=\lambda_A(f(0))=\lambda_A(0)$

$\mu_A(x*(y*(y*x)))=\mu_A(f(a*(b*(b*a))))=\mu_A^f(a*(b*(b*a)))$

$\geq\min\{\mu^f{}_A((a*b)*c),\mu_A^f(c)\}$

$=\min\{\mu_A(((f(a)*f(b))*f(c)))\ \mu_A(f(c))\}=\min\{\mu_A((x*y)*z),\mu_A(z)\}$.

$\lambda_A(x*(y*(y*x)))=\lambda_A(f(a*(b*(b*a))))=\lambda_A^f(a*(b*(b*a)))\leq\max\{\lambda^f{}_A((a*b)*c),\lambda_A^f(c)\}$

$=\max\{\lambda_A(((f(a)*f(b))*f(c)))\ \lambda_A(f(c))\}=\max\{\lambda_A((x*y)*z),\lambda_A(z)\}$.

Thus Proving that $A=(\mu_A,\lambda_A)$ is a bifuzzy commutative ideal of X_2

Algorithms for fuzzy and bifuzzy subset of BCK - algebras

Here we give some algorithms for studding the structure of the fuzzy and bifuzzy BCK -algebras.

Algorithm for BCK-algebras

Input (X: set, *: binary operation)

Output ("X is a BCK- algebra or not");

Begin

IfX=ϕ then

go to (1.);

EndIf

If0∉X then

go to (1.);

EndIf

Stop:=false;

i:=1;

While i≤|X| and not (Stop) do

If $x_i*x_i\neq o$ then

Stop:=true;

EndIf

If $0*x_i\neq 0$ then

Stop:=true;

EndIf

j:=1;

While j≤|X| and not (Stop) do

k:=1;

While k≤|X| and not (Stop) do

If $(x_i*(x_i*y_j))*y_j\neq 0$ then

Stop:=true;

EndIf

```
If (x_i*y_j=0) and (y_j*x_i=0) then
If x_i≠y_j then
Stop:=true;
EndIf
EndIf
k:=1;
 While k≤|X| and not (Stop) do
 If ((x_i*y_j )*(x_i*z_k ))*(z_k*y_j )≠0then
Stop:=true;
EndIf
Endwhile
Endwhile
Endwhile
 If Stop then
 (1.) Output is a ("Xis not BCK – algebra " )
 Else
 Output ("X is BCK – algebra " )
EndIf
End
```

Algorithm for subalgebras of BCK - Algebras

```
Input(X:BCK - algebra, *: binary operation,I: subset ofX);
Output("I is subalgebra ofXor not");
Begin
 IfI=ϕ then
go to (1.);
EndIf
 Stop:=false;
i:=1;
 While i≤|X| and not (Stop) do
j:=1;
 While j≤|X| and not (Stop) do
 If x_i∈Iand y_j∈I then
 If x_i*y_j∉I
 Stop:=true;
EndIf
EndIf
Endwhile
Endwhile
 If Stop then
 Output ("Iis asubalgebraof X")
 Else
 (1.) Output ("Iis not a subalgebraof X")
EndIf
 End
```

Algorithm for ideals of BCK - Algebras

```
Input(X:BCK - algebra, *: binary operation,I: subset ofX);
Output("I is an ideal ofXor not");
Begin
 IfI=ϕ then
go to (1.);
EndIf
```

```
If0∉I then
go to (1.);
EndIf
 Stop:=false
i:=1;
 While i≤|X| and not (Stop) do
j:=1;
 While j≤|X| and not (Stop) do
 If x_i*y_j∈Iand y_j∈I then
 If x_i∉I
Stop:=true;
EndIf
EndIf
Endwhile
Endwhile
 If Stop then
 Output ("Iis an ideal of X")
 Else
 (1.) Output ("Iis not an ideal of X")
EndIf
 End
```

Algorithm for fuzzy set

```
Input(X:BCKI-algebra, A:X→R);
Output("A is a fuzzy set in Xor not");
Begin
 Stop:=false;
i:=1;
 While i≤|X| and not (Stop) do
 If (A(x_i)<0) or (A(x_i)>1) then
 Stop:=true;
EndIf
Endwhile
If Stop then
 Output ("Ais not a fuzzy set in X")
 Else
 Output ("Ais a fuzzy set in X")
EndIf
End
```

Algorithm for bifuzzy set

```
Input(X:BCKI-algebra, A=(μ_A,λ_A ):X→[0,1]×[0,1]);
Output ("A is a bifuzzy set in Xor not");
Begin
 Stop:=false;
i:=1;
 While i≤|X| and not (Stop) do
 If μ_A (x_i )+λ_A (x_i)>1 then
 Stop:=true;
EndIf
Endwhile
If Stop then
 Output ("Ais not a bifuzzy set of X")
```

Else

Output (“Ais a bifuzzy set of X”)

EndIf

End

Algorithm for fuzzy ideals of BCK-Algebra

Input(X: BCK-algebra,*: binary operation, A: fuzzy set inX);

Output (“A is a fuzzy ideal ofXor not”);

Begin

Stop:=false;

i:=1;

While i≤|X| and not (Stop) do

If $A(0)<A(x_i)$ then

Stop:=true;

EndIf

j:=1;

While j≤|X| and not (Stop) do

If,$\mu(x_i)<\min\{\mu(x_i*y_j),\mu(y_j)\}<$

Stop:=true;

EndIf

EndIf

Endwhile

Endwhile

If Stop then

Output (“Ais not a fuzzy ideal of X”)

Else

Output (“Ais a fuzzy ideal of X”)

EndIf

End

Algorithm for bifuzzy ideals of BCK-Algebra

Input(X: BCK-algebra,*: binary operation, A: bifuzzy set inX);

Output (“A is a bifuzzy ideal ofXor not”);

Begin

Stop:=false;

i:=1;

While i≤|X| and not (Stop) do

If $(\mu_A(0)<\mu_A(x_i))$or $(\lambda_A(0)>\lambda(x_i))$, then

Stop:=true;

EndIf

j:=1;

While j≤|X| and not (Stop) do

If,$(\mu_A(x_i)<\min\{\mu_A(x_i*y_j),\mu_A(y_j)\})or(\lambda_A(x_i)>\min\{\lambda_A(x_i*y_j),\lambda_A(y_j)\})$

Stop:=true;

EndIf

EndIf

Endwhile

Endwhile

If Stop then

Output (“Ais not a bifuzzy ideal of X”)

Else

Output (“Ais a bifuzzy ideal of X”)

EndIf

End

Algorithm for fuzzy subalgebras of BCK-Algebra

Input(X: BCK-algebra,*: binary operation, A: fuzzy subset ofX);

Output ("A is a fuzzy subalgebra ofXor not");

Begin

Stop:=false;

i:=1;

While i≤|X| and not (Stop) do

If $A(0)<A(x_i)$ then

Stop:=true;

EndIf

j:=1;

While j≤|X| and not (Stop) do

If,$\mu(x_i*y_j)<\min\{\mu(x_i),\mu(y_j)\}$

Stop:=true;

EndIf

EndIf

Endwhile

Endwhile

If Stop then

Output ("Ais not a fuzzy subalgebraof X")

Else

Output ("Ais a fuzzy subalgebraof X")

EndIf

End

Algorithm for bifuzzy subalgebras of BCK-Algebra

Input(X: BCK-algebra,*: binary operation, A: bifuzzy subset ofX);

Output ("A is a bifuzzysubalgebra ofXor not");

Begin

Stop:=false;

i:=1;

While i≤|X| and not (Stop) do

If $(\mu_A(0)<\mu_A(x_i))$or $(\lambda_A(0)>\lambda_A(x_i))$ then

Stop:=true;

EndIf

j:=1;

While j≤|X| and not (Stop) do

If$(\mu_A(x_i*y_j)<\min\{\mu_A(x_i),\mu(y_j)\})or(\lambda_A(x_i*y_j)>\min\{\lambda_A(x_i),\mu(y_j)\})$

Stop:=true;

EndIf

EndIf

Endwhile

Endwhile

If Stop then

Output ("Ais not a bifuzzysubalgebraof X")

Else

Output ("Ais a bifuzzysubalgebraof X")

EndIf

End

Algorithm for commutative ideals of BCK-Algebra

Input(X: BCK-algebra, *: binary operation,I: subset ofX);

```
Output ("I is a commutative ideal ofXor not");
Begin
 IfI=ϕ then
go to (1.);
EndIf
 If0∉I then
go to (1.);
EndIf
 Stop:=false;
i:=1;
 While i≤|X| and not (Stop) do
j:=1;
 While j≤|X| and not (Stop) do
k:=1;
 While k≤|X| and not (Stop) do
 If (x_i*y_j )*z_k∈I and z_k∈I then
 If x_i*(y_j*(y_j*x_j ))∉I
 Stop:=true;
EndIf
EndIf
Endwhile
Endwhile
Endwhile
 If Stop then
 Output ("Iis a commutative ideal ofX")
 Else
 (1.) Output ("Iis not a commutative ideal ofX")
EndIf
 End
```

Algorithm for fuzzy commutative ideals of BCK-Algebra

```
Input(X: BCK-algebra,*: binary operation, A: fuzzy subset ofX);
Output ("A is a fuzzy commutative ideal ofXor not");
Begin
 Stop:=false;
i:=1;
 While i≤|X| and not (Stop) do
 If A(0)<A(x_i) then
 Stop:=true;
EndIf
j:=1;
 While j≤|X| and not (Stop) do
k:=1;
 While k≤|X| and not (Stop) do
IfA(x_i*(y_j*(y_j*x_i )))<Min(A((x_i*y_j )*z_k ),A(z_k ))
 Stop:=true;
EndIf
Endwhile
Endwhile
Endwhile
 If Stop then
 Output ("Ais not a fuzzy commutative ideal of X")
```

Else

Output ("Ais a fuzzy commutative ideal of X")

EndIf

End

Algorithm for bifuzzy commutative ideals of BCK-Algebra

Input(X:BCK-algebra,*: binary operation, A: bifuzzy subset ofX);

Output("A is a bifuzzy commutative ideal ofXor not");

Begin

Stop:=false;

i:=1;

While i≤|X| and not (Stop) do

If (μ_A (0)<μ_A (x_i))or (λ_A (0)>λ_A (x_i)) then

Stop:=true;

EndIf

j:=1;

While j≤|X| and not (Stop) do

k:=1;

While k≤|X| and not (Stop) do

If(μ_A (x_i*(y_j*(y_j*x_i)))<Min(μ_A ((x_i*y_j)*z_k),μ_A (z_k)))or(λ_A (x_i*(y_j*(y_j*x_i)))>Min(λ_A ((x_i*y_j)*z_k),λ_A (z_k)))

Stop:=true;

EndIf

Endwhile

Endwhile

Endwhile

If Stop then

Output ("Ais not a bifuzzycommutative ideal of X")

Else

Output ("Ais a bifuzzycommutative ideal of X")

EndIf

End.

Conclusion and Future Research

In this paper, we introduce the new notion of bifuzzy commutative ideals of BCK-algebras, and investigate some of their properties. We hope that this work would be the foundation for further study of the theory of bifuzzy ideals in BCK/BCI - algebras.

In our future study of bifuzzy ideals in BCK/BCI algebras, maybe the following topics should be considered:

- Developing the properties of bifuzzy ideals of BCK/BCI algebras.
- Finding useful results on other structures of bifuzzy theory of ideals of BCK/BCI algebras.
- Constructing the related logical properties of such structures.
- May also apply this concept to study some applications in many fields such as artificial intelligence, signal processing, multiagent system, pattern recognition, robotics, computer networks, expert system, decision making, automata theory, and medical diagnosis.

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