

Bifuzzy Commutative Ideals in BCK-algebras

Mahasin A Ahmed^{1*} and Esmat A Amhed²¹College of Science, Department of Mathematics, Sudan University of Science and Technology, Khartoum, Republic of Sudan²College of Electrical Engineering and IT Department of Electrical Engineering, Onaizah Colleges, KSA***Corresponding Author:** Mahasin A Ahmed, College of Science, Department of Mathematics, Sudan University of Science and Technology, Khartoum, Republic of Sudan.**Received:** April 01, 2021**Published:** August 18, 2021© All rights are reserved by **Mahasin A Ahmed and Esmat A Amhed.****Abstract**

The tricky point when studying bifuzzy mathematics lies in how to carry out the ordinary concept to bifuzzy case. In other words, how to pick out the ordered pair of the rational generalization from the large ordered pairs available in $[0,1] \times [0,1]$. The elements of the bifuzzy sets are featured by an additional degree which is called the degree of uncertainty. This paper aims to introduce a new notion of bifuzzy commutative ideals in BCK-algebras, and various properties of these notions in the context of bifuzzy sets are established.

Introduction

The notion of BCK/BCI-algebras was introduced by Isèki [2,3]. Since then, a great deal of literature has been produced on the theory of BCK/BCI-algebras. The notion of fuzzy sets was introduced by Zadeh [11]. In 1991, Xi [10] applied the concept to BCK-algebras. In 2020 Muhiuddin and Jun [8] give further Results of Neutrosophic subalgebras in BCK/BCI -algebras based on Neutrosophic points.

In 1983, Atanassov introduced the notion of intuitionistic fuzzy sets as a generalization of fuzzy sets. In 1995, Gerstenkorn and Mańko [4] re-named the intuitionistic fuzzy sets as bifuzzy sets. The elements of the bifuzzy sets are featured by an additional degree which is called the degree of uncertainty. Bifuzzy sets have also been defined by Takeuti and Titani in [9]. Takeuti and Titanti considered bifuzzy logic in the narrow sense and derived a set theory from logic which they called bifuzzy set theory. The bifuzzy sets have drawn the attention of many researchers in the last de-

ades. This is mainly because bifuzzy sets are consistent with human behavior, by reflecting and modeling the hesitancy present in real-life situations. In 2019 Muhiuddin, Kim and Jun [7] discussed Implicative N – ideals of BCK – algebras based on Neutrosophic N – structures. Bifuzzy sets give kind of fuzzy set extensions in the fuzzy set theory whose membership degree range in thus the elements of bifuzzy sets are featured by an additional degree which is called the degree of uncertainty which is an essential tool for giving applications in mathematics and computer science. Bifuzzy sets takes the advantage of fuzzy sets to handle information with various facts of uncertainty such as fuzziness and randomness the bifuzzy set has become a formal and useful tool for computer science to deal with bifuzzy information and uncertain information. In this paper, we introduce the notions of bifuzzy commutative ideal of BCK-algebras and investigate their properties.

Preliminaries

An algebra $(X; *, 0)$ of type $(2, 0)$ is called a BCI-algebra if it satisfies the following axioms:

(BCI -1), $((x*y)*(x*z))*(z*y)=0$

(BCI -2), $(x*(x*y))*y=0$

(BCI -3), $x*x=0$

(BCI -4) $x*y=0$ and $y*x=0$ impl $x=y$ for all $x,y,z \in X$.

A BCI-algebras is said to be a BCK-algebra if it satisfies:

BCK-5. $0*x=0$

In a BCI-algebra X , a partially ordered relation \leq can be defined by $x \leq y$ if and only if $x*y=0$, then (X, \leq) is a partially ordered set with least element 0.

In any BCI-algebra X , the following hold:

$(x*y)*z=(x*z)*y$,

$x*0=x$,

$0*(x*y)=(0*x)*(0*y)$,

$0*(0*(x*y))=0*(x*y)$,

$(x*z)*(y*z) \leq x*y$,

$x*y=0$ implies $x*z \leq y*zz*y \leq z*x$ and

For all $x, y, z \in X$.

Definition 2.1 [5]

A nonempty subset I of a BCK-algebra X is called an ideal of X if it

$0 \in I$,

$x*y \in I$ and $y \in I$ imply $x \in I$.

Definition 2.2 [2]

A nonempty subset I of a BCK-algebra X , is called a commutative ideal X if it satisfies $0 \in I$;

$\forall x,y,z \in X, (x*y)*z \in I$ and $z \in I$ imply $x*(y*(y*x)) \in I$

Definition 2.3 ([5])

A partially ordered set (X, \leq) is said to be a lower semi lattice if every pair of elements X has a greatest lower bound (meet (\wedge)); it is called to be an upper semi lattice if every pair of elements in X

has the least upper bound (Join (\vee)) (X, \leq) . It is both an upper and a lower semi lattice, then it is called a lattice.

Bifuzzy commutative ideals of BCK-algebras

Definition 3.1 [1]

A mapping $A=(\mu_A, \lambda_A): X \rightarrow [0,1] \times [0,1]$ is called bifuzzy set in X if $\mu_A(x) + \lambda_A(x) \leq 1$ for all $x \in X$, where the mappings $\lambda_A: X \rightarrow [0,1]$ and denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\lambda_A(x)$) of each element $x \in X$ to A respectively.

Definition 3.2

A bifuzzy set $A=(\mu_A, \lambda_A): X \rightarrow [0,1] \times [0,1]$ is called a bifuzzy ideal of BCK - algebra X if $\forall x,y,z \in X$ the following conditions hold:

$\mu_A(0) \geq \mu_A(x), \lambda_A(0) \leq \lambda_A(x)$,

$\mu_A(x) \geq \min\{\mu_A(x*y), \mu_A(y)\}$,

$\lambda_A(x) \leq \max\{\lambda_A(x*y), \lambda_A(y)\}$.

Example 3.3

Let $X=\{0,a,b,c\}$ in which $*$ is defined by

| | | | | |
|---|---|---|---|---|
| * | 0 | a | b | c |
| 0 | 0 | 0 | 0 | 0 |
| a | a | 0 | a | c |
| b | b | b | 0 | a |
| c | c | c | b | 0 |

Figure a

Then $(X, *, 0)$ is BCK-algebra.

We define a bifuzzy set $\mu_A(0)=0.7$ by, $\mu_A(x)=0.01$ and $\lambda_A(0)=0.02$, $\lambda_A(x)=0.6$ for all $x \neq 0$ in X . By routine computations, we can easily verify that the bifuzzy set A is a bifuzzy ideal of a BCK-algebra X .

Definition 3.4

A bifuzzy set $A=(\mu_A, \lambda_A):X \rightarrow [0,1] \times [0,1]$ is called a bifuzzy commutative ideal of BCK - algebra X if $\forall x,y,z \in X$ the following conditions hold:

$$\mu_A(0) \geq \mu_A(x), \lambda_A(0) \leq \lambda_A(x), \mu_A(x^*(y^*(y^*x))) \geq \min\{\mu_A((x^*y)^*z), \mu_A(z)\},$$

$$\lambda_A(x^*(y^*(y^*x))) \leq \max\{\lambda_A((x^*y)^*z), \lambda_A(z)\}.$$

If we put $y=0$, then follows that $A=(\mu_A, \lambda_A):X \rightarrow [0,1] \times [0,1]$ is bifuzzy ideal of X . Thus every bifuzzy commutative ideal of BCK - algebra X is a bifuzzy ideal X .

Example 3.5

Let $X=\{0,1,2,3,4\}$ be a BCK-algebra with Cayley table as follows:

| | | | | | |
|---|---|---|---|---|---|
| * | 0 | 1 | 2 | 3 | 4 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 |
| 2 | 2 | 2 | 0 | 0 | 0 |
| 3 | 3 | 3 | 3 | 0 | 0 |
| 4 | 4 | 4 | 4 | 3 | 0 |

Figure b

We define a bifuzzy set $\mu_A(0)=0.67$ by, $\mu_A(x)=0.03$ and $\lambda_A(0)=0.06$, $\lambda_A(x)=0.73$ for all $x \neq 0$ in X . By routine computations, we can easily verify that the bifuzzy set A is a bifuzzy commutative ideal of a BCK-algebra X .

Definition 3.6 [1]

Let $A=(\mu_A, \lambda_A)$ be a bifuzzy set on X and let $(\alpha, \beta) \in [0,1] \times [0,1]$ with $\alpha + \beta \leq 1$. Then the set

$X_{(\alpha, \beta)A} = \{x \in X / \alpha \leq \mu_A(x), \lambda_A(x) \leq \beta\}$ is called an (α, β) -level subset A . The set of all $(\alpha, \beta) \in \text{Im}(\mu_A) \times \text{Im}(\lambda_A)$ such that $\alpha + \beta \leq 1$ is called the image of $A=(\mu_A, \lambda_A)$.

We notice that, $X_{(\alpha, \beta)A} = \{x \in X / \mu_A(x) \geq \alpha, \lambda_A(x) \leq \beta\}$

$$= \{x \in X / \mu_A(x) \geq \alpha\} \cap \{x \in X / \lambda_A(x) \leq \beta\}$$

$$= \mu_A A \cap \lambda_{\beta A}.$$

Where $\mu_{\alpha A} = \{x \in X / \mu_A(x) \geq \alpha\}$ and $\lambda_{\beta A} = \{x \in X / \lambda_A(x) \leq \beta\}$ are level subsets of μ_A, λ_A respectively.

Theorem 3.7

Let $A=(\mu_A, \lambda_A)$ be a bifuzzy commutative ideal of BCK - algebra X , then the following are satisfies:

$$\mu_A(x^*(y^*(y^*x))) \geq \mu_A(x^*y) \text{ and } \lambda_A(x^*(y^*(y^*x))) \leq \lambda_A(x^*y).$$

(ii) If $\mu_{\alpha_1}, \mu_{\alpha_2}$ two-level commutative ideals (with $\alpha_1 < \alpha_2$) of Aare equal if and only if there is no $x \in X$ such that $\alpha_1 < \mu_A(x) < \alpha_2$. And if $\lambda_{\beta_1}, \lambda_{\beta_2}$ two-level commutative ideals (with $\beta_1 < \beta_2$) of Aare equal if and only if there is no $x \in X$ such that $\beta_1 < \lambda_A(x) < \beta_2$.

Proof.(i) That $A=(\mu_A, \lambda_A)$ is a bifuzzy commutative ideal of BCK - algebra X , then $\mu_A(x^*(y^*(y^*x))) \geq \min\{\mu_A(x^*y), \mu_A(0)\} = \mu_A(x^*y)$, thus $\mu_A(x^*(y^*(y^*x))) \geq \mu_A(x^*y)$.

$\lambda_A(x^*(y^*(y^*x))) \leq \max\{\lambda_A(x^*y), \lambda_A(0)\} = \lambda_A(x^*y)$, thus $\lambda_A(x^*(y^*(y^*x))) \leq \lambda_A(x^*y)$.

(ii) Assume that $\mu_{\alpha_1} = \mu_{\alpha_2}$ for $\alpha_1 < \alpha_2$ and that there exists $x \in X$ such that $\alpha_1 < \mu_A(x) < \alpha_2$ then μ_{α_1} is a proper subset μ_{α_2} , which contradicts the hypothesis.

Conversely, suppose that there is no $x \in X$ such that $\alpha_1 < \mu_A(x) < \alpha_2$. Since $\alpha_1 < \alpha_2$ we have $\mu_{\alpha_1} \subseteq \mu_{\alpha_2}$. Let $x \in \mu_{\alpha_1}$ then $\mu_A(x) \geq \alpha_1$ and hence $\mu_A(x) \geq \alpha_2$, because $\mu_A(x)$ does not lie between α_1 and α_2 . Hence $x \in \mu_{\alpha_2}$ which implies $\mu_{\alpha_1} \subseteq \mu_{\alpha_2}$, this completes the proof in case $\mu_{\alpha_1}, \mu_{\alpha_2}$. By the same method we can prove in case $\lambda_{\beta_1}, \lambda_{\beta_2}$.

Theorem 3.8

A bifuzzy set $A=(\mu_A, \lambda_A)$ of BCK - algebra X is a bifuzzy commutative ideal of X if, for all $(\alpha, \beta) \in [0,1] \times [0,1]$, $X_{(\alpha, \beta)A}$ is either empty or a commutative ideal of X .

Proof. Suppose that $A=(\mu_A, \lambda_A)$ is a bifuzzy commutative ideal of X and $X \neq \emptyset$ for any $(\alpha, \beta) \in [0,1] \times [0,1]$. it is clear that $0 \in \mu_{\alpha A}$ since $\mu_A(0) \geq \alpha$. Let $(x^*y)^*z \in \mu_{\alpha A}$ and $z \in \mu_{\alpha A}$, then $\mu_A((x^*y)^*z) \geq \alpha$ and $\mu_A(z) \geq \alpha$, since $A=(\mu_A, \lambda_A)$ is a bifuzzy commutative ideal we have $\mu_A(x^*(y^*(y^*x))) \geq \min\{\mu_A((x^*y)^*z), \mu_A(z)\} \geq \alpha$

Thus $x^*(y^*(y^*x)) \in \mu_{\alpha A}$, this shows that $\mu_{\alpha A}$ is a commutative ideal X .

It is clear that $0 \in \lambda_{\beta A}$ since $\lambda_A(0) \leq \beta$. Let $(x^*y)^*z \in \lambda_{\beta A}$ and $z \in \lambda_{\beta A}$, then $\lambda_A((x^*y)^*z) \leq \beta$ and $\lambda_A(z) \leq \beta$, since $A=(\mu_A, \lambda_A)$ is a bifuzzy is a commutative ideal we have $\lambda_A(x^*(y^*(y^*x))) \leq \max\{\lambda_A((x^*y)^*z), \lambda_A(z)\} \leq \beta$, thus $x^*(y^*(y^*x)) \in \lambda_{\beta A}$, this shows $\lambda_{\beta A}$ is a positive implicative ideal

of X . Since that $X_{(\alpha,\beta)A} = \{x \in X / \mu_A(x) \geq \alpha, \lambda_A(x) \leq \beta\} = \mu_A \wedge \lambda_{\beta A}$, thus $X_{(\alpha,\beta)A}$ is a commutative ideal of X .

Conversely, suppose that for each $(\alpha, \beta) \in [0, 1] \times [0, 1]$, $X_{(\alpha,\beta)A}$ is either empty or a commutative ideal of X . For any $x \in X$ let $\mu_A(x) = \alpha$, then $x \in \mu_{\alpha A}$. Since $\mu_{\alpha A} (\neq \emptyset)$ is a commutative ideal of X , therefore $0 \in \mu_{\alpha A}$ and hence $\mu_A(0) \geq \alpha = \mu_A(x)$, thus $\mu_A(0) \geq \mu_A(x)$ for all $x \in X$. Now we only need to show that $\mu_A(x^*(y^*(y^*x))) \geq \min\{\mu_A((x^*y)^*z), \mu_A(z)\}$. If not, then there exists $x', y', z' \in X$ such that $\mu_A(x^*(y^*(y^*x'))) < \min\{\mu_A((x^*y')^*z'), \mu_A(z')\}$

Taking $\alpha_0 = 1/2 \{\mu_A(x^*(y^*(y^*x'))) + \mu_A((x^*y')^*z')\}$, then we have $\mu_A(x^*(y^*(y^*x'))) < \alpha_0 < \min\{\mu_A((x^*y')^*z'), \mu_A(z')\}$. Hence

$x^*(y^*(y^*x')) \notin \mu_{\alpha_0 A}, (x^*y')^*z' \in \mu_{\alpha_0 A}, z' \in \mu_{\alpha_0 A}$ i.e. $\mu_{\alpha_0 A}$ is not a commutative ideal of X , which is a contradiction therefore

$$\mu_A(x^*(y^*(y^*x))) \geq \min\{\mu_A((x^*y)^*z), \mu_A(z)\}.$$

For $(\alpha, \beta) \in [0, 1] \times [0, 1]$. For any $x \in X$ let $\lambda_A(x) = \beta$, then $x \in \lambda_{\beta A}$. Since $\lambda_{\beta A} (\neq \emptyset)$ is a positive implicative ideal of X , therefore $0 \in \lambda_{\beta A}$ and hence $\lambda_A(0) \leq \beta = \lambda_A(x)$, thus $\lambda_A(0) \leq \lambda_A(x)$ for all $x \in X$. Now we also only need to show that $\lambda_A(x^*z) \leq \max\{\lambda_A((x^*y)^*z), \lambda_A(y^*z)\}$. If not, then there exists $x', y', z' \in X$ such that $\lambda_A(x^*(y^*(y^*x'))) > \min\{\lambda_A((x^*y')^*z'), \lambda_A(z')\}$.

Taking $\beta_0 = 1/2 \{\lambda_A(x^*(y^*(y^*x'))) + \max\{\lambda_A((x^*y')^*z'), \lambda_A(z')\}\}$, then we have $\lambda_A(x^*(y^*(y^*x'))) > \beta_0 > \max\{\lambda_A((x^*y')^*z'), \lambda_A(z')\}$. Hence $x^*(y^*(y^*x')) \notin \lambda_{\beta_0 A}, ((x^*y')^*z') \in \lambda_{\beta_0 A}, z' \in \lambda_{\beta_0 A}$ i.e. $\lambda_{\beta_0 A}$ is not a commutative ideal of X , which is a contradiction therefore $\lambda_A(x^*(y^*(y^*x))) \leq \max\{\lambda_A((x^*y)^*z), \lambda_A(z)\}$, thus $A = (\mu_A, \lambda_A)$ a bifuzzy commutative ideal.

Theorem 3.9

Let $\{A_i / i \in I\}$ be a family of bifuzzy commutative ideals of BCK-algebra X then $\{A_i, \subset\}$ is ordering set under the ordering of bifuzzy set inclusion \subset .

Proof. Let $\{A_i / i \in I\}$ is a family of bifuzzy commutative ideals X . Since $[0, 1]$ is a completely distributive lattice for the usual ordering $[0, 1]$, it is sufficient to show that $\bigcap_{i \in I} A_i = (\bigwedge_{i \in I} \mu_{A_i}, \bigvee_{i \in I} \lambda_{A_i})$ is a bifuzzy commutative ideal X .

For any $x \in X$, where \vee and \wedge denoted the operation join and meet respectively.

$$(\bigwedge_{i \in I} \mu_{A_i})(0) = \inf_{i \in I} \mu_{A_i}(0) \geq \inf_{i \in I} \mu_{A_i}(x) = (\bigwedge_{i \in I} \mu_{A_i})(x)$$

and

$$(\bigvee_{i \in I} \lambda_{A_i})(0) = \sup_{i \in I} \lambda_{A_i}(0) \leq \sup_{i \in I} \lambda_{A_i}(x) = (\bigvee_{i \in I} \lambda_{A_i})(x)$$

Let $x, y, z \in X$ then

$$\begin{aligned} (\bigwedge_{i \in I} \mu_{A_i})(x * (y * (y * x))) &= \inf_{i \in I} \mu_{A_i}(x * (y * (y * x))) = \inf_{i \in I} \{\mu_{A_i}(x * (y * (y * x))) / i \in I\} \\ &\geq \inf_{i \in I} \{\min\{\mu_{A_i}((x * y) * z), \mu_{A_i}(z)\} / i \in I\} \end{aligned}$$

$$= \min\{\inf_{i \in I} \{\mu_{A_i}((x * y) * z), \mu_{A_i}(z)\} / i \in I\},$$

$$= \min\{\inf_{i \in I} \mu_{A_i}((x * y) * z), \inf_{i \in I} \mu_{A_i}(z)\} / i \in I, = \min\{\bigwedge_{i \in I} \mu_{A_i}((x * y) * z), \bigwedge_{i \in I} \mu_{A_i}(z)\} / i \in I$$

$$(\bigvee_{i \in I} \lambda_{A_i})(x * (y * (y * x))) = \sup_{i \in I} \lambda_{A_i}(x * (y * (y * x))) = \sup_{i \in I} \{\lambda_{A_i}(x * (y * (y * x))) / i \in I\}$$

$$\geq \sup_{i \in I} \{\max\{\lambda_{A_i}((x * y) * z), \lambda_{A_i}(z)\} / i \in I\}$$

$$= \max\{\sup_{i \in I} \{\lambda_{A_i}((x * y) * z), \lambda_{A_i}(z)\} / i \in I\},$$

$$= \max\{\sup_{i \in I} \lambda_{A_i}((x * y) * z), \sup_{i \in I} \lambda_{A_i}(z)\} / i \in I, = \max\{\bigvee_{i \in I} \lambda_{A_i}((x * y) * z), \bigvee_{i \in I} \lambda_{A_i}(z)\} / i \in I$$

Hence $\bigcap_{i \in I} A_i = (\bigvee_{i \in I} \mu_{A_i}, \bigwedge_{i \in I} \lambda_{A_i})$ is a bifuzzy commutative ideal of X

Theorem 3.10

Let I be a commutative ideal of a BCK-algebra X . Then there exists a bifuzzy commutative ideal $A = (\mu_A, \lambda_A)$ of X such that $X_{\alpha, \beta A} = I$ $(\alpha, \beta) \in [0, 1] \times [0, 1]$ was with $\alpha + \beta \leq 1$.

Proof. Define $\mu_A: X \rightarrow [0, 1]$ by

$$\mu_A(x) = \begin{cases} \alpha & ; x \in I \\ 0 & ; x \notin I \end{cases}$$

Where α is a fixed number $(0, 1]$ and define

$\lambda_A: X \rightarrow [0, 1]$ by

$$\lambda_A(x) = \begin{cases} 0 & ; x \in I \\ \beta & ; x \notin I, \end{cases}$$

Where β is a fixed number $(0, 1]$. Since $0 \in I$, $\mu_A(0) = \alpha \geq \mu_A(x)$ for all $x \in X$, also since $0 \in I$, $\lambda_A(0) = 0 \leq \lambda_A(x)$ for all $x \in X$. For μ_A . Since I is a commutative ideal of X , we have $(x^*y)^*z \in I$ and $z \in I$ then $x^*(y^*(y^*x)) \in I$. Hence $\mu_A((x^*y)^*z) = \mu_A(z) = \mu_A(x^*(y^*(y^*x))) = \alpha$, thus

$$\mu_A(x^*(y^*(y^*x))) = \min\{\mu_A((x^*y)^*z), \mu_A(z)\} = \alpha.$$

If at least one $(x^*y)^*z$ and z is not in I , then at least one $\mu_A((x^*y)^*z)$ and $\mu_A(z)$ is 0.

Therefore

$$\mu_A(x^*(y^*(y^*x))) \geq \min\{\mu_A((x^*y)^*z), \mu_A(z)\}.$$

For λ_A . Since I is commutative of X , if $(x^*y)^*z \in I$ and $z \in I$ then $x^*(y^*(y^*x)) \in I$.

Hence $\lambda_A(((x*y)*z)) = \lambda_A(z) = \lambda_A(x*(y*(y*x))) = \beta$, thus

$$\lambda_A(x*(y*(y*x))) = \max\{\lambda_A(((x*y)*z)), \lambda_A(z)\} = \beta.$$

If at least one $(x*y)*z$ and z is not in I , then at least one $\lambda_A(((x*y)*z))$ and $\lambda_A(z)$ is β .

Therefore

$$\lambda_A(x*(y*(y*x))) \leq \max\{\lambda_A(((x*y)*z)), \lambda_A(z)\}.$$

Thus $A = (\mu_A, \lambda_A)$ is a commutative ideal of X . It is clear that

$$I = \mu_\alpha A \cap \lambda_{\beta A} = X_{(\alpha, \beta)A} \text{ and so the result follows } \square$$

Definition 3.11 [1]

Let X be a nonempty set. Then we call mapping

$$A = (\mu_A, \lambda_A): X \times X \rightarrow [0,1] \times [0,1]$$

a bifuzzy relation on X if $\mu_A(x,y) + \lambda_A(x,y) \leq 1 \forall (x,y) \in X \times X$.

Definition 3.12 [1]

Let $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ be a bifuzzy sets on a set X , then $A = (\mu_A, \lambda_A)$ is a bifuzzy relation on $B = (\mu_B, \lambda_B)$ if

$$\mu_A(x,y) \leq \min\{\mu_B(x), \mu_B(y)\} \text{ and}$$

$$\lambda_A(x,y) \geq \max\{\lambda_B(x), \lambda_B(y)\}.$$

Definition 3.13 [1]

Let $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ be two bifuzzy sets on a set X . Then the Cartesian product $A \times B$ is defined as follows

$$A \times B = (\mu_A, \lambda_A) \times (\mu_B, \lambda_B) = (\mu_A \times \mu_B, \lambda_A \times \lambda_B), \text{ where}$$

$$\lambda_A \times \lambda_B(x,y) = \max\{\lambda_A(x), \lambda_B(y)\}$$

We note that the Cartesian product $A \times B$ is always a bifuzzy set in $X \times X$, if

$$0 \leq \min\{\mu_A(x), \mu_B(y)\} + \max\{\lambda_A(x), \lambda_B(y)\} \leq 1.$$

Definition 3.14 [1]

Let $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ be a bifuzzy set on X . then the strongest bifuzzy relation on $B = (\mu_B, \lambda_B)$ is A_B , defined by $A_B = (\mu_{A \times B}, \lambda_{A \times B})$ where $\lambda_{A \times B}(x,y) = \min\{\lambda_B(x), \lambda_B(y)\}$ and.

Theorem 3.15

If $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ are two bifuzzy commutative ideal of BCK – algebra X , Then $A \times B$ is a bifuzzy commutative idea of $X \times X$.

Proof. Let

$$0 = (0, 0), x = (x_1, x_2), (x*y)*z = ((x_1*y_1)*z_1, (x_2*y_2)*z_2), z = (z_1, z_2)$$

$$x*(y*(y*x)) = (x_1*(y_1*(y_1*x_1)), x_2*(y_2*(y_2*x_2))) \text{ and, any elements of.}$$

$$X \times X \text{ Then we have } (\mu_A \times \mu_B)(0) = (\mu_A \times \mu_B)((0,0))$$

$$= \min\{\mu_A(0), \mu_B(0)\} \geq \min\{\mu_A(x_1), \mu_B(x_2)\}$$

$$= (\mu_A \times \mu_B)((x_1, x_2)) = (\mu_A \times \mu_B)(x).$$

$$(\lambda_A \times \lambda_B)(0) = (\lambda_A \times \lambda_B)((0,0))$$

$$= \max\{\lambda_A(0), \lambda_B(0)\} \leq \max\{\lambda_A(x_1), \lambda_B(x_2)\}$$

$$= (\lambda_A \times \lambda_B)((x_1, x_2)) = (\lambda_A \times \lambda_B)(x).$$

$$(\mu_A \times \mu_B)(x*(y*(y*x))) = (\mu_A \times \mu_B)((x_1*(y_1*(y_1*x_1)), x_2*(y_2*(y_2*x_2)))) = \min\{\mu_A(x_1*(y_1*(y_1*x_1))), \mu_B(x_2*(y_2*(y_2*x_2)))\}$$

$$\geq \min\{\min\{\mu_A((x_1*y_1)*z_1), \mu_A(z_1)\}, \min\{\mu_B((x_2*y_2)*z_2), \mu_B(z_2)\}\}$$

$$= \min\{\min\{\mu_A((x_1*y_1)*z_1), \mu_B((x_2*y_2)*z_2)\}, \min\{\mu_A(z_1), \mu_B(z_2)\}\}$$

$$= \min\{(\mu_A \times \mu_B)((x_1*y_1)*z_1, (x_2*y_2)*z_2), (\mu_A \times \mu_B)((z_1, z_2))\}$$

$$= \min\{(\mu_A \times \mu_B)((x*y)*z), (\mu_A \times \mu_B)(z)\}.$$

$$(\lambda_A \times \lambda_B)(x*(y*(y*x))) = (\lambda_A \times \lambda_B)((x_1*(y_1*(y_1*x_1)), x_2*(y_2*(y_2*x_2))))$$

$$= \max\{\lambda_A(x_1*(y_1*(y_1*x_1))), \lambda_B(x_2*(y_2*(y_2*x_2)))\}$$

$$\leq \max\{\max\{\lambda_A((x_1*y_1)*z_1), \lambda_A(z_1)\}, \max\{\lambda_B((x_2*y_2)*z_2), \lambda_B(z_2)\}\}$$

$$= \max\{\max\{\lambda_A((x_1*y_1)*z_1), \lambda_B((x_2*y_2)*z_2)\}, \max\{\lambda_A(z_1), \lambda_B(z_2)\}\}$$

$$= \max\{(\lambda_A \times \lambda_B)((x_1*y_1)*z_1, (x_2*y_2)*z_2), (\lambda_A \times \lambda_B)((z_1, z_2))\}$$

$$= \max\{(\lambda_A \times \lambda_B)((x*y)*z), (\lambda_A \times \lambda_B)(z)\}.$$

Hence $A \times B$ is a bifuzzy commutative ideal of $X \times X$ \square

Theorem 3.16

Let $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ be a bifuzzy sets in BCK – algebra X and A_B the strongest bifuzzy relation on X . Then $B = (\mu_B, \lambda_B)$ is a bifuzzy commutative ideal of X if and only if A_B is a bifuzzy commutative ideal of $X \times X$.

Proof. Let

$0=(0,0), x=(x_1, x_2), z=(z_1, z_2), (x*y)*z=((x_1*y_1)*z_1, (x_2*y_2)*z_2)$ and

$x*(y*(y*x))=(x_1*(y_1*(y_1*x_1)), x_2*(y_2*(y_2*x_2)))$

, any elements of $X \times X$ and let $B=(\mu_B, \lambda_B)$ be a bifuzzy positive implicative ideal of X .

$$\mu_{A_{\mu B}}(0) = \mu_{A_{\mu B}}((0,0))$$

$$= \min \{ \mu_B(0), \mu_B(0) \}$$

$$\geq \min \{ \mu_B(x), \mu_B(x) \}$$

$$= \mu_{A_{\mu B}}((x_1, x_2))$$

$$= \mu_{A_{\mu B}}(x).$$

$$\lambda_{A_{\mu B}}(0) = \lambda_{A_{\mu B}}((0,0))$$

$$= \max \{ \lambda_B(0), \lambda_B(0) \}$$

$$\leq \max \{ \lambda_B(x), \lambda_B(x) \}$$

$$= \lambda_{A_{\mu B}}((x_1, x_2))$$

$$= \lambda_{A_{\mu B}}(x).$$

$$\mu_{A_{\mu B}}(x*(y*(y*x))) = \mu_{A_{\mu B}}((x_1*(y_1*(y_1*x_1)), x_2*(y_2*(y_2*x_2))))$$

$$= \min \{ \mu_B(x_1*(y_1*(y_1*x_1))), \mu_B(x_2*(y_2*(y_2*x_2))) \}$$

$$\geq \min \{ \min \{ \mu_B((x_1*y_1)*z_1), \mu_B(z_1) \}, \min \{ \mu_B((x_2*y_2)*z_2), \mu_B(z_2) \} \} = \min \{ \min \{ \mu_A((x_1*y_1)*z_1), \mu_B((x_2*y_2)*z_2) \}, \min \{ \mu_A(z_1), \mu_B(z_2) \} \}$$

$$= \min \{ \mu_{A_{\mu B}}(((x_1*y_1)*z_1, (x_2*y_2)*z_2)), \mu_{A_{\mu B}}((z_1, z_2)) \}$$

$$= \min \{ \mu_{A_{\mu B}}((x*y)*z), \mu_{A_{\mu B}}(z) \}.$$

$$\lambda_{A_{\mu B}}(x*(y*(y*x))) = \lambda_{A_{\mu B}}((x_1*(y_1*(y_1*x_1)), x_2*(y_2*(y_2*x_2))))$$

$$= \max \{ \lambda_B(x_1*(y_1*(y_1*x_1))), \lambda_B(x_2*(y_2*(y_2*x_2))) \}$$

$$\leq \max \{ \max \{ \lambda_B((x_1*y_1)*z_1), \lambda_B(z_1) \}, \max \{ \lambda_B((x_2*y_2)*z_2), \lambda_B(z_2) \} \} = \max \{ \max \{ \lambda_A((x_1*y_1)*z_1), \lambda_B((x_2*y_2)*z_2) \}, \max \{ \lambda_A(z_1), \lambda_B(z_2) \} \}$$

$$= \max \{ \lambda_{A_{\mu B}}(((x_1*y_1)*z_1, (x_2*y_2)*z_2)), \lambda_{A_{\mu B}}((z_1, z_2)) \}$$

$$= \max \{ \lambda_{A_{\mu B}}((x*y)*z), \lambda_{A_{\mu B}}(z) \}$$

This shows that $A_{\mu B}$ is a bifuzzy commutative ideal $X \times X$.

Conversely, suppose that $A_{\mu B}=(\mu_{A_{\mu B}}, \lambda_{A_{\mu B}})$ is a bifuzzy commutative ideal $X \times X$. Then $\min \{ \mu_B(0), \mu_B(0) \} = \mu_{A_{\mu B}}((0,0)) \geq \mu_{A_{\mu B}}((x,y)) = \min \{ \mu_B(x), \mu_B(y) \}$

$$\max \{ \lambda_B(0), \lambda_B(0) \} = \mu_{A_{\mu B}}((0,0)) \leq \lambda_{A_{\mu B}}((x,y)) = \max \{ \mu_B(x), \mu_B(y) \}.$$

For all, it follows that $\mu_B(x) \leq \mu_B(0)$ and $\lambda_B(x) \geq \lambda_B(0)$.

For any $x*(y*(y*x))=(x_1*(y_1*(y_1*x_1)), x_2*(y_2*(y_2*x_2))) \in X \times X$ and $(x*y)*z = ((x_1*y_1)*z_1, (x_2*y_2)*z_2), z=(z_1, z_2) \in X \times X$

$$\min \{ \mu_B(x_1*(y_1*(y_1*x_1))), \mu_B(x_2*(y_2*(y_2*x_2))) \} = \mu_{A_{\mu B}}((x_1*(y_1*(y_1*x_1)), x_2*(y_2*(y_2*x_2)))) = \mu_{A_{\mu B}}(x*(y*(y*x))) \geq \min \{ \mu_{A_{\mu B}}((x*y)*z), \mu_{A_{\mu B}}(z) \} =$$

$$= \min \{ \mu_{A_{\mu B}}(((x_1*y_1)*z_1, (x_2*y_2)*z_2)), \mu_{A_{\mu B}}((z_1, z_2)) \} = \min \{ \min \{ \mu_B((x_1*y_1)*z_1), \mu_B((x_2*y_2)*z_2) \}, \min \{ \mu_B(z_1), \mu_B(z_2) \} \} \text{ Putting } x_2=z_2=0 \text{ gives}$$

$$\mu_B(x_1*(y_1*(y_1*x_1))) \geq \min \{ \mu_B((x_1*y_1)*z_1), \mu_B(z_1) \}.$$

$$\max \{ \lambda_B(x_1*(y_1*(y_1*x_1))), \lambda_B(x_2*(y_2*(y_2*x_2))) \} = \lambda_{A_{\mu B}}((x_1*(y_1*(y_1*x_1)), x_2*(y_2*(y_2*x_2)))) = \lambda_{A_{\mu B}}(x*(y*(y*x))) \leq \max \{ \lambda_{A_{\mu B}}((x*y)*z), \lambda_{A_{\mu B}}(z) \} =$$

$$= \max \{ \lambda_{A_{\mu B}}(((x_1*y_1)*z_1, (x_2*y_2)*z_2)), \lambda_{A_{\mu B}}((z_1, z_2)) \} = \max \{ \max \{ \lambda_B((x_1*y_1)*z_1), \lambda_B((x_2*y_2)*z_2) \}, \max \{ \lambda_B(z_1), \lambda_B(z_2) \} \} \text{ Putting } x_2=z_2=0 \text{ gives}$$

$$\lambda_B(x_1*(y_1*(y_1*x_1))) \geq \max \{ \lambda_B((x_1*y_1)*z_1), \lambda_B(z_1) \}.$$

Hence $B=(\mu_B, \lambda_B)$ is a bifuzzy commutative ideal of X \square

Definition 3.17 [1]

Let a mapping $f: X_1 \rightarrow X_2$ from BCK - algebra X_1 into BCK - algebra X_2 and let $A=(\mu_A, \lambda_A)$ be a bifuzzy set X_2 . The map $A=(\mu_A, \lambda_A)$ is called the pre-image of $A=(\mu_A, \lambda_A)$ under $f: X_1 \rightarrow X_2$, if

$$\mu_A^f(x) = \mu_A(f(x)) \text{ and } \lambda_A^f(x) = \lambda_A(f(x)) \forall x \in X_1.$$

Theorem 3.18

Let a mapping $f: X_1 \rightarrow X_2$ be a homomorphism from BCK - algebra X_1 into BCK - algebra X_2 such that $f(0)=0$, then $A=(\mu_A, \lambda_A)$ is a bifuzzy

commutative ideal $X_2 X_1$

Proof. Let

$A=(\mu_A, \lambda_A)$ is a bifuzzy commutative ideal X_2 . Then for any $x \in X_1$, we have $\mu_A^f(0) = \mu_A(f(0)) = \mu_A(0) \geq \mu_A(f(x)) = \mu_A^f(x)$ and

$$\lambda_A^f(0) = \lambda_A(f(0)) = \lambda_A(0) \geq \lambda_A(f(x)) = \lambda_A^f(x).$$

For any $x, y, z \in X_1$, since $A=(\mu_A, \lambda_A)$ is a bifuzzy commutative ideal of X_2 , $\mu_A^f(x*(y*(y*x))) = \mu_A(f(x*(y*(y*x)))) \geq \min\{\mu((f(x)*f(y))*f(z))*\mu(f(z))\}$

$$= \min\{\mu_A((f(x*y)*z)), \mu_A(f(z))\}$$

$$= \min\{\mu_A^f((x*y)*z), \mu_A^f(z)\}.$$

$$\lambda_A^f(x*(y*(y*x))) = \lambda_A(f(x*(y*(y*x)))) \leq \max\{\lambda((f(x)*f(y))*f(z))*\lambda(f(z))\}$$

$$= \max\{\lambda_A((f(x*y)*z)), \lambda_A(f(z))\}$$

$$= \max\{\lambda_A^f((x*y)*z), \lambda_A^f(z)\}$$

Proving that $A^f=(\mu_A^f, \lambda_A^f)$ is a bifuzzy commutative ideal X_1 .

Conversely Let $A^f=(\mu_A^f, \lambda_A^f)$ is a bifuzzy commutative ideal X_1 , $x, y, z \in X_2$. For any, there exists $a, b, c \in X_1$ such that $x=f(a)$, $y=f(b)$, $z=f(c)$ follow that

$$\mu_A(x) = \mu_A(f(a)) = \mu_A^f(a) \leq \mu_A^f(0) = \mu_A(f(0)) = \mu_A(0)$$

$$\lambda_A(x) = \lambda_A(f(a)) = \lambda_A^f(a) \geq \lambda_A^f(0) = \lambda_A(f(0)) = \lambda_A(0)$$

$$\mu_A(x*(y*(y*x))) = \mu_A(f(a*(b*(b*a)))) = \mu_A^f(a*(b*(b*a)))$$

$$\geq \min\{\mu_A^f((a*b)*c), \mu_A^f(c)\}$$

$$= \min\{\mu_A(((f(a)*f(b))*f(c))) \mu_A(f(c))\} = \min\{\mu_A((x*y)*z), \mu_A(z)\}.$$

$$\lambda_A(x*(y*(y*x))) = \lambda_A(f(a*(b*(b*a)))) = \lambda_A^f(a*(b*(b*a))) \leq \max\{\lambda_A^f((a*b)*c), \lambda_A^f(c)\}$$

$$= \max\{\lambda_A(((f(a)*f(b))*f(c))) \lambda_A(f(c))\} = \max\{\lambda_A((x*y)*z), \lambda_A(z)\}.$$

Thus Proving that $A=(\mu_A, \lambda_A)$ is a bifuzzy commutative ideal of X_2

Algorithms for fuzzy and bifuzzy subset of BCK - algebras

Here we give some algorithms for studding the structure of the fuzzy and bifuzzy BCK -algebras.

Algorithm for BCK-algebras

Input (X: set, *: binary operation)

Output (“X is a BCK- algebra or not”);

Begin

If $X = \emptyset$ then

go to (1.);

EndIf

If $0 \notin X$ then

go to (1.);

EndIf

Stop:=false;

i:=1;

While $i \leq |X|$ and not (Stop) do

If $x_i * x_i \neq 0$ then

Stop:=true;

EndIf

If $0 * x_i \neq 0$ then

Stop:=true;

EndIf

j:=1;

While $j \leq |X|$ and not (Stop) do

k:=1;

While $k \leq |X|$ and not (Stop) do

If $(x_i * (x_i * y_j)) * y_j \neq 0$ then

Stop:=true;

EndIf

| | |
|--|--|
| If $(x_i * y_j = 0)$ and $(y_j * x_i = 0)$ then | Stop:=false; |
| If $x_i \neq y_j$ then | i:=1; |
| Stop:=true; | While $i \leq X $ and not (Stop) do |
| EndIf | j:=1; |
| EndIf | While $j \leq X $ and not (Stop) do |
| k:=1; | If $x_i \in I$ and $y_j \in I$ then |
| While $k \leq X $ and not (Stop) do | If $x_i * y_j \notin I$ |
| If $((x_i * y_j) * (x_i * z_k)) * (z_k * y_j) \neq 0$ then | Stop:=true; |
| Stop:=true; | EndIf |
| EndIf | EndIf |
| Endwhile | Endwhile |
| Endwhile | Endwhile |
| Endwhile | If Stop then |
| If Stop then | Output ("I is a subalgebra of X") |
| (1.) Output is a ("X is not BCK - algebra ") | Else |
| Else | (1.) Output ("I is not a subalgebra of X") |
| Output ("X is BCK - algebra ") | EndIf |
| EndIf | End |
| End | Algorithm for ideals of BCK - Algebras |
| Algorithm for subalgebras of BCK - Algebras | Input(X:BCK - algebra, *: binary operation,I: subset ofX); |
| Input(X:BCK - algebra, *: binary operation,I: subset ofX); | Output("I is an ideal ofX or not"); |
| Output("I is subalgebra ofX or not"); | Begin |
| Begin | If I= ϕ then |
| If I= ϕ then | go to (1.); |
| go to (1.); | EndIf |
| EndIf | |

| | |
|--|--|
| If $0 \notin I$ then | $i:=1;$ |
| go to (1.); | While $i \leq X $ and not (Stop) do |
| EndIf | If $(A(x_i) < 0)$ or $(A(x_i) > 1)$ then |
| Stop:=false | Stop:=true; |
| $i:=1;$ | EndIf |
| While $i \leq X $ and not (Stop) do | Endwhile |
| $j:=1;$ | If Stop then |
| While $j \leq X $ and not (Stop) do | Output ("A is not a fuzzy set in X") |
| If $x_i * y_j \in I$ and $y_j \in I$ then | Else |
| If $x_i \notin I$ | Output ("A is a fuzzy set in X") |
| Stop:=true; | EndIf |
| EndIf | End |
| EndIf | Algorithm for bifuzzy set |
| Endwhile | Input(X:BCKI-algebra, $A=(\mu_A, \lambda_A):X \rightarrow [0,1] \times [0,1]$); |
| Endwhile | Output ("A is a bifuzzy set in X or not"); |
| If Stop then | Begin |
| Output ("I is an ideal of X") | Stop:=false; |
| Else | $i:=1;$ |
| (1.) Output ("I is not an ideal of X") | While $i \leq X $ and not (Stop) do |
| EndIf | If $\mu_A(x_i) + \lambda_A(x_i) > 1$ then |
| End | Stop:=true; |
| Algorithm for fuzzy set | EndIf |
| Input(X:BCKI-algebra, $A:X \rightarrow R$); | Endwhile |
| Output("A is a fuzzy set in X or not"); | If Stop then |
| Begin | Output ("A is not a bifuzzy set of X") |
| Stop:=false; | |

| | |
|--|---|
| Else | EndIf |
| Output ("A is a bifuzzy set of X") | End |
| EndIf | Algorithm for bifuzzy ideals of BCK-Algebra |
| End | Input(X: BCK-algebra, *: binary operation, A: bifuzzy set in X); |
| Algorithm for fuzzy ideals of BCK-Algebra | Output ("A is a bifuzzy ideal of X or not"); |
| Input(X: BCK-algebra, *: binary operation, A: fuzzy set in X); | Begin |
| Output ("A is a fuzzy ideal of X or not"); | Stop:=false; |
| Begin | i:=1; |
| Stop:=false; | While i ≤ X and not (Stop) do |
| i:=1; | If ($\mu_A(0) < \mu_A(x_i)$) or ($\lambda_A(0) > \lambda(x_i)$), then |
| While i ≤ X and not (Stop) do | Stop:=true; |
| If $A(0) < A(x_i)$ then | EndIf |
| Stop:=true; | j:=1; |
| EndIf | While j ≤ X and not (Stop) do |
| j:=1; | If ($\mu_A(x_i) < \min\{\mu_A(x_i * y_j), \mu_A(y_j)\}$) or ($\lambda_A(x_i) > \min\{\lambda_A(x_i * y_j), \lambda_A(y_j)\}$) |
| While j ≤ X and not (Stop) do | Stop:=true; |
| If $\mu(x_i) < \min\{\mu(x_i * y_j), \mu(y_j)\} <$ | EndIf |
| Stop:=true; | EndIf |
| EndIf | Endwhile |
| EndIf | Endwhile |
| Endwhile | If Stop then |
| Endwhile | Output ("A is not a bifuzzy ideal of X") |
| If Stop then | Else |
| Output ("A is not a fuzzy ideal of X") | Output ("A is a bifuzzy ideal of X") |
| Else | EndIf |
| Output ("A is a fuzzy ideal of X") | |

| | |
|---|---|
| End | Input(X: BCK-algebra, *: binary operation, A: bifuzzy subset of X); |
| Algorithm for fuzzy subalgebras of BCK-Algebra | Output ("A is a bifuzzysubalgebra of X or not"); |
| Input(X: BCK-algebra, *: binary operation, A: fuzzy subset of X); | Begin |
| Output ("A is a fuzzy subalgebra of X or not"); | Stop:=false; |
| Begin | i:=1; |
| Stop:=false; | While i≤ X and not (Stop) do |
| i:=1; | If ($\mu_A(0) < \mu_A(x_i)$) or ($\lambda_A(0) > \lambda_A(x_i)$) then |
| While i≤ X and not (Stop) do | Stop:=true; |
| If $A(0) < A(x_i)$ then | EndIf |
| Stop:=true; | j:=1; |
| EndIf | While j≤ X and not (Stop) do |
| j:=1; | If ($\mu_A(x_i * y_j) < \min\{\mu_A(x_i), \mu(y_j)\}$) or ($\lambda_A(x_i * y_j) > \min\{\lambda_A(x_i), \mu(y_j)\}$) |
| While j≤ X and not (Stop) do | Stop:=true; |
| If $\mu(x_i * y_j) < \min\{\mu(x_i), \mu(y_j)\}$ | EndIf |
| Stop:=true; | EndIf |
| EndIf | Endwhile |
| EndIf | Endwhile |
| Endwhile | If Stop then |
| Endwhile | Output ("A is not a bifuzzysubalgebra of X") |
| If Stop then | Else |
| Output ("A is not a fuzzy subalgebra of X") | Output ("A is a bifuzzysubalgebra of X") |
| Else | EndIf |
| Output ("A is a fuzzy subalgebra of X") | End |
| EndIf | Algorithm for commutative ideals of BCK-Algebra |
| End | Input(X: BCK-algebra, *: binary operation, I: subset of X); |
| Algorithm for bifuzzy subalgebras of BCK-Algebra | |

```

Output ("I is a commutative ideal ofXor not");
Begin
IfI=φ then
go to (1.);
EndIf
If0∉I then
go to (1.);
EndIf
Stop:=false;
i:=1;
While i≤|X| and not (Stop) do
j:=1;
While j≤|X| and not (Stop) do
k:=1;
While k≤|X| and not (Stop) do
If (xi*yj)*zk∈I and zk∈I then
If xi*(yj*(yj*xi))∉I
Stop:=true;
EndIf
EndIf
Endwhile
Endwhile
Endwhile
If Stop then
Output ("Iis a commutative ideal ofX")
Else

```

```

(1.) Output ("Iis not a commutative ideal ofX")
EndIf
End
Algorithm for fuzzy commutative ideals of BCK-Algebra
Input(X: BCK-algebra,*: binary operation, A: fuzzy subset ofX);
Output ("A is a fuzzy commutative ideal ofXor not");
Begin
Stop:=false;
i:=1;
While i≤|X| and not (Stop) do
If A(0)<A(xi) then
Stop:=true;
EndIf
j:=1;
While j≤|X| and not (Stop) do
k:=1;
While k≤|X| and not (Stop) do
IfA(xi*(yj*(yj*xi)))<Min(A((xi*yj)*zk),A(zk))
Stop:=true;
EndIf
Endwhile
Endwhile
Endwhile
If Stop then
Output ("Ais not a fuzzy commutative ideal of X")

```

```

Else
Output ("Ais a fuzzy commutative ideal of X")
EndIf
End
Algorithm for bifuzzy commutative ideals of BCK-Algebra
Input(X:BCK-algebra,*: binary operation, A: bifuzzy subset ofX);
Output("A is a bifuzzy commutative ideal ofXor not");
Begin
Stop:=false;
i:=1;
While i≤|X| and not (Stop) do
If ( $\mu_A(0) < \mu_A(x_i)$ ) or ( $\lambda_A(0) > \lambda_A(x_i)$ ) then
Stop:=true;
EndIf
j:=1;
While j≤|X| and not (Stop) do
k:=1;
While k≤|X| and not (Stop) do
If( $\mu_A(x_i*(y_j*(y_j*x_i))) < \text{Min}(\mu_A((x_i*y_j)*z_k), \mu_A(z_k))$ ) or ( $\lambda_A(x_i*(y_j*(y_j*x_i))) > \text{Min}(\lambda_A((x_i*y_j)*z_k), \lambda_A(z_k))$ )
Stop:=true;
EndIf
Endwhile
Endwhile
Endwhile
If Stop then

```

```

Output ("Ais not a bifuzzycommutative ideal of X")
Else
Output ("Ais a bifuzzycommutative ideal of X")
EndIf
End.

```

Conclusion and Future Research

In this paper, we introduce the new notion of bifuzzy commutative ideals of BCK-algebras, and investigate some of their properties. We hope that this work would be the foundation for further study of the theory of bifuzzy ideals in BCK/BCI - algebras.

In our future study of bifuzzy ideals in BCK/BCI algebras, maybe the following topics should be considered:

- Developing the properties of bifuzzy ideals of BCK/BCI algebras.
- Finding useful results on other structures of bifuzzy theory of ideals of BCK/BCI algebras.
- Constructing the related logical properties of such structures.
- May also apply this concept to study some applications in many fields such as artificial intelligence, signal processing, multiagent system, pattern recognition, robotics, computer networks, expert system, decision making, automata theory, and medical diagnosis.

Bibliography

1. M Akram. "Bifuzzy ideals of K -algebras". *WSEAS Transactions on Mathematics* 7.5 (2008): 313-322.
2. K Iséki and S Tanaka. "Ideal theory of BCK-algebras". *Japan Mathematics Journals* 21 (1976): 351-366.
3. K Iséki and S Tanaka. "An introduction to the theory of BCK-algebras". *Japan Mathematics Journals* 23 (1978): 1-26.
4. T Gerstenkornand and J Manko. "Bifuzzy probabilistic sets". *Fuzzy Sets and Systems* 71 (1995): 207-214.
5. Y B Jun and J Meng. "BCK-algebra, by Kyung Moon". SACo SeoulKorea (1994).

6. Y B Jun., *et al.* "Fuzzy Implicative ideals of BCK – algebras". *Fuzzy sets, and systems* 89 (1997): 243-248.
7. Muhiuddin G., *et al.* "Implicative N - ideals of BCK - algebras based on Neutrosophic N – structures". *Discrete Mathematics, Algorithms, and Applications* 11.1 (2019): 1950011.
8. G Muhiuddin and YB Jun. "Further Results of Neutrosophic-subalgebras in BCK/BCI -algebras based on Neutrosophic points". *TWMS Journal of Applied and Engineering Mathematics* 10.2 (2020): 232-240.
9. G Takeutis Titans. "Intuitionistic fuzzy logic and Intuitionistic fuzzy set Theory". *Journal Symbolic Logic* 49 (1984): 851-866.
10. G Xi. "Fuzzy BCK – algebras". *Japan Mathematics Journals* 36 (1991): 935-942.
11. L A Zadeh. "Fuzzy sets". *Inform. and Control* 8 (1965): 338-353.

Volume 3 Issue 9 September 2021

© All rights are reserved by Mahasin A Ahmed and Esmat A Amhed.