Acta Scientific COMPUTER SCIENCES

Volume 3 Issue 6 June 2021

Case Study

# Analysis of Inertial Torque Acting on a Rotating Body

**Ryspek Usubamatov***

*Kyrgyz State Technical University After I. Razzakov, Bishkek, Kyrgyzstan*

***Corresponding Author:** Ryspek Usubamatov, Kyrgyz State Technical University After I. Razzakov, Bishkek, Kyrgyzstan.

**Received:** April 15, 2021

**Published:** May 06, 2021



## Abstract

The mathematical model for the rotation of a body about a fixed point is a classical topic of physics mechanics. Physics considers planar rigid-body motion that is presented by rotation about a fixed point and its curvilinear motion. Analysis of the motions demonstrates the object subjected by the radial and angular accelerations and hence the action of the inertial force and torques. The textbooks of classical mechanics do not describe the angular acceleration of objects and are confined only by the radial one. The angular acceleration of objects generates the inertial torques acting on the rotating object. Engineering practice requests exact computing of inertial forces and torques acting on the mechanical components for the quality of machine work. This short communication considers the physical interpretation of the angular acceleration of a rotating body about the fixed point that generates the inertial torques acing around its center mass and fixed point.

**Keywords**: Radial and Angular Acceleration; Inertial Force and Torque; Rotating Body

## Introduction

All textbooks of engineering mechanics contain the chapter that considers the rotation of the body with the constant angular velocity about a fixed axis and about the fixed point when the body's axis is offset on some distance r [1-7]. The mathematical model for the rotation of the body about a fixed axis is a follows: $J_b\varepsilon = T$ where $J_b$ (kg·m$^2$) is the mass moment of inertia of the body, ε (rad/s$^2$) is the angular acceleration, and T (N·m$^2$) is the external torque acing on the rotating body. The left side of this expression Jε presents the inertial torque.

Analysis of the rotating body about the fixed point yields the radial inertial acceleration and hence the radial force that depends on the values of the angular velocity and radius of rotation.

The radial acceleration of rotating body about the fixed point is presented by the following equation $a = r\omega^2$ where a is the radial acceleration, *r* is the radius of rotation of the body relatively of a fixed point of rotation and $\omega$ is the constant angular velocity of its rotation about a fixed point and the action of the external torque [8-15]. The left and right of the expression is components of the centrifugal and centripetal forces respectively according the textbooks. The textbooks present the centrifugal force that is $F_{ct} = ma$ (kg·m/s$^2$), where m is the mass and a is the radial acceleration of the rotating body about the fixed point. The centripetal force is presented by the following: $F_{cp} = mr\omega^2$ (kg·m/s$^2$), where all components are as specified above. Two components of the expression have different physical interpretations, i.e., there is dual presentations of the one physical term. Such dualism contradicts mathematical logic and mathematics that is exact science and does not allow ambiguous interpretations. The practice of the rotation of the body about the fixed point demonstrates its turn about its own center mass and hence the action of the inertial torque. This short communication describer the physics of the turn of the body about its center mass at the process of rotation about the fixed point.

## Methodology

Analysis of the radial acceleration ($a = r\omega^2$) yields the following: the angular velocity ω presents the scalar product of the angular velocity that express the angular acceleration, i.e., $\omega^2 = \varepsilon$. This angular acceleration does not relate to the rotation of the body about the fixed point as far as it rotates with the constant angular velocity ω about the fixed point. Logically, the angular acceleration ε relates only to the rotation of the body about its center mass that locates on the distance r from its fixed point of rotation. The angular acceleration ε of the body means it rotates about its center mass and subjected to the action of the external torque $T_b$ that expresses by the following: $T_b = J_b\varepsilon$, where $J_b$ is the mass moment of inertia of the body about its center mass.

The second mathematical proves to establish the proposed solution is presented by the following. The textbooks equation of the circular motion of the body under the action of the external torque T that rotates it about the fixed point is as follows: $T = J_\varepsilon = (J_b + mr^2)\varepsilon$, where $J = J_b + mr^2$ is defined by the parallel axis theorem, other parameters are as specified above. This equation can be expressed by the following: $T_b = J_b\varepsilon + mr^2\varepsilon$, where the first item of the right side is $T_b = J_b\varepsilon$ that is the torque rotating the body about the center mass. The second item is presented by the following expression: $T_F = mr(r\varepsilon) = ma_t r = Fr$, where $a_t = r\varepsilon$ is the tangential acceleration of the rotating body, $F = ma_t$ is the inertial force, and $T_F$ is the torque acting on the body about the fixed point.

The rotating body about the fixed point is subjected to the action of the centrifugal force $T_{ct}$ of the radial direction, the inertial torque $T_b$ that turns the body about its center mass, and inertial torque $T_F$ acting on the body about the fixed point. These inertial torques were missed from consideration by the physics that present the fundamental principles of classical mechanics. All rotating bodies always turn around their center mass with the angular velocity of their rotation about the fixed point. This statement is validated by the circular motion of the moon that always shows its one side toward the earth.

## Case Study

The disc of the radius 0.02 m, the mass of 0.1 kg that located on the length 0.4 m from the fixed point rotates with the constant angular velocity of 5 rad/s. Determine the values of the centrifugal force and the inertial torque acting on the disc.

## Solution

The value of the centrifugal force is as follows:

$F = ma = mr\omega^2 = 0{,}1 \times 0{,}4 \times 5^2 = 1{,}0$ N

The disc turns around its center mass under the action of the inertial torque of the value:

$$T = J\omega^2 = (ml^2/2 + mr^2)\omega^2 = (0{,}1\times 0{,}02^2/2 + 0{,}1\times 0{,}4^2)\times 5^2 = 0{,}4005 \text{ Nm}$$

Where $J$ is the disc mass moment of inertia about the fixed point.

## Results and Discussion

The mathematical models for the radial and angular accelerations of the rotating body about a fixed point are used for computing the inertial forces and torques. An analysis of the rotation of the body around the fixed point demonstrated, apart from the centrifugal force, on the body are acting the inertial torques that turn one about the center mass and fixed point at the process of its circular motion. The conducted analysis explains the physics of the inertial torques acting on the body rotating around the fixed point that validated by practice. The inertial torques are generated by the kinetic energy of the body's rotation. These inertial torques do not describe in the textbooks and present the missed components in the analytical approach and interpretation of the mathematical models for the rotating body.

## Conclusion

The textbooks of engineering mechanics consider the action only of the centrifugal force on the body rotating about the fixed point. The detailed analysis of the rotation of the body around a fixed point revealed the action of the new two inertial torques. The first inertial torque turns the body about the center mass and the second one acting on the body about the fixed point. These inertial torques are generated by the kinetic energy of the body with circular motion. The physics mechanics missed the two inertial torques acting on a rotating body about the fixed point. These inertial torques can be considered as the fundamental principles of classical mechanics. The new inertial torques should be presented in the textbooks and used by practitioners in engineering.

## Bibliography

1. Taylor JR. "Classical Mechanics". Sausalito CA: University Science Books (2004).

2. Gregory DR. “Classical Mechanics”. Cambridge University Press, New York (2006).

3. Hibbeler RC. “Engineering Mechanics - Statics and Dynamics”. 12th ed. Prentice Hall, Pearson, Singapore (2010).

4. Dreizler R. “Theoretical Mechanics: Theoretical Physics 1”. Springer, Boca Raton (2010).

5. Jewett J and Serway RA. “Physics for Scientists and Engineers, Cengage Learning”. 10 ed. USA, Boston (2018).

6. Smith P F and Longley WR. “Theoretical Mechanics”. Franklin Classics, Boston (2018).

7. Germain P., *et al*. “Theoretical and Applied Mechanics”. Elsevier (2012).

8. Nolting W. “Theoretical Physics 1: Classical Mechanics”. Springer, Cham, Switzerland (2016).

9. Aardema MD. “Analytical Dynamics”. Theory and Application. Academic/Plenum Publishers, New York (2005).

10. Thornton M. “Classical dynamics of particles and systems”. (5th ed.). Brooks/Cole (2004).

11. Baumann G. Mathematica for Theoretical Physics, Springer, New York (2003)

12. Millard B F. “Principles of Engineering Mechanics: Dynamics-The Analysis of Motion”. Springer (2006).

13. Kevin Russell., *et al*. “Kinematics and Dynamics of Mechanical Systems”. 2nd ed.: Implementation in MATLAB and Sim Mechanics, Taylor and Francis, Boca Raton (2019).

14. Müller-Kirsten Harald JW. “Classical Mechanics and Relativity”. *World Scientific* (2008).

15. Knauf, Andreas. “Mathematical Physics: Classical Mechanics”. Springer (2018).

16. https://www.scientificamerican.com/article/is-it-just-a-coincidence/

**Volume 3 Issue 6 June 2021**