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Review Article

Theoretical Mathematical Modeling of Heat Transfer in Straight Round Rough Pipes Based on a Modified Three-Layer Model of Turbulent Boundary Layers

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Abstract

Objectives: A method of theoretical computational determination of hydraulic resistance and heat transfer for round pipes with rough walls based on multilayer models of a turbulent boundary layer is developed, which differs significantly from existing theories by taking into account the proportion of the volume of depressions in the sublayer and the ratio of rough and smooth channel surfaces.

Method: The calculation was carried out on the basis of a theoretical method based on a three-layer modified mathematical model of a turbulent boundary layer, which was complicated by taking into account the ratio of smooth and rough surfaces of the pipe surface, as well as the introduction of the volume coefficient of depressions, reflecting the proportion of the volume of depressions in the sublayer.

Result: The results of calculating the integrated heat transfer for round rough pipes for an extended range of determining parameters that correspond well to the existing experiment are obtained. The maximum differences between the calculated data obtained by this method and the calculated data obtained by the method typical for pipes with turbulators Nu (10)/Nu [16-24], with Reynolds numbers greater than the critical Reqr, characteristic of developed roughness modes, are quite noticeable and are: h/R_0 = 0.016: -17.1%; h/R_0 = 0.020: -14.4%; h/R_0 = 0.027: -12.3%; h/R_0 = 0.028: -13.4%; h/R_0 = 0.033: -13.0%; h/R_0 = 0.037: -15.4%; h/R_0 = 0.042: -12.3%; h/R_0 = 0.043: -12.0%; h/R_0 = 0.046: -11.3%; h/R_0 = 0.048: -10.8%; h/R0 = 0.053: -10.8%; h/R_0 = 0.066: -7.2%; h/R_0 = 0.073: -10.0%; h/R_0 = 0.078: -5.7%; h/R_0 = 0.107: -4.9%; h/R_0 = 0.160: -11.7%.

Conclusion: The obtained results of the calculation of the intensified heat transfer for round rough pipes for an extended range of determining parameters correspond well to the existing experiment. It is shown that the proposed model describes heat transfer for rough pipes more accurately than the theory typical for pipes with turbulators, for example [16-24], since it takes into account the specific features of heat transfer in pipes with rough walls, while the theory for pipes with turbulators was developed for conditions when the distances between turbulators are large enough. It is revealed that the advantage of the proposed specific heat transfer model for rough pipes over the model for pipes with turbulators is especially noticeable in the developed roughness mode. The results obtained for calculating the average heat transfer for round rough pipes for an extended range of determining parameters differ significantly from the corresponding data for smooth round pipes, but indirectly indicate the level of heat transfer intensification due to the use of rough pipes instead of smooth ones. The developed specific method of theoretical computational determination of averaged heat transfer for round pipes with rough walls based on three-layer modified mathematical modeling of a turbulent boundary layer differs significantly from existing theories and it must be applied in the calculation of intensified heat transfer for these conditions, despite the certain higher complexity. but indirectly indicate the level of heat transfer intensification due to the use of rough pipes instead of smooth ones. The developed specific method of theoretical computational determination of averaged heat transfer for round pipes with rough walls based on three-layer modified mathematical modeling of a turbulent boundary layer differs significantly from existing theories and it must be applied in the calculation of intensified heat transfer for these conditions, despite the certain higher complexity. but indirectly indicate the level of heat transfer intensification due to the use of rough pipes instead of smooth ones. The developed specific method of theoretical computational determination of averaged heat transfer for round pipes with rough walls based on three-layer modified mathematical modeling of a turbulent boundary layer differs significantly from existing theories and it must be applied in the calculation of intensified heat transfer for these conditions, despite the certain higher complexity.

Keywords: Analytical; Intensification; Channel; Mathematical; Modeling; Model; Relative; Boundary Layer; Scheme; Heat Transfer; Pipe; Turbulent; Turbulator; Three-Layer

Introduction

The relevance of studying the patterns of flows in rough pipes is based on the fact that roughness can be an intensifier of heat transfer. Revealing the theoretical regularities of flow and heat transfer in a turbulent regime in rough tubes is important, since these channels are used in heat exchangers and devices used in various fields of technology.

In this work, heat transfer in straight round pipes with rough walls is theoretically investigated, the standard defined as a set of surface irregularities (for example, a system of protrusions and depressions) with relatively small steps, identified using the base length [15]. Pipes with turbulators, where the steps between the turbulators are relatively large, are not studied in this work.

The distribution of roughness irregularities in the pipe is assumed to be uniform, and the distances between the protrusions are relatively small, so they have a mutual influence.

It is known that the surface roughness has practically no effect on the flow when the thickness of the viscous sublayer is greater than the height of the ridges. On the contrary, for the developed roughness regime, characterized by the self-similarity of the hydraulic resistance coefficient on the Reynolds number, the height of the protrusions is much greater than the thickness of the viscous sublayer. In some experimental works, the generation of isolated vortex-like flows in the basins is observed [1]. If we consider the flow around a single turbulizer, then at low Reynolds numbers a continuous flow is observed; with an increase in the Reynolds number, a single vortex is generated, which loses its stability and becomes turbulent with a further increase in the Reynolds number. Due to turbulent mixing, the vortexes diffuse into the core of the flow.

A similar picture takes place in the trough of roughness: at low Reynolds numbers, the flow near a rough wall practically does not differ from a smooth one, since the height of the protrusions is less than the viscous sublayer. With an increase in the Reynolds number, the thickness of the viscous sublayer decreases and becomes comparable with the height of the ridges. The flow occurs at a speed comparable to the average flow rate; therefore, a vortex is generated in the trough, which characterizes the onset of a transient flow regime. With a further increase in the Reynolds number, perturbations in the outer core of the flow increase, which penetrate into the trough, the flow in the trough becomes unstable, which characterizes the developed turbulent flow regime.

The turbulent core of the main stream [1] is characterized by an increase in the intensity of pulsations in the presence of roughness. The maximum pulsations are in the region of the tops of the protrusions and are tenths of the average flow velocity (the pulsation velocity normal to the wall can even be half the average velocity in the channel [1]). Consequently, transverse pulsations penetrating into the depressions of roughness will intensively stir the coolant in them. The flow of the coolant from the core of the flow to the vortex zones between the roughness ridges, as well as in the opposite direction, can be unstable and take place in the form of random turbulent exchange.

In the flow core, the scale of perturbations is of the order of the characteristic size of the channel, and the intrinsic pulsations in the trough are of the order of the trough height. Consequently, if the height of the depression is small in comparison with the characteristic size of the channel, then the mixing processes in the depression and the heat transfer processes are determined mainly by external pulsations, due to which the coolant will be ejected from the roughness cavity. Based on this, the viscous sublayer in the depression is not formed from the averaged flow in it, but is determined by the interaction of external pulsations with the wall. If the height of the protrusion is comparable to the size of the channel, then the mixing processes in it are determined not only by external, but also by their own pulsations; moreover, in the depth of the roughness trough in the immediate vicinity of the wall, the influence of own pulsations is decisive.

Experimental and theoretical studies of the patterns of flow and heat transfer in rough pipes indicate that they are fundamentally different from the corresponding patterns for tubes with turbulators ([1] and [2-6], respectively).

The existing theoretical studies of flow and heat transfer in rough pipes are based on the use of a logarithmic velocity profile, which greatly simplifies the mathematical model, which leads to additional discrepancies for large relative (relative to the pipe diameter) roughness.

A large relative roughness can be realized, for example, in pipes with small diameters, which can be compared with the corresponding conditions for pipes of small diameters with turbulators [7].

Mathematical modeling of flow and heat transfer in rough pipes was carried out in a relatively small number of studies (a fairly complete list of works on this topic is contained in monographs [8-10]), which do not go beyond the logarithmic velocity profile. In this study, more complex patterns of heat transfer for rough pipes were obtained than existing ones, therefore they are more justified and can be used for a wider determining range. Earlier, in a theoretical study of heat transfer for pipes with turbulators [2-6], more complex dependencies were obtained than when using a logarithmic velocity profile.

The available experimental studies of the flow and heat transfer in rough channels indicate that, with relatively large roughness ridges, the turbulent flow is significantly different with respect to the flows in smooth-tube channels.

In experimental studies of flow and heat transfer in rough pipes, which are analyzed in [11-13], four flow regimes are distinguished:

1) What occurs according to Poiseuille's law at low Reynolds numbers Re, self-similar to the roughness height - laminar flow;
2) Occurring according to the law of hydraulic resistance for smooth pipes at intermediate Reynolds numbers Re - turbulent flow;
3) Occurring according to the law of hydraulic resistance, which is a function of the intermediate Reynolds numbers Re and relative roughness $\overline{h} = h/R_0$(the ratio of the average height of the roughness protrusions to the radius of the pipe; $D = 2R_0$ is the larger inner diameter of the pipe) - turbulent flow;
4) Occurring according to the law of hydraulic resistance, which is a function of only the relative roughness at high Reynolds numbers, Re is a self-similar flow.

In the case of a large relative roughness height, the elimination of the region with a turbulent regime takes place, the regularity for which is characteristic of smooth pipes; a similar elimination also takes place for pipes with turbulators [1-6].

Heat transfer during flow in channels of coolants with constant thermophysical properties for conditions of intensified heat transfer in straight round rough pipes is modeled by a multilayer scheme of a turbulent boundary layer based on the fact that the value of turbulent viscosity and the velocity profiles of the turbulent boundary layer are already deterministic.

A similar scheme for calculating the intensified heat transfer was used in [2-6] to calculate heat transfer in pipes with turbulators, which makes it possible to use it in the future when calculating heat transfer in pipes with rough walls, subject to the appropriate restrictions [2-6], since the process conditions heat transfer are similar.

The solution to the problem of intensified heat transfer in this work is obtained using the Lyon integral when making the assumption (is the ratio of the axial component of the velocity to the average flow rate) which, as shown by theoretical studies [2-6] for round pipes and channels of non-circular cross-section with turbulators, has little effect on averaged intensified heat transfer: $w/\overline{w}_x \cong 1$.

$$Nu = \frac{2}{\int_0^1 \left(\frac{R^3}{1+\frac{Pr}{Pr_T}\frac{\mu_T}{\mu}} \right) dR}, (1)$$

where R = r/R_0 is the dimensionless pipe radius (the ratio of the distance from the pipe axis r to the pipe radius R_0); Pr and Pr_T - molecular and turbulent Prandtl numbers; Nu is the Nusselt number;μ and μ_T - molecular and turbulent dynamic viscosities.

Unlike a smooth pipe, in a rough pipe, the thickness of the viscous sublayer will be inconsistent across the surface of the ridges and depressions. Therefore, it is necessary to enter the average thickness of each sub-layer. The heat flux density over the thickness of the viscous sublayer can be considered almost constant.

Let us introduce the following notation: n_F = Fsm/Ft (F_{sm} and F_t are the areas of smooth and rough surfaces, respectively).

For a viscous sublayer, the heat flux density q is: q = q_w (F_{sm}/F_t) = $q_w n_F$ (is the density of the heat flux into the wall), since the thickness of the viscous sublayer is infinitely small compared to the height of the rib.

For a heat flow in a depression, the heat flow density qvp depends on the shape of the depression and is variable over the depth of the depression. In practice, it can be assumed that heat transfer takes place at a constant heat flux through a flat layer from the thickness of the viscous sublayer to the sum of the turbulator height and the thickness of the viscous sublayer.

At the boundary with the flow core, the heat flow density is equal to:

q_w/[1 - (h + h_v)/R)], where hv is the thickness of the viscous sublayer.

At the interface with a viscous sublayer, the heat flux density is equal to:

q_w (F_{sm}/F_t).

The average of the above values can be taken as the density of the heat flow in the depression, namely:

$$\frac{q_w}{2}\left(\frac{1}{1-\frac{h+h_t}{R}} + \frac{F_{sm}}{F_t} \right).$$

Since only part of the surface is occupied by the depressions, and in the considered sublayer only part of the volume of this sublayer is accounted for by the depressions, the smaller the depressions on the heat exchange surface, the smaller their volume, the less thermal resistance falls on the depressions; and vice versa: the more depressions on the heat exchange surface, the larger their volume, the greater the contribution to the total thermal resistance falls on the depressions. The above change in the thermal resistance of the depressions can be taken into account by introducing the volume coefficient n_V, which reflects the fraction of the volume of the depressions in the sublayer.

Coefficients n_F and n_V for rough pipes are calculated either on the basis of known geometric parameters of roughness, or from processing pipe profilograms.

For example, for a roughness in the form of a metric thread, the above factors are: n_F=0.58 and n_V = 0.50.

If we refer to the heat transfer coefficient to a smooth surface, then, and if to a rough surface, then (is the average total temperature head): $\alpha_{sm} = \frac{q_w}{\Delta\overline{T}} \alpha_t = \frac{q_w}{\Delta\overline{T}}\frac{Fsm}{Ft} = \frac{q_w}{\Delta\overline{T}} n_F \Delta\overline{T}$.

The dynamic velocity ("friction velocity") for a rough surface in this case will differ from the corresponding value for a smooth surface: $w_*^t = w_*^{sm}\sqrt{n_F}$.

Let's consider some empirical relationships for hydraulic resistance for rough pipes.

For pipes with roughness with relatively small heights of protrusions, the asymptotic behavior of the hydraulic resistance coefficient is described by the well-known empirical Nikuradze dependence:

$$\xi = \frac{1}{\left\{1{,}74+2\lg\left[\frac{1}{\left(\frac{h}{R_0}\right)}\right]\right\}^2} (2).$$

Dependence of the hydraulic resistance coefficient for rough pipes not only on the relative roughness, but also on the number Reynolds $\xi = f\left(\frac{h}{R_0}; Re\right)$, is described empirically best by the Colebrook formula, which can be written as follows:

$$\frac{1}{\sqrt{\xi}} = 1{,}74 - 2\lg\left(\frac{18{,}7}{Re\sqrt{\xi}} + \frac{h}{R_0}\right) (3).$$

Consequently, in the empirical relations for the coefficient of hydraulic resistance during flow in rough pipes, a logarithmic velocity profile is used.

Three-layer modified mathematical modeling of heat exchange in pipes with rough walls

The values of the hydraulic resistance in straight round rough surfaces should be used to calculate the heat transfer for these intensification conditions, since the flow stratification depends on the hydraulic resistance.

Heat transfer during flow in channels of coolants with constant thermophysical properties for conditions of intensified heat transfer in straight round rough pipes is modeled by a multilayer scheme of a turbulent boundary layer on the basis that the value of the turbulent viscosity and the velocity profiles of the turbulent boundary layer are assumed to be deterministic.

Now we should proceed to a direct detailed examination of each of the above sublayers.

Viscous sublayer

The viscous sublayer is located in the following neighborhood: $R\epsilon\left[1 - \frac{\eta_1}{Re\sqrt{n_F}}\sqrt{\frac{32}{\xi}} - -\frac{h}{R_0}; 1\right]\eta_1 = 5$, where is a constant characterizing the dimensionless thickness of the viscous sublayer [14].

In the region of the viscous sublayer, it is assumed that:

$$\frac{\mu}{\mu_T} = \beta\frac{\eta^3}{\eta_1^2} = \frac{\beta}{\eta_1^2}Re^3 n_F^{\frac{3}{2}}(1-R)^3\left(\frac{\xi}{32}\right)^{\frac{3}{2}}; (4)$$

$$\frac{w_x}{\bar{w}_x} = \frac{\xi}{16}Re\sqrt{n_F}(1-R), (5)$$

Whereμ/μ_T- the ratio of turbulent and molecular dynamic viscosities; $w_x/\bar{w}_x$- the ratio of the axial component of the speed to the average consumption; $\eta = (1-R)^3 Re\sqrt{n_F}\sqrt{\frac{\xi}{32}}$ - dimensionless coordinate; β - constant in the law of "third degree": $\frac{\mu}{\mu_T} = \beta\frac{\eta^2}{\eta_1^2}$ [14].

Intermediate subcoat

The intermediate sublayer is located in the following neighborhood: $R\epsilon\left[1 - -\frac{\eta_2}{Re\sqrt{n_F}}\sqrt{\frac{32}{\xi}} - \frac{h}{R_0}; 1 - \frac{\eta_1}{Re\sqrt{n_F}}\sqrt{\frac{32}{\xi}} - \frac{h}{R_0}\right]\eta_1 = 30$, where is a constant characterizing the dimensionless thickness of the buffer (intermediate) sublayer [14].

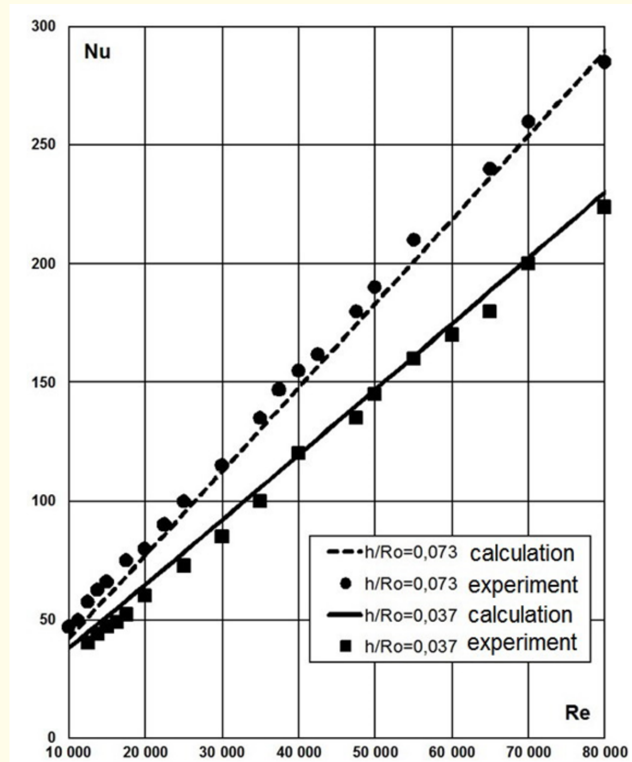

Figure 1: Comparison of theoretical data on heat transfer in rough pipes with experiment [1] for h/R_0 = 0.037 (lower values) and h/R_0 = 0.073 (upper values) for different Reynolds numbers Re = $10^4 \div 10^5$.

In the area of the intermediate sublayer, it is assumed that:

$$\frac{\mu}{\mu_T} = \frac{\eta}{5} - 1 = \frac{1}{5}(1-R)Re\sqrt{n_F}\sqrt{\frac{\xi}{32}} - 1; (6)$$

$$\frac{w_x}{\bar{w}_x} = 5\sqrt{\frac{\xi}{8}}\left(1 + \ln\left(\frac{\eta}{5}\right)\right) = 5\sqrt{\frac{\xi}{8}}\left(1 + \ln\left(\frac{1}{5}(1-R)Re\sqrt{n_F}\sqrt{\frac{\xi}{32}}\right)\right); (7)$$

Turbulent core

The turbulent core is located in the following neighborhood:

$$R\epsilon\left[0; 1 - -\frac{\eta_2}{Re\sqrt{n_F}}\sqrt{\frac{32}{\xi}} - \frac{h}{R_0}\right].$$

In the region of the turbulent core, it is assumed that:

$$\frac{\mu}{\mu_T} = \frac{2}{5}R(1-R)Re\sqrt{\frac{\xi}{32}};\ (8)$$

$$\frac{w_x}{\bar{w}_x} = \left(1{,}325\sqrt{\xi}+1\right)(1-R)^{\sqrt{\xi}}.\ (9)$$

In accordance with the given flow stratification, the averaged heat transfer (1) will be as follows:

$$\mathrm{Nu} = 2\left[\int_{1-\frac{\eta_1}{Re\sqrt{n_F}}\sqrt{\frac{32}{\xi}}-\frac{h}{R_0}}^{1}\left(\frac{R^3}{1+\frac{Pr}{Pr_T}\frac{\beta}{\eta_1^2}Re^3 n_F^{\frac{3}{2}}(1-R)^3\left(\frac{\xi}{32}\right)^{\frac{3}{2}}}\right)dR+\right.$$

$$+n_V\frac{\left(n_F+\frac{1}{1-\frac{\eta_1}{Re\sqrt{n_F}}\sqrt{\frac{32}{\xi}}-\frac{h}{R_0}}\right)}{2}\int_{1-\frac{\eta_2}{Re\sqrt{n_F}}\sqrt{\frac{32}{\xi}}-\frac{h}{R_0}}^{1-\frac{\eta_1}{Re\sqrt{n_F}}\sqrt{\frac{32}{\xi}}-\frac{h}{R_0}}\left(\frac{R^3}{1+\frac{Pr}{Pr_T}\left(\frac{1}{5}(1-R)Re\sqrt{n_F}\sqrt{\frac{\xi}{32}}-1\right)}\right)dR+\ (10)$$

$$\left.+\int_0^{1-\frac{\eta_2}{Re\sqrt{n_F}}\sqrt{\frac{32}{\xi}}-\frac{h}{R_0}}\left(\frac{R^3}{1+\frac{Pr}{Pr_T}\frac{2}{5}R(1-R)Re\sqrt{n_F}\sqrt{\frac{\xi}{32}}}\right)dR\right]^{-1}.$$

It is possible to obtain analytical dependences for integrals (10) in exactly the same way as in works [16-24], in which the problem of heat transfer for tubes with turbulators was solved.

Calculations of heat transfer for rough pipes with roughness in the form of a triangular thread, characteristic of [1], were carried out using the above method. The maximum differences between the calculated data obtained by this method and the calculated data obtained by the method typical for pipes with Nu (10)/Nu turbulators [16-24], at Reynolds numbers large for the critical Recr, characteristic of developed roughness modes, are quite noticeable and are: h/R_0 = 0.016: -17.1%; h/R_0 = 0.020: -14.4%; h/R_0 = = 0.027: -12.3%; h/R_0 = 0.028: -13.4%; h/R_0 = 0.033: -13.0%; h/R_0 = 0.037: -15.4%; h/R_0 = 0.042: -12.3%; h/R_0 = 0.043: -12.0%; h/R_0 = 0.046: -11.3%; h/R_0 = 0.048: -10.8%; h/R_0 = 0.053: -10.8%; h/R_0 = 0.066: -7.2%; h/R_0 = 0.073: -10.0%; h/R_0 = 0.078: -5.7%; h/R_0=0.107: -4.9%; h/R_0 = 0.160: -11.7%.

Thus, the introduction of surface roughness corrections nF and nV into the model with a correspondingly changed flow stratification to a certain extent clarifies the values of the Nusselt number for rough pipes in comparison with the method typical for pipes with turbulators [16-24], which determines its preferential use (especially for the developed roughness regime) for these specific conditions.

As an example, consider the comparison of this theory with experiment, given in [1]. For roughness in the form of a triangular thread with h/R_0 = 0.037, n_F = 0.58, n_V = 0.50, Pr = 0.7, Re = 87300, the Nusselt number is Nu = 251; the calculation according to the proposed method with the determination of the hydraulic resistance according to the formula (2) gives a value of 250.4, which corresponds to the experiment with an error of a quarter of a percent. Comparison of theory with experiment [1] for h/R_0 = 0.037 (lower values) and h/R_0= 0.073 (upper values) is shown for different Reynolds numbers in figure 1, which shows a very good correlation between them.

Consequently, the proposed theory more accurately describes heat transfer for rough pipes than the theory characteristic of pipes with turbulators [16-24], due to the specific stratification of the turbulent boundary layer (somewhat different from the stratification in pipes with turbulators), as well as due to the introduction of corrections to change the surface of rough pipes.

Key findings

1. A method of theoretical computational determination of intensified heat transfer for round pipes with rough walls based on a three-layer modified mathematical modeling of a turbulent boundary layer has been developed.
2. The predominant specific difference of this technique from the existing one, which is typical for pipes with turbulators, consists in a modified flow stratification, taking into account the change in the friction rate for rough pipes.
3. To calculate heat transfer in rough pipes, the three-layer modified mathematical model of the turbulent boundary layer was complicated by taking into account the ratio of smooth and rough areas of the pipe surface, as well as by introducing the volume factor of the depressions, which reflects the fraction of the volume of the depressions in the sublayer.
4. The obtained results of the calculation of the intensified heat transfer for round rough pipes for an extended range of determining parameters are in good agreement with the existing experiment.
5. It is shown that the proposed model more accurately describes heat transfer for rough pipes than the theory typical for pipes with turbulators, for example [16-24], since it takes into account the specific features of heat transfer in pipes with rough walls, while the theory for pipes with turbulators

was developed for conditions where the distance between the turbulators is large enough.

6. It has been revealed that the advantage of the proposed specific model of heat transfer for rough pipes over the model for pipes with turbulators is especially noticeable in the regime of developed roughness.
7. The obtained results of calculating the averaged heat transfer for round rough pipes for an extended range of governing parameters differ significantly from the corresponding data for smooth round pipes, but indirectly indicate the level of heat transfer intensification due to the use of rough pipes instead of smooth ones.
8. The developed specific method of theoretical computational determination of averaged heat transfer for round pipes with rough walls based on a three-layer modified mathematical modeling of a turbulent boundary layer mainly differs from existing theories and must be used in calculating intensified heat transfer for these conditions, despite its definitely higher complexity.

Conclusion

The obtained results of the calculation of the intensified heat transfer for round rough pipes for an extended range of determining parameters correspond well to the existing experiment.

Bibliography

1. EK Kalinin., *et al*. "Effective heat transfer surfaces". Moscow: Energoatomizdat (1998): 408.
2. Lobanov IE. "Mathematical modeling of intensified heat transfer in turbulent flow in channels". Dissertation for the degree of Doctor of Technical Sciences. M (2005): 632.
3. Lobanov IE and Stein LM. "Perspective heat exchangers with intensified heat exchange for metallurgical production. (General theory of intensified heat transfer for heat exchangers used in modern metallurgical production)". In 4 volumes. Volume I. Mathematical modeling of intensified heat transfer in turbulent flow in channels using basic analytical and numerical methods. M: Publishing House of the Association of Construction Universities (2009): 405.
4. Lobanov IE and Stein LM. "Perspective heat exchangers with intensified heat exchange for metallurgical production. (General theory of intensified heat transfer for heat exchangers used in modern metallurgical production)". In 4 volumes. Volume II. Mathematical modeling of intensified heat transfer in turbulent flow in channels using non-basic analytical and numerical methods. M: Publishing House of the Association of Construction Universities (2010): 290.
5. Lobanov IE and Stein LM. "Perspective heat exchangers with intensified heat exchange for metallurgical production. (General theory of intensified heat transfer for heat exchangers used in modern metallurgical production)". In 4 volumes. Volume III. Mathematical modeling of intensified heat transfer in turbulent flow in channels using multilayer, super-multilayer and compound models of a turbulent boundary layer. M: Mgakhis (2010): 288.
6. Lobanov IE and Stein LM. "Perspective heat exchangers with intensified heat exchange for metallurgical production. (General theory of integrated heat transfer for heat exchangers used in modern metallurgical production)". In 4 volumes Volume IV. Special aspects of mathematical modeling of hydrogas dynamics, heat transfer, and heat transfer in heat exchangers with intensified heat exchange. M: Mgakhis (2011): 343.
7. Lobanov IE and Dotsenko AI. "Mathematical modeling of limiting heat transfer for turbulized flow in channels". *M Mikkhis* (2008): 194.
8. Ievlev VM. "Numerical modeling of turbulent flows". Moscow: Nauka (1990): 215.
9. Lyakhov VK. "The method of relative correspondence in the calculations of turbulent near-night flows". Saratov: Saratov University Press (1975):123.
10. Lyakhov VK and Migalin VK. "Effect of thermal or diffusive roughness". Saratov: Saratov University Press, (1989): 176.
11. Millionshchikov MD. "Turbulent flows in the boundary layer and in pipes". Moscow: Nauka (1969): 52.
12. Millionshchikov MD. "Turbulent flows in the wall layer and in pipes" 28.3 (1970): 207-220.
13. Millionshchikov MD. "Turbulent heat and mass transfer in pipes with smooth and rough walls". *Atomic Energy* 31.3 (1971): 199-204.

14. Kutateladze SS. “Fundamentals of the theory of heat transfer”. M: Atomizdat (1979) 416.
15. Yakushev AI., *et al.* “Interchangeability, standardization and technical measurements”. Moscow: Mashinostroenie (1986): 352.
16. Lobanov IE. “Theory of intensified heat transfer in turbulent flow in channels based on a four-layer scheme of a turbulent boundary layer” 3 (2010): 81-89.
17. Lobanov IE and Stein LM. “Mathematical modeling of the intensified heat exchange at turbulent flow in pipes with turbulators for heat exchangers of modern metallurgical production using a four-layer model of a turbulent boundary layer” 3 (2010): 67-77.
18. Lobanov IE. “Theory of intensified heat transfer in turbulent flow in channels based on a four-layer scheme of a turbulent boundary layer for relatively high flow turbulators”. *Actual Problems of Modern Science* 6 (2010): 248-252.
19. Lobanov IE. “General theory of intensified heat transfer in turbulent flow in round pipes with turbulators using a four-layer model of a turbulent boundary layer” 5 (2011): 25-32.
20. Lobanov IE. “Mathematical modeling of intensified heat transfer in turbulent flow in pipes with turbulators for heat exchangers of modern metallurgical production using a four-layer model of a turbulent boundary layer”. *Almanac of Modern Science and Education* 9.52 (2011): 29-35.
21. Lobanov IE. “General theory of intensified heat transfer in turbulent flow in round pipes with high turbulators based on a four-layer model of a turbulent boundary layer”. *Moscow Scientific Review* 10 (2011): 10-15.
22. Lobanov IE. “Exact solution of the problem of intensified heat exchange in turbulent flow in channels with relatively low flow turbulators based on a four-layer scheme of a turbulent boundary layer” 2 (2012): 26-37.
23. Lobanov IE. “General theory of intensified heat transfer in turbulent flow in round pipes with relatively high turbulators using a four-layer model of a turbulent boundary layer”. *Branch Aspects of Technical Sciences* 10 (2013): 7-13.
24. Lobanov IE. “Four-layer theory of intensified heat transfer for pipes with relatively low flow turbulators”. *Branch Aspects of Technical Sciences* 11 (2012): 3-6.