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Review Article

Quantum Teleportation of a Tripartite Entangled Coherent State Using a Nonmaximally Entangled Quantum Channel

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Abstract

Various studies are based on successful teleportation of maximally entangled coherent states. In many cases it has been found that teleportation is not very successful with non-maximally entangled states giving very less fidelity. In the present paper, we propose a scheme to teleport a tripartite entangled coherent state using a non-maximally entangled 4-partite state as a quantum channel. Our scheme involves Bell State measurements and the use of beam splitters and phase shifters. We have separated the vacuum state from even states and find that, even by taking a non-maximally entangled quantum state, we can get an almost perfect teleportation for an appreciable mean number of photons. The average fidelity of teleportation is found to be almost equal to unity when the mean number of photons is 2.

Keywords: Entanglement; Fidelity; Photon Density; Teleportation

Introduction

Teleportation addresses the problem of transmission of quantum information over arbitrary distances using a classical channel. Quantum information is encoded [1] in states that exist in a continuous space. It follows that describing a quantum state completely would require an infinite amount of both time and classical bits. To circumvent this, Bennett., *et al.* [2] proposed a teleportation scheme that exploits entanglement [3]. The original scheme calls for two parties, Alice and Bob, to share an EPR state. In addition, Alice possesses an unknown qubit that she wishes to send to Bob. She performs a Bell-state measurement on her two particles: the unknown qubit and one half of the entangled pair. The measurement destroys Alice's quantum states and returns two classical bits. Bob's previously-entangled particle acquires information about Alice's unknown qubit and is now a unitary transformation away from it. Alice then transmits the classical bits to Bob using a classical channel. These help him pick a suitable transformation, using which he recovers the original state of Alice's unknown qubit. The procedure has since been demonstrated experimentally [4,5].

Van Enk and Hirota [6] suggested a scheme to teleport a Schrödinger cat state using bipartite entangled coherent states. Wang [7] generalised it to teleport a bipartite state using both maximally and non-maximally entangled states as quantum channels, and to teleport a tripartite state using a maximally entangled state as a channel. Prakash., *et al.* [8] modified Wang's scheme, for teleporting a bipartite state using a maximally entangled channel state and predicted nearly perfect teleportation for an appreciable mean number of photons. We extend Wang's scheme by teleporting a tripartite entangled coherent state using a non-maximally entangled channel. We also alter it by using the modified photon counting scheme used by N Chandra., *et al.* [9], which yields a better minimum average fidelity than the one used by Wang. Prakash and Mishra [10] used a non-maximally entangled channel to teleport a superposed coherent state. They also studied the effect of decreasing entanglement on average fidelity of teleportation [11] and reported an increase in average fidelity on using a non-maximally entangled channel to teleport superposed coherent states [12]. For this reason, we decided to investigate the usefulness of a non-maximally entangled quantum channel in teleporting a tripartite entangled coherent state.

In general, a pure state cannot be easily prepared, and generally mixed states are obtained. As in the case of pure states, we can divide entangled mixed states into two categories: (i) maximally entangled states which are the states with maximum entanglement and (ii) non-maximally entangled states i.e. the states which are not entangled.. The motivation for this work lies in performing a study of non-maximally entangled states which can be used in quantum teleportation.

Teleportation of a tripartite entangled coherent state

We consider two communicating parties, Alice and Bob. Alice is in possession of the information state

$$|I\rangle_{1,2,3} = \varepsilon_+ |\alpha,\alpha,\alpha\rangle_{1,2,3} + \varepsilon_- |-\alpha,-\alpha,-\alpha\rangle_{1,2,3}, \quad (1)$$

Existing in modes 1 - 3, where the complex coefficients obey the normalisation condition

$$|\varepsilon_+|^2 + |\varepsilon_-|^2 + 2e^{-6|\alpha|^2}\,\mathrm{Re}(\varepsilon_+^*\varepsilon_-) = 1. \quad (2)$$

The mean number of photons in a mode, calculated using the number operator, comes out to be

$$\bar{n} = |\alpha|^2 [1 - 4e^{-2|\alpha|^2}\,\mathrm{Re}(\varepsilon_+^*\varepsilon_-)] \quad (3)$$

And is nearly equal to $|\alpha|^2$ for $|\alpha|^2 > 1$. Alice and Bob are required to share a quantum channel, for which we pick the non-maximally entangled state

$$|E\rangle_{4,5,6,7} = \frac{1}{\sqrt{2(1-x^8)}}[|\alpha,\alpha,\alpha,\alpha\rangle_{4,5,6,7} - |-\alpha,-\alpha,-\alpha,-\alpha\rangle_{4,5,6,7}] \quad (4)$$

involving modes 4 – 7, where $x = e^{-|\alpha|^2}$. The initial global state of the combined system is given by

$$|\psi\rangle_{1,2,3,4,5,6,7} = |I\rangle_{1,2,3}|E\rangle_{4,5,6,7}, \quad (5)$$

Out of which modes 1 – 4 are with Alice. Bob has modes 5 – 7, which he will use to recover Alice's information state.

Alice uses a phase shifter on mode 2, changing $|\alpha\rangle_2$ to $|-i\alpha\rangle_8$. She then mixes the output with mode 1 using a lossless 50:50 beam splitter. The operation is denoted by $|\beta\rangle_1|\gamma\rangle_8 \rightarrow \left|\frac{\beta+i\gamma}{\sqrt{2}}\right\rangle_9 \left|\frac{\gamma+i\beta}{\sqrt{2}}\right\rangle_{10}$. The global state becomes

$$|\psi\rangle_{9,10,3,4,5,6,7} = \left[\varepsilon_+ |\sqrt{2}\alpha,0,\alpha\rangle_{9,10,3} + \varepsilon_- |-\sqrt{2}\alpha,0,-\alpha\rangle_{9,10,3}\right]|E\rangle_{4,5,6,7}. \quad (6)$$

The separable vacuum state $|0\rangle_{10}$ can be ignored, so that Alice is left with three modes: 9, 3 and 4. She uses a phase shifter on mode 3, which changes $|\alpha\rangle_3$ to $|-i\alpha\rangle_{11}$, and a second lossless 50:50 beam splitter to mix modes 11 and 4 thus: $|\beta\rangle_{11}|\gamma\rangle_4 \rightarrow \left|\frac{\beta+i\gamma}{\sqrt{2}}\right\rangle_{12} \left|\frac{\gamma+i\beta}{\sqrt{2}}\right\rangle_{13}$. Alice then uses a phase shifter on mode 12, which changes $|\alpha\rangle_{12}$ to $|-i\alpha\rangle_{14}$. The global state can be written as:

$$\begin{aligned}|\psi\rangle_{9,14,13,5,6,7} = \frac{1}{\sqrt{2(1-x^8)}}\Big[&\varepsilon_+ |\sqrt{2}\alpha,0,\sqrt{2}\alpha\rangle_{9,14,13}\,|\alpha,\alpha,\alpha\rangle_{5,6,7} \\ &- \varepsilon_+ |\sqrt{2}\alpha,-\sqrt{2}\alpha,0\rangle_{9,14,13}\,|-\alpha,-\alpha,-\alpha\rangle_{5,6,7} \\ &+ \varepsilon_- |-\sqrt{2}\alpha,\sqrt{2}\alpha,0\rangle_{9,14,13}\,|\alpha,\alpha,\alpha\rangle_{5,6,7} \\ &- \varepsilon_- |-\sqrt{2}\alpha,0,-\sqrt{2}\alpha\rangle_{9,14,13}\,|-\alpha,-\alpha,-\alpha\rangle_{5,6,7}\Big]. \end{aligned} \quad (7)$$

Alice is now required to perform a photon counting measurement on modes 9, 14 and 13. For this purpose, we shall rewrite her entangled state in a different measurement basis.

Wang used the $\{|EVEN,\alpha\rangle, |ODD,\alpha\rangle\}$ basis [13], where

$$|EVEN,\alpha\rangle = \frac{|\alpha\rangle + |-\alpha\rangle}{\sqrt{2(1+x^2)}} \text{ and } |ODD,\alpha\rangle = \frac{|\alpha\rangle - |-\alpha\rangle}{\sqrt{2(1-x^2)}}. \quad (8)$$

Using this, the information state (1) becomes

$$\begin{aligned}|I\rangle_{1,2,3} &= A_+ |EVEN,\alpha,\alpha,\alpha\rangle_{1,2,3} + A_- |ODD,\alpha,\alpha,\alpha\rangle_{1,2,3} \\ &= \cos\frac{\theta}{2} |EVEN,\alpha,\alpha,\alpha\rangle_{1,2,3} + \sin\frac{\theta}{2} e^{i\phi} |ODD,\alpha,\alpha,\alpha\rangle_{1,2,3}\end{aligned} \quad (9)$$

Where, $A_+ = \cos\frac{\theta}{2}$, $A_- = \sin\frac{\theta}{2}e^{i\phi}$, and the two scalar coefficients can be expanded as

$$A_\pm = (\varepsilon_+ \pm \varepsilon_-)\sqrt{\frac{1 \pm x^6}{2}}. \qquad (10)$$

For finer consideration, we shall use the modified measurement basis proposed by N. Chandra *et al.* [9]. In this basis, $|EVEN,\alpha\rangle$ can be expressed as a superposition of the vacuum state, $|0\rangle$, and the nonzero even photon state,

$$|NZE,\alpha\rangle = \frac{|\alpha\rangle + |-\alpha\rangle - 2\sqrt{x}|0\rangle}{\sqrt{2}(1-x)}, \qquad (11)$$

So that the coherent state can be expanded as

$$|\pm\alpha\rangle = \sqrt{x}|0\rangle + \frac{1-x}{\sqrt{2}}|NZE,\alpha\rangle \pm \sqrt{\frac{1-x^2}{2}}|ODD,\alpha\rangle, \qquad (12)$$

Resulting in

$$\left|\pm\sqrt{2}\alpha\right\rangle = x|0\rangle + \frac{1-x^2}{\sqrt{2}}\left|NZE,\sqrt{2}\alpha\right\rangle \pm \sqrt{\frac{1-x^4}{2}}\left|ODD,\sqrt{2}\alpha\right\rangle. \quad (13)$$

Rewriting the global state (7) using the $\left\{|0\rangle, \left|NZE,\sqrt{2}\alpha\right\rangle, \left|ODD,\sqrt{2}\alpha\right\rangle\right\}$ basis, and dropping the "$\sqrt{2}\alpha$" label for the sake of clarity, we get

$$\begin{aligned}
|\psi\rangle_{9,14,13,5,6,7} = \frac{1}{\sqrt{2(1-x^8)}}\Big[& x^2|0,0,0\rangle_{9,14,13}\left((\varepsilon_+ + \varepsilon_-)|\alpha,\alpha,\alpha\rangle_{5,6,7} - (\varepsilon_+ + \varepsilon_-)|-\alpha,-\alpha,-\alpha\rangle_{5,6,7}\right) \\
& + \frac{x(1-x^2)}{\sqrt{2}}|NZE,0,0\rangle_{9,14,13}\left((\varepsilon_+ + \varepsilon_-)|\alpha,\alpha,\alpha\rangle_{5,6,7} - (\varepsilon_+ + \varepsilon_-)|-\alpha,-\alpha,-\alpha\rangle_{5,6,7}\right) \\
& + x\sqrt{\frac{1-x^4}{2}}|ODD,0,0\rangle_{9,14,13}\left((\varepsilon_+ - \varepsilon_-)|\alpha,\alpha,\alpha\rangle_{5,6,7} - (\varepsilon_+ - \varepsilon_-)|-\alpha,-\alpha,-\alpha\rangle_{5,6,7}\right) \\
& + \frac{x(1-x^2)}{\sqrt{2}}|0,0,NZE\rangle_{9,14,13}\left(\varepsilon_+|\alpha,\alpha,\alpha\rangle_{5,6,7} - \varepsilon_-|-\alpha,-\alpha,-\alpha\rangle_{5,6,7}\right) \\
& + x\sqrt{\frac{1-x^4}{2}}|0,0,ODD\rangle_{9,14,13}\left(\varepsilon_+|\alpha,\alpha,\alpha\rangle_{5,6,7} - \varepsilon_-|-\alpha,-\alpha,-\alpha\rangle_{5,6,7}\right) \\
& + \frac{(1-x^2)^2}{2}|NZE,0,NZE\rangle_{9,14,13}\left(\varepsilon_+|\alpha,\alpha,\alpha\rangle_{5,6,7} - \varepsilon_-|-\alpha,-\alpha,-\alpha\rangle_{5,6,7}\right) \\
& + \frac{1-x^4}{2}|ODD,0,ODD\rangle_{9,14,13}\left(\varepsilon_+|\alpha,\alpha,\alpha\rangle_{5,6,7} - \varepsilon_-|-\alpha,-\alpha,-\alpha\rangle_{5,6,7}\right) \\
& + \frac{x(1-x^2)}{\sqrt{2}}|0,NZE,0\rangle_{9,14,13}\left(\varepsilon_-|\alpha,\alpha,\alpha\rangle_{5,6,7} - \varepsilon_+|-\alpha,-\alpha,-\alpha\rangle_{5,6,7}\right)
\end{aligned}$$

Alice's photon counting measurement yields one out of 15 possible results, listed in (14), which have nonzero probability of occurrence. Consequently, she obtains three classical bits of information that she transmits to Bob using a classical channel. We group Alice's measurement results into six outcomes, each corresponding to a distinct teleported state $|T\rangle_{5,6,7}$ into which modes 5 - 7 get projected. Using the classical bits he receives, Bob applies a unitary transformation on $|T\rangle_{5,6,7}$ to recover the original information state. We discuss the course of action Bob would take for each of these possible outcomes.

Outcome I

When Alice gets $|0,0,0\rangle_{9,14,13}$ or $|NZE,0,0\rangle_{9,14,13}$, the (unnormalised) teleported state with Bob is

$$+x\sqrt{\frac{1-x^4}{2}}|0,ODD,0\rangle_{9,14,13}\left(\varepsilon_-|\alpha,\alpha,\alpha\rangle_{5,6,7}-\varepsilon_+|-\alpha,-\alpha,-\alpha\rangle_{5,6,7}\right)$$
$$+\frac{(1-x^2)^2}{2}|NZE,NZE,0\rangle_{9,14,13}\left(\varepsilon_-|\alpha,\alpha,\alpha\rangle_{5,6,7}-\varepsilon_+|-\alpha,-\alpha,-\alpha\rangle_{5,6,7}\right)$$
$$-\frac{1-x^4}{2}|ODD,0,0\rangle_{9,14,13}\left(\varepsilon_-|\alpha,\alpha,\alpha\rangle_{5,6,7}-\varepsilon_+|-\alpha,-\alpha,-\alpha\rangle_{5,6,7}\right)$$
$$+\frac{(1-x^2)\sqrt{1-x^4}}{2}|NZE,0,ODD\rangle_{9,14,13}\left(\varepsilon_+|\alpha,\alpha,\alpha\rangle_{5,6,7}+\varepsilon_-|-\alpha,-\alpha,-\alpha\rangle_{5,6,7}\right)$$
$$+\frac{(1-x^2)\sqrt{1-x^4}}{2}|ODD,0,NZE\rangle_{9,14,13}\left(\varepsilon_+|\alpha,\alpha,\alpha\rangle_{5,6,7}+\varepsilon_-|-\alpha,-\alpha,-\alpha\rangle_{5,6,7}\right)$$
$$+\frac{(1-x^2)\sqrt{1-x^4}}{2}|NZE,ODD,0\rangle_{9,14,13}\left(\varepsilon_-|\alpha,\alpha,\alpha\rangle_{5,6,7}+\varepsilon_+|-\alpha,-\alpha,-\alpha\rangle_{5,6,7}\right)$$
$$\left.+\frac{(1-x^2)\sqrt{1-x^4}}{2}|ODD,NZE,0\rangle_{9,14,13}\left(\varepsilon_-|\alpha,\alpha,\alpha\rangle_{5,6,7}+\varepsilon_+|-\alpha,-\alpha,-\alpha\rangle_{5,6,7}\right)\right]. \quad (14)$$

$$|T\rangle_{5,6,7}\sim(\varepsilon_++\varepsilon_-)|\alpha,\alpha,\alpha\rangle_{5,6,7}-(\varepsilon_++\varepsilon_-)|-\alpha,-\alpha,-\alpha\rangle_{5,6,7} \quad (15)$$

That, in the $\{|EVEN,\alpha\rangle,|ODD,\alpha\rangle\}$ basis, becomes

$$|T\rangle_{5,6,7}\sim 2A_+\sqrt{\frac{1-x^6}{1+x^6}}|ODD,\alpha,\alpha,\alpha\rangle_{5,6,7} \quad (16)$$

Where $x=e^{-|\alpha|^2}$ and normalisation constant $N_I=\left[4|A_+|^2\left(\frac{1-x^6}{1+x^6}\right)\right]^{-1/2}$. Teleportation fails for this outcome, so we need not apply a unitary transformation to $|T\rangle_{5,6,7}$. Fidelity of Bob's teleported state is

$$F_I=\sin^2\frac{\theta}{2} \quad (17)$$

And the probability of occurrence of the outcome is

$$P_I=\frac{2x^2+x^2(1-x^2)^2}{4(1-x^8)}. \quad (18)$$

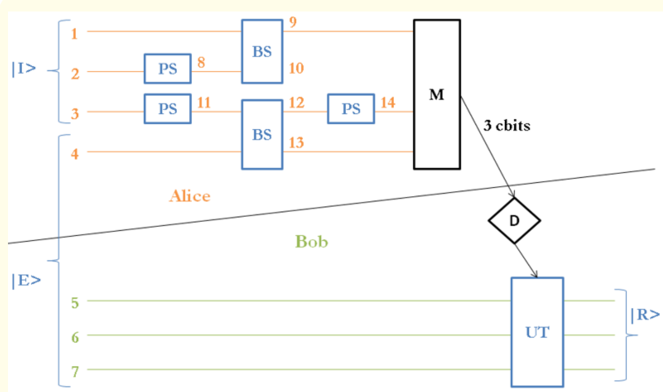

Figure 1: Numerals 1 – 14 refer to modes. The arrow of time goes from left to right. The distribution of modes belonging to the information state, , and the quantum channel, , is shown. (i) Alice uses phase shifters on states in modes 2 and 3. (ii) She mixes the outputs with modes 1 and 4 respectively, using lossless 50:50 beam splitters. (iii) She uses a phase shifter on mode 12. (iii) She performs a photon counting measurement on modes 9, 14, and 13, and obtains 3 classical bits that she sends to Bob. (iv) Bob uses these classical bits to decide which unitary transformation would help him recover the original information state at his end. denotes the recovered state.

Outcome II

When Alice gets $|ODD,0,0\rangle_{9,4,3}$, the teleported state with Bob is

$$|T\rangle_{5,6,7} \sim (\varepsilon_{+}-\varepsilon_{-})|\alpha,\alpha,\alpha\rangle_{5,6,7} - (\varepsilon_{+}-\varepsilon_{-})|-\alpha,-\alpha,-\alpha\rangle_{5,6,7} \quad (19)$$

That, in the $\{|EVEN,\alpha\rangle, |ODD,\alpha\rangle\}$ basis, becomes

$$|T\rangle_{5,6,7} \sim 2A_{-}|ODD,\alpha,\alpha,\alpha\rangle_{5,6,7} \quad (20)$$

With normalisation constant $N_{II} = \left[4|A_{+}|^{2}\right]^{-1/2}$. Teleportation fails for this outcome, so we need not apply a unitary transformation to $|T\rangle_{5,6,7}$. Fidelity of Bob's state is

$$F_{II} = F_{I} = \sin^{2}\frac{\theta}{2} \quad (21)$$

And the probability of occurrence of the outcome is

$$P_{II} = \frac{x^{2}(1-x^{4})}{4(1-x^{8})}. \quad (22)$$

Outcome III

When Alice gets $|0,0,NZE\rangle_{9,4,3}$, $|0,0,ODD\rangle_{9,4,3}$, $|NZE,0,NZE\rangle_{9,4,3}$, or $|ODD,0,ODD\rangle_{9,4,3}$, the teleported state with Bob is

$$|T\rangle_{5,6,7} \sim \varepsilon_{+}|\alpha,\alpha,\alpha\rangle_{5,6,7} - \varepsilon_{-}|-\alpha,-\alpha,-\alpha\rangle_{5,6,7} \quad (23)$$

That, in the $\{|EVEN,\alpha\rangle, |ODD,\alpha\rangle\}$ basis, becomes

$$|T\rangle_{5,6,7} \sim A_{-}\sqrt{\frac{1+x^{6}}{1-x^{6}}}|EVEN,\alpha,\alpha,\alpha\rangle_{5,6,7} + A_{+}\sqrt{\frac{1-x^{6}}{1+x^{6}}}|ODD,\alpha,\alpha,\alpha\rangle_{5,6,7} \quad (24)$$

With normalisation constant $N_{III} = \left[|A_{+}|^{2}\left(\frac{1-x^{6}}{1+x^{6}}\right) + |A_{-}|^{2}\left(\frac{1+x^{6}}{1-x^{6}}\right)\right]^{-1/2}$. Bob applies the unitary transformation

$$\hat{U}_{III} = |ODD,\alpha,\alpha,\alpha\rangle\langle EVEN,\alpha,\alpha,\alpha| + |EVEN,\alpha,\alpha,\alpha\rangle\langle ODD,\alpha,\alpha,\alpha| \quad (25)$$

To convert $|T\rangle_{5,6,7}$ to the recovered information state

$$|R\rangle_{5,6,7} \sim A_{+}\sqrt{\frac{1-x^{6}}{1+x^{6}}}|EVEN,\alpha,\alpha,\alpha\rangle_{5,6,7} + A_{-}\sqrt{\frac{1+x^{6}}{1-x^{6}}}|ODD,\alpha,\alpha,\alpha\rangle_{5,6,7}. \quad (26)$$

Fidelity of the recovered information state is

$$F_{III} = \frac{\left(\cos^{2}\frac{\theta}{2}\sqrt{\frac{1-x^{6}}{1+x^{6}}} + \sin^{2}\frac{\theta}{2}\sqrt{\frac{1+x^{6}}{1-x^{6}}}\right)^{2}}{\cos^{2}\frac{\theta}{2}\left(\frac{1-x^{6}}{1+x^{6}}\right) + \sin^{2}\frac{\theta}{2}\left(\frac{1+x^{6}}{1-x^{6}}\right)}, \quad (27)$$

Which is almost unity for $|\alpha|^{2} = 1$, and the probability of occurrence of the outcome is

$$P_{III} = \frac{2x^{2}(1-x^{2})^{2} + 2x^{2}(1-x^{4}) + (1-x^{2})^{4} + (1-x^{4})^{2}}{8(1-x^{8})}. \quad (28)$$

Outcome IV

When Alice gets $|0,NZE,0\rangle_{9,4,3}$, $|0,ODD,0\rangle_{9,4,3}$, $|NZE,NZE,0\rangle_{9,4,3}$ or $|ODD,ODD,0\rangle_{9,4,3}$, the teleported state with Bob is

$$|T\rangle_{5,6,7} \sim \varepsilon_{-}|\alpha,\alpha,\alpha\rangle_{5,6,7} - \varepsilon_{+}|-\alpha,-\alpha,-\alpha\rangle_{5,6,7} \quad (29)$$

That, in the $\{|EVEN,\alpha\rangle, |ODD,\alpha\rangle\}$ basis, becomes

$$|T\rangle_{5,6,7} \sim -A_{-}\sqrt{\frac{1+x^{6}}{1-x^{6}}}|EVEN,\alpha,\alpha,\alpha\rangle_{5,6,7} + A_{+}\sqrt{\frac{1-x^{6}}{1+x^{6}}}|ODD,\alpha,\alpha,\alpha\rangle_{5,6,7} \quad (30)$$

With normalisation constant $N_{IV} = \left[|A_{+}|^{2}\left(\frac{1-x^{6}}{1+x^{6}}\right) + |A_{-}|^{2}\left(\frac{1+x^{6}}{1-x^{6}}\right)\right]^{-1/2}$. Bob applies the unitary transformation

$$\hat{U}_{IV} = |EVEN,\alpha,\alpha,\alpha\rangle\langle ODD,\alpha,\alpha,\alpha| - |ODD,\alpha,\alpha,\alpha\rangle\langle EVEN,\alpha,\alpha,\alpha| \quad (31)$$

To convert $|T\rangle_{5,6,7}$ to the recovered information state

$$|R\rangle_{5,6,7} \sim A_{+}\sqrt{\frac{1-x^{6}}{1+x^{6}}}|EVEN,\alpha,\alpha,\alpha\rangle_{5,6,7} + A_{-}\sqrt{\frac{1+x^{6}}{1-x^{6}}}|ODD,\alpha,\alpha,\alpha\rangle_{5,6,7}, \quad (32)$$

Which is same as the one we obtain for outcome III. Fidelity of the recovered information state is

$$F_{IV} = F_{III} = \frac{\left(\cos^2\frac{\theta}{2}\sqrt{\frac{1-x^6}{1+x^6}} + \sin^2\frac{\theta}{2}\sqrt{\frac{1+x^6}{1-x^6}}\right)^2}{\cos^2\frac{\theta}{2}\left(\frac{1-x^6}{1+x^6}\right) + \sin^2\frac{\theta}{2}\left(\frac{1+x^6}{1-x^6}\right)}, \quad (33)$$

Which is almost unity for $|\alpha|^2 = 1$, and the probability of occurrence of the outcome is

$$P_{IV} = P_{III} = \frac{2x^2(1-x^2)^2 + 2x^2(1-x^4) + (1-x^2)^4 + (1-x^4)^2}{8(1-x^8)}. \quad (34)$$

Outcome V

When Alice gets $|NZE,0,ODD\rangle_{9,4,3}$ or $|ODD,0,NZE\rangle_{9,4,3}$, the teleported state with Bob is

$$|T\rangle_{5,6,7} \sim \varepsilon_+|\alpha,\alpha,\alpha\rangle_{5,6,7} + \varepsilon_-|-\alpha,-\alpha,-\alpha\rangle_{5,6,7} \quad (35)$$

That, in the $\{|EVEN,\alpha\rangle, |ODD,\alpha\rangle\}$ basis, becomes

$$|T\rangle_{5,6,7} \sim A_+|EVEN,\alpha,\alpha,\alpha\rangle_{5,6,7} + A_-|ODD,\alpha,\alpha,\alpha\rangle_{5,6,7} \quad (36)$$

With the normalisation condition $|A_+|^2 + |A_-|^2 = 1$. Bob applies the unitary transformation

$$\hat{U}_V = \hat{I} \quad (37)$$

To convert $|T\rangle_{5,6,7}$ to the recovered information state

$$|R\rangle_{5,6,7} \sim A_+|EVEN,\alpha,\alpha,\alpha\rangle_{5,6,7} + A_-|ODD,\alpha,\alpha,\alpha\rangle_{5,6,7}, \quad (38)$$

Which is same as the information state $|I\rangle_{1,2,3}$. Fidelity of the recovered information state is

$$F_V = 1 \quad (39)$$

And the probability of occurrence of the outcome is

$$P_V = \frac{(1-x^4)(1-x^2)^2}{4(1-x^8)}. \quad (40)$$

Outcome VI

When Alice gets $|NZE,ODD,0\rangle_{9,4,3}$ or $|ODD,NZE,0\rangle_{9,4,3}$, the teleported state with Bob is

$$|T\rangle_{5,6,7} \sim \varepsilon_-|\alpha,\alpha,\alpha\rangle_{5,6,7} + \varepsilon_+|-\alpha,-\alpha,-\alpha\rangle_{5,6,7} \quad (41)$$

That, in the $\{|EVEN,\alpha\rangle, |ODD,\alpha\rangle\}$ basis, becomes

$$|T\rangle_{5,6,7} \sim A_+|EVEN,\alpha,\alpha,\alpha\rangle_{5,6,7} - A_-|ODD,\alpha,\alpha,\alpha\rangle_{5,6,7} \quad (42)$$

With the normalisation condition $|A_+|^2 + |A_-|^2 = 1$. Bob applies the unitary transformation

$$\hat{U}_{VI} = |EVEN,\alpha,\alpha,\alpha\rangle\langle EVEN,\alpha,\alpha,\alpha| - |ODD,\alpha,\alpha,\alpha\rangle\langle ODD,\alpha,\alpha,\alpha| \quad (43)$$

To convert $|T\rangle_{5,6,7}$ to the recovered information state

$$|R\rangle_{5,6,7} \sim A_+|EVEN,\alpha,\alpha,\alpha\rangle_{5,6,7} + A_-|ODD,\alpha,\alpha,\alpha\rangle_{5,6,7}, \quad (44)$$

Which is same as the information state $|I\rangle_{1,2,3}$. Fidelity of the recovered information state is

$$F_{VI} = F_V = 1 \quad (45)$$

And the probability of occurrence of the outcome is

$$P_{VI} = P_V = \frac{(1-x^4)(1-x^2)^2}{4(1-x^8)}. \quad (46)$$

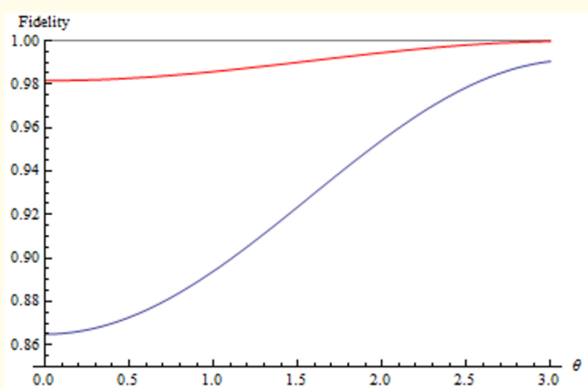

Figure 2: The horizontal grey line denotes unit fidelity (for the case of perfect teleportation, shown here for comparison). The red and blue curves show the variation in average fidelity, , for and respectively, for different values of .

Conclusion

We get perfect teleportation – with unit fidelity – when Alice obtains $|0\rangle$, $\left|NZE,\sqrt{2}\alpha\right\rangle$, and $\left|ODD,\sqrt{2}\alpha\right\rangle$, in any order, in her three modes. For two vacuum states, two nonzero even states, or two odd states – in modes 9 and 14, or in modes 9 and 13 – we obtain almost perfect teleportation when $|\alpha|^2 = 1$. Teleportation fails when Alice finds vacuum states in modes 14 and 13.

We define average fidelity, F_{av}, as the sum of over all products of individual fidelities (of the six outcomes discussed above) and their corresponding probabilities of occurrence.

$$F_{av} = \sum_i (F_i P_i)$$

$$= \left[\frac{(1-x^4)(1-x^2)^2}{2(1-x^8)}\right] + \left(\sin^2\frac{\theta}{2}\right) \times \left[\frac{x^2(2-x^2)}{2(1-x^8)}\right]$$

$$+ \left[\frac{\left(\cos^2\frac{\theta}{2}\sqrt{\frac{1-x^6}{1+x^6}} + \sin^2\frac{\theta}{2}\sqrt{\frac{1+x^6}{1-x^6}}\right)^2}{\cos^2\frac{\theta}{2}\left(\frac{1-x^6}{1+x^6}\right) + \sin^2\frac{\theta}{2}\left(\frac{1+x^6}{1-x^6}\right)}\right] \times \left[\frac{4x^2(1-x^2)+(1-x^2)^4+(1-x^4)^2}{4(1-x^8)}\right], \quad (47)$$

Where $x = e^{-|\alpha|^2}$ and i = 1, 2,..., 6. We are interested in the minimum average fidelity, $F_{av,\min}$, which comes out to be 0.865 for $|\alpha|^2 = 1$ and 0.9816 for $|\alpha|^2 = 2$, which is almost equal to unity. Figure 2 shows the variation in F_{av}, for different values of $|\alpha|^2$, as a function of θ.

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