ACTA SCIENTIFIC COMPUTER SCIENCES

Volume 2 Issue 7 July 2020

Review Article

Conceptualization of the Concept of Early Algebra and Early Algebar Opinion

Vishnu Narayan Mishra*

Department of Mathematics, Indira Gandhi National Tribal University, India

***Corresponding Author:** Vishnu Narayan Mishra, Department of Mathematics, Indira Gandhi National Tribal University, India.

Received: June 17, 2020

Published: June 01, 2020

Abstract

Main goal of this work should be getting familiar with terms early algebra and early algebraic thinking, as well as introducing some basic ideas of implementation algebra in early school grades. While arithmetic calculation mainly deals with concrete numbers, algebra deals with processes and their abstract concepts. For most students, transition between concrete and abstract is a big problem. Early algebra means involving algebraic contents together with arithmetic contents in early mathematic teaching. To know early algebra is to know well algebraic thinking and its components. Insight into the nature of algebraic thinking will facilitate implementation of early algebra in teaching mathematics. This is accomplished primarily by encouraging early algebraic thinking of students, algebraization of teaching materials and creating positive classroom climate where they will nurture student's modeling, representation, reasoning and interpretation of mathematical problems in different ways.

Keywords: Algebra; Arithmetic; Early Algebra; Early Algebraic Thinking

Introduction

Practice has shown that students have great difficulty understanding formal algebra in higher education classes. The cause of this problem is primarily the insufficient connection of arithmetic (which is studied in younger school age) and algebraic content that is taught mainly in the third triad of primary school education. This transition should be facilitated by early algebra (a concept that has long existed in theory, but its application in practice is accompanied by many difficulties). There is a consensus among mathematical researchers education that algebraic content through early algebra can be included in teaching from the very beginning primary schooling (children aged 6 - 7). This implies many questions that still do not there are unambiguous and simple answers. What all belongs to the domain of early algebra and early algebra opinion? Where does arithmetic end and early algebra begin? What content is suitable for involvement algebra? What is the role of the teacher's instruction? Are classroom teachers trained for realization of this type of teaching? What is the role and significance of early algebra? How learning early algebra affects later student understanding of mathematics? The answer to this and many other questions can be reached only by researching teaching practice in order to improve the quality of mathematics teaching as well as the quality student knowledge.

Arithmetics and algebra

Algebra (from the Arabic word Al-jabr which can be translated as assembling broken parts) is part mathematics which we can most simply say deals with the mathematical symbols and rules that are related to the manipulation of these symbols [1]. Although some forms of algebra have been known since ancient times (Babylonia, Greece, Egypt), we connect the beginning of algebra in Europe with the works of French mathematician Francois Viet from the 16th century. He calls allegory analytical art. Great contribution the development of algebra (as can be seen from its name) was given by Arab mathematicians. Al Khwarzimi u to his most famous work he gives a compilation of the rules for solving linear quadratic equations and problems of geometry and proportion. In its name the word al jabr is mentioned thanks to which algebra bears today's name. There are different definitions of algebra, and therefore different opinions about what exactly it belongs to domain algebra. The simplest, algebra can be divided into elementary algebra (deals with properties mathematical operations) and abstract algebra (studies algebraic structures such as groups, rings, and fields).

According to Weyl (2005) the subject of algebra is the study of different systems of coordination of concrete (numbers, polynomials, functions...) or abstract (rings, groups, fields, etc.) mathematical entities. On the other hand, arithmetic (Greek arithmos

- numbers, techne - art) is a branch of mathematics that deals with computational operations with numbers. With arithmetic and its basic arithmetic operations (addition, subtraction, multiplication and division) students meet in the first grades of primary school. Problems arise when one moves from arithmetic to learning algebra. The question is what are the essential differences between arithmetic and algebra? What is it that causes problems? First of all, arithmetic deals calculations with known numbers, while algebra implies reasoning about unknowns and by changing quantities and recognizing the differences between specific and generalized situations (Van Amer, 2003). In algebra we deal with processes with abstract concepts. This is where knowledge comes to the fore and understanding of concepts (conceptual mathematical knowledge), but also knowledge of understanding the process with it concepts (process mathematical knowledge) [2]. Arithmetic does not operate on the same level as abstract algebra although both involve writing symbols and understanding operations. Arithmetic is limited by numbers and numerical calculations [3]. However, what does algebra and algebraic thinking special? Luis Radford (2006) highlights the following three essential elements of algebra and algebraic thinking:

- The meaning of uncertainty, which is a property of one of the basic algebraic objects, known as unknown, variable or parameter;
- This indeterminacy is an object that is treated analytically (Vieta considered algebra to be analytical art);
- Algebraic thinking, in a characteristic symbolic way, means the studied mathematical objects.

There are differences in the interpretation of letters, symbols and expressions between arithmetic and algebra. While in arithmetic the letters are mostly abbreviations, the algebraic letters serve as variable or unknown numbers (Van Amer, 2003). The interpretation of the sign of equality is also different. In elementary school mathematics (where arithmetic dominates) the sign of equality is seen only as a signal that we need to execute some kind of calculation. Only later in the teaching of mathematics are students brought into situations to give a sign equations are viewed in a whole new way, as a sign that the left and right sides of the expression are equal (comparability of two statements). Different interpretation of mathematical symbols in different periods of schooling it could be the cause of many problems that arise when students are introduced to algebraic ideas.

It has long been believed that arithmetic must precede algebra and that there is no place in the initial teaching of mathematics algebraic ideas. It was considered that students were not able to adopt even the simplest algebraic ones concepts. Such a belief has led to many headaches (student but also teacher) when learning and the most basic algebraic ideas in the older grades of elementary school. Many studies have shown that students are very capable of adopting algebraic concepts from the very beginning of their schooling, of course with appropriate instruction and guidance from the teacher. That doesn't mean we'll have students in first grade elementary school talk about formal algebra. Kieran (2004) indicates that the transformation of arithmetic to algebra in many students takes place with difficulty since algebra requires students significant adjustments. She argues that an acceptable transformation from arithmetic to algebra requires:

- Focusing on the interrelationships between numbers, and not just on calculating with those numbers;
- Focusing on operations, but also on their inversions, as well as on the concepts of what should or should not should work;
- Focusing on understanding what the problem is, not just how to solve it;
- Focusing on numbers and terms (letters), and not only on numbers as was the case before;
- Re-Examining the meaning of the sign of equality.

Kaput [4] proposed the integration of algebraic reasoning in all classes in order to achieve it stability and depth in school learning, and to avoid too late, isolated and artificial algebra which is studied in the upper grades of primary school and in secondary school. Carpenter and Levy [5] consider that the artificial division of primary education into the arithmetic and algebraic part prevents students from developing powerful thought patterns in the early grades and makes learning difficult in the later grades. Algebra is common considered a language, a tool, a generalization of arithmetic (Lee, 2005). Algebra is no longer thought of as the subject as a way of thinking but also acting on mathematical objects [3]. Tu na the scene enters a term about which many papers have been written, but in practice it is still insufficiently researched (especially in our country) - early algebra. National Council of Mathematics Teachers (NCMT, 2000) and many Mathematics education researchers agree that this concept has the potential to enrich mathematics activity and to serve as a guide to promote learning with understanding. Early algebra should be a solution to many problems that arise in mathematics education. We can do early algebra viewed as a bridge between arithmetic and formal algebra. What all belongs to early algebra? What is it early algebraic thinking? What is the nature of early algebraic thinking? On this and some other questions we will try to give an answer below.

Early algebra and early algebar thinking

Simply put, by early algebra we mean the process of involving algebraic ones content together with the usual arithmetic content in the lower grades of primary school. That wound algebra is not the same formal algebra that is taught in higher grades. This is more about the partial integrating these two domains, that is, inserting algebraic tools into arithmetic concepts wherever that possible [3]. This concept should be supported not only by the line ministry but also the academic community of implementers of mathematics in our primary schools, since algebra has potential to enrich mathematical activities, and to serve as a guide to promote learning with understanding [3]. The problem arises when the question is asked what exactly belongs to the domain of the wound algebra? How can the set goals be achieved through early algebra? How it should look curriculum in such teaching? Is it necessary to create a new program or is it possible to customize an existing one? What should you pay special attention to? There is agreement to work in the domain of wounds algebra implies the construction, understanding, description, and justification of generalizations of arithmetic concepts, acceptance and understanding of representations of mathematical ideas, but also their writing down using symbols, as well as the use of these symbols to understand and solve problems [2]. According to Kaput (2000), early algebra means:

- Generalization of patterns and relations (special generalization of arithmetic and quantitative reflections);
- Functional thinking;
- Modeling;
- Syntactic guidance of formula manipulation;
- Study of structures.

On the other hand, Van Ameram (2003) distinguishes four basic perspectives from which we can observe algebra:

- Algebra as a generalization of arithmetic;
- Algebra as a problem solving tool;
- Algebra as a study of relations;
- Algebra as a study of structures.

Carpenter and Levy [5] are of the opinion that early algebra is identified through two concepts:

- Generalization;
- Use of symbols to represent mathematical ideas and solve mathematical problems.

One of the most important elements of early algebra is introducing students to algebraic notation. Although they are already from the first grades they got acquainted with letters in mathematics, only with the help of early algebra students can discover the true meaning of algebraic notation. Research (Carraher, 2000) has shown that students have the most difficulty in understanding that the letter n (or any other letter) can denote any number.

The reason is primarily that students in arithmetically dominant teaching are accustomed to dealing with the concrete cases, while on the other hand in the teaching of algebra, general cases predominate. The whole purpose of the early introducing algebraic notation is to accelerate student progress and create a good foundation that will facilitate future teachings of algebra and mathematics. It can be said (Carraher, 2000) that the essence of early algebra deepening arithmetic teaching. When we talk about early algebra, we can't help but wonder what makes up early algebraic thinking. Early algebraic thinking is part of mathematical thinking. What's the difference between arithmetic and algebraic thinking? Kaput [6] characterized algebraic thinking through the following two aspects:

- Observing and expressing generalizations through a formal and conventional system symbols;
- Reasoning through these symbolic forms, including synthetically guided manipulations these symbolic forms.

Arithmetic is primarily response-oriented and insufficiently focused on representations and relations. An example from the book Adding It Up (Kilpatrick, Swafford and Findell, 2001) sums it up best. Students who have just started studying algebra will put a number in the blank space in task 8 + 5 = _ + 913 because 8 + 5 is their signal to perform the calculation, instead of putting the correct value 4. When the sign the equality present they treat it as a separator between problem and solution, taking it as a signal to perform the calculation indicated on the left side of the sign. Many mathematical problems, especially those that can be characterized as linearly simple tasks and linearly complex tasks (within the SOLO taxonomy pre-structural level tasks and unistructural level tasks) do not require a deeper understanding of the sign equality. On the other hand, tasks that we characterize as nonlinearly complex and non-standard tasks, i.e. as multistructural complex tasks, tasks at the relational level, i.e. at the abstract level (within the SOLO taxonomy) require that the equals sign represent comparability two statements [3]. According to Lins [7] to think algebraically implies the following three aspects:

- Think arithmetically, which means modeling with numbers;
- Think introspectively which means that only operations and relations of equality are taken into account and that elements observe and evaluate elements of number fields and arithmetic operations;
    - Think analytically, what needs to be understood as what needs to be unknown in order to be treated as known One of the most famous structures of algebraic thinking was given by Shelly Kriegler in her work Just What is Algebraic Thinking from 1997. According to her, the two main components of algebraic thinking are:
- Tools of mathematical thinking and fundamental algebraic ideas. The tools of mathematical thinking are organized into three categories:
    - Problem solving skills;
    - Representation skills;
    - Reasoning skills (inferences).

These thinking tools are important in all aspects of life and most people use them on a daily basis in different ways life situations. By problem-solving skill we mean the use of strategies for problem solving and exploring different approaches and ways of solving one problem.

The essence of successful problem solving is that the student knows what to do in a situation when he does not know what to do to do. Students who possess at least one strategy will find it easier to deal with the problem than students who do not possess any such strategy. Further, students who have learned to use the above-mentioned strategies more easily will cope not only in solving school mathematical tasks but also in everyday life situations that do not have an unambiguous template solution. Using presentation skills (representations) students are able to represent different relationships using visuals (diagrams, pictures, graphs), symbolic, numerical (tables, various lists, tables) and verbal representations.

Different connections can be further established between the representations thus obtained, as well as different ones interpretations within representations. Ability to create, interpret and transition from one representation on the other it gives students powerful thinking tools. Finally, the skill of reasoning contains analysis problems and inductive and deductive reasoning. Induction implies a logical method or type conclusions in which one starts from the individual to the general. Based on induction by testing individual case and pattern discovery we come to a general rule. Unlike induction, deduction represents a logical procedure in which one starts from the general to the individual. That means by studying problem structures we can come up with individual examples. These tools of mathematical thinking are very essential for the integration of algebra into the early grades, so it is necessary to build student understanding on them mathematics.

The fundamental algebraic ideas listed here are intended for learning in concrete and familiar situations to help students develop a strong conceptual basis for later abstract learning mathematics. They help students better understand the rigorous conceptual foundations of algebra. You algebraic ideas can be observed in three ways:

- Algebra as a generalization of arithmetic;
- Algebra as a language;
- Algebra as a tool for functional and mathematical modeling.

Algebra is very often defined as a generalization of arithmetic. When we talk about algebra as in the generalization of arithmetic, we primarily think of examining the properties of both the meaning of numbers and the sense computational operations between them. Strong development of arithmetic thinking in students in those years can be the basis for the emergence and further development of algebraic thinking [3]. If we routinely encourage conceptual approaches when learning algebraic procedures, students will develop a network of structures from which they can draw knowledge when they start learning formal algebra [8].

Algebra is a language (Usiskin, 1997). In order to understand this language, we must first understand the concept variables i.e. what the variables represent, as well as the meaning of the solution. Algebra as a language implies ability to read, write and manipulate numerical and symbolic representations in formulas, expressions, equations and inequalities. Fluency in algebraic language requires an understanding of the meaning within algebraic vocabulary (understanding of symbols and variables) and the ability to apply flexibly grammatical rules of algebraic language. This flexibility implies the use of standard algebraic ones convention e.g. the expression 14x means 14 multiplied by x, further, the expression x14 is also completely correct (denotes the same thing), but 14x is preferred because there is a convention, an unwritten rule that the coefficient writes in the first place. Also, the variables used in algebra can have different meanings in contextual dependencies. For example, in the equation 3 + x = 7, x is unknown and 4 is the solution of the equation. While in the expression y + x = x + y, y represents any number.

Finally, we can look at algebra as a tool for analyzing functions and mathematics modeling. If we look at algebraic thinking in this way, it shows the student the importance of and the usefulness of algebra in real life situations. It involves searching, expressing and generalizing patterns and rules in everyday context, representing mathematical ideas using equations, tables and graphs, working with output and input patterns and developing coordination skills graphs. Functions and mathematical models provide the context for the application of these algebraic ideas (Kriegler, 1997).

Teaching algebra in the lower grades of primary school

Teachers who teach math in the elementary grades of elementary school should do well know the structure of mathematical operations and the essence of working with natural numbers [3]. Others in words, teachers must be proficient in arithmetic and algebra. Unfortunately, practice has shown that this is often not the case and that the cause of many student problems in teaching mathematics is poor teacher instruction and insufficient knowledge of the essence and structure of computational operations, i.e. poor knowledge of arithmetic relations and algebra. It is very difficult, almost impossible to achieve the goals we are talking about in such conditions we speak in this text. According to Stevanović and Romano [9] the most appropriate stimuli of the early algebraic the considerations to be subjected to lower grade students are as follows: generalization of arithmetic (regularity observation), functional connection observation (generalization of numerical models), observation relational connections, related thinking, and questioning the meaning of the sign of equality. When teaching mathematical content should focus on students mastering the real fundamental skills in generalizing, recognizing, understanding and using representations of objects, concepts, processes and properties, but also to assess the expediency of mathematical generalizations [10]. The construction of abstract concepts is the goal of mathematical learning. Ability to work meaningfully in algebra, as well as fluent manipulation of algebraic notation implies that students first develops the ability to semantically understand arithmetic [11]. One of the main aspects understanding arithmetic operations and relations is understanding the internal structure of operations and relations among them, along with the ability to relate these elements to real situations. To make students developed the necessary semantic understanding of arithmetic should insist on the use of related thinking.

We can say that someone thinks related, that is, uses related thinking when questioning two or more mathematical ideas or objects, alternatively looking at their interrelationships, analyzing or using these relationships to solve a problem, form a claim, or learn more about the situation and the concepts that are there included (Molina, Castro and Ambrose, 2005). This thinking is very important in teaching mathematics since many fundamental mathematical ideas contain relations between different representations of numbers and mutual operations [12]. We can encourage connected thinking using various activities, e.g. helping students pay attention to the relationships between operations and numbers and shifting the focus from simple addition and student focus solely on the answer (Molina, Castro and Ambrose, 2005). For example, related thinking could be encouraged by using the following tasks where students do not have to come to a solution by mere addition but it is possible to accomplish and in other way. In example 27 + 48 - 48 = _ students can understand that after we add and subtraction number 27 remain unchanged and will thus avoid performing operations. Next, in example 12 + 7 = 7+ _ students can come up with a solution by noticing that the order of additions is reversed, instead of add 12 and 7 and then solve problem 19 = 7 + _. Also, the problem 8 + 4 = _ + 5 can be solved so that the student will notice that five is one greater than four, and the second addition must be one less than eight. To think this way, the student must look at equations as objects to analyze, rather than as processes to be performed (Molina, Castro and Ambrose, 2005). Focusing on addition and subtraction identified eight main features and relationships to pay attention to when solving such tasks:

- Complementarity of addition and subtraction;
- Commutativity of addition;
- The sum of two numbers does not change when the same number is added to one addition and subtracted from the other collection;
- The result of confiscation does not change when the same number is added or subtracted from both members;
- Zero is neutral for addition;
- When the number is subtracted from itself, zero is obtained;
- Associativity of addition;
- Each number can be displayed in several ways as the sum of two numbers.

Even though the teacher knows algebra and arithmetic very well, there are many other problems with which can be encountered when teaching algebra in lower grades. According to Molini (2005) Kindt cites three main problems of learning algebra:

- Lack of attention to generalizations of arithmetic concepts;
- The practice of teaching formal algebra too early;
- Failure to consider the methodological reasons for when and for what it is useful to involve algebra in the lower ones elementary school classes.

According to Blanton and Kaput (2003) teachers need to create such a classroom atmosphere that will respect student modeling, research, discussion, presentation, and testing of student methods. Too, a greater degree of interaction is needed both between teacher and student and between students. You interactions can be initiated by the teacher often asking such questions to help students to verbalize and expand their opinion, such as: What do you think at the moment? Can this problem to solve in a different way? How do you know this solution is correct? Does this way of dealing always give accurate results? In addition to creating a positive algebraic classroom culture, Blanton and Kaput (2003) in their the paper cites the algebraization of instructional material and the encouragement of students' algebraic thinking as important teaching activities in the strategy of involving algebra in the lower grades.

When teaching is based on student mathematical ideas and substantiates theirs curiosity, students often express algebraic ways of thinking in arithmetic, geometry and measurement [13]. The main goal of such teaching is to promote advanced mathematics opinions, but at the same time taking into account students 'mental capacities. Teachers should nurture algebraic thinking while learning other mathematical concepts by helping students to pay attention to mathematical proportions, relations and patterns (Molina, Castro and Ambrose, 2005). Also, the current way, in which we teach lower primary school students to use the letter as of algebraic variables and unknowns comes down to using the variable x to describe a space and arithmetic notations, and solving simple equations and inequalities in a set of natural numbers, no encourages the development of algebraic thinking in students [3]. There is also talk of arithmetic structures, and thus about arithmetic thinking.

Conclusion

The incorporation of algebra into the teaching of mathematics in the lower grades of primary school is not at all simple and easy task. Our education system faces a great challenge. It is necessary that the Ministry of Education and the academic community in our country understand the extraordinary importance of early algebra.

The benefits of this fresh concept are great, starting with a better understanding of arithmetic, arithmetic operations as well as mathematics in general, facilitating the learning of algebra in the higher grades of elementary schools, all the way to a clearer understanding and better coping in everyday problems and life situations, which is, after all, one of the most important goals of teaching mathematics. Perhaps the biggest obstacle the introduction of algebra in this way in the initial grades is a weak qualification of the primary school teachers to conduct this type of teaching. Many math teachers have problems with themselves insufficient understanding of arithmetic structures and relationships between computational operations, which is largely number of cases due to poor teacher instruction and artificial division of mathematics teaching in primary schools on the arithmetic and algebraic part. There is a great responsibility on the institutions that deal with it by educating future teachers to train them to teach early algebra and thus break this enchanted round. Of course, the institutions that adopt the curriculum should also follow the modern ones tendencies that occur in mathematical education and react accordingly to the common good students, teachers, parents but also society as a whole.

Bibliography

1. Herstein IN. "Topics in Algebra". Blaisdell Pub. Co., Mass (1964).
2. Crvenkovic C and Romano AD. "Early algebra and early algebraic thinking, In: Methodological aspects of teaching mathematics". Third International Conference MATM (2014).
3. Romano DA. "What is algebraic thinking?" *MAT-KOL* 25.2 (2009): 19-29.
4. Kaput JJ. "A Research Base Supporting Long Term Algebra Reform?". In DT Owens, MK Reed and GM Millsaps (Editions.), Proceedings of the 17th Annual Meeting of PME-NA 1 (1995): 71-94.
5. Carpenter TP and Levi L. "Developing conceptions of algebraic reasoning in the primarygrades". Research report No. 00-2. Wisconsin, Madison: National Center for improving studentlearning and achievement in mathematics and science (2000).
6. Kaput JJ. "What is algebra? What is algebraic reasoning?". In: James J. Kaput, David W. Carraher, Maria L. Blanton (Editions.) Algebra in the Early Grades (Studies in Mathematical Thinking and Learning Series (2008): 235-272.
7. Lins R. "Algebraic and non-algebraic algebra". In (W. Geeslin and K. Graham, Editions.) Proceeding of the Sixteenth International Conference for the Psychology of Mathematics Education 2 (1992): 56-63.

8. Kriegler S. "Just what is algebraic thinking. Submitted for Algebraic Concepts in the Middle School". A special edition of Mathematics Teaching in the Middle School (2008).

9. Stevanovic S., *et al*. "An example of analysis of arithmetic and early algebraic thinking". *Inovacije U Nastavi* 27.1 (2014): 118-134.

10. Kaput JJ and Blanton M. "Algebrafying the elementary mathematics experience". In Chick H., Stacey, K., Vincent, J., and Vincent, J. (2001). Proceedings of the 12th ICMI Study Conference (Editions). The Future of the Teaching and Learning of Algebra 1 (2001): 344-352.

11. Booth LR. "Seeing the Pattern: Approaches to Algebra". *Australian Mathematics Teacher* 45.3 (1989): 12-13.

12. Carpenter TP., *et al.* "Thinking mathematically: Integrating arithmetic and algebra in elementary school". Heinemann, Portsmouth (2003).

13. Russell SJ., *et al*. "Is it 2 more or 2 less?" Algebra in the elementary classroom". *Connect* 19.3 (2006): 1-3.