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Review Article

Inertial Torques Acting on a Spinning Circular Cone

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Abstract

In engineering most movable mechanisms contain rotating components as the disc, cone, sphere, toroid, etc. that manifest the action of inertial torques. The known gyroscope theories do not give a correct explanation of the nature of inertial torques except the action of the torques originated by the change in the angular momentum. The physics of the inertial torques generated by the spinning objects is more complex than represented by the new research. Any rotating objects produce the system of the inertial torques that resulting in their motions. This system of inertial torques forms the fundamental principles of gyroscope theory, which are used for developing the mathematical models for the motions of gyroscopic devices. This work represents formulation of the several inertial torques generated by the spinning circular cone. The obtained mathematical models for inertial torques acting on for the spinning cone enables for computing its motion in space.

Keywords: Gyroscopic Effects; Spinning Cone; Torque; Force

Abbreviations

fct, f cr., fin: Centrifugal, Coriolis and inertial forces, respectively, generated by mass elements of a spinning objects; *J:* Mass Moment of Inertia; *M:* Mass of a Circular Cone; *M:* Mass Element of a Circular Cone; *R:* External Radius of a Circular Cone; *T:* Load External Torque; *Tct, Tcr., Tin. Tam:* Torque generated by centrifugal, Coriolis and inertial forces and a change in the angular momentum, respectively; *t:* Time; *yc, ym:* Centroid and distance of the location of the mass element along with axis; $\Delta\alpha$, α: Increment angle and angle of the turn for a circular cone around own axis; $\Delta\delta$: Incremental angle of the circular cone a mass element; $\Delta\gamma$: Angle of inclination of a circular cone plane; ω: Angular velocity of a circular cone; ωx: Angular velocity of precession around axis *ox*

Introduction

The theory of dynamics of the rotating objects appeared in the twentieth century with an intensification of the machine's work [1-3]. The action of the inertial torques of spinning objects attracted researchers that tried to describe gyroscopic effects by simplified mathematical models that not well-matching practice [4-6]. The unsolved problems of the dynamics of rotating objects are still hanging off the day and waiting for analytical solutions. The known textbooks and publications of engineering mechanics cannot describe adequately the action of inertial torques that manifest gyroscopic effects [7-9]. All known publications in the area of gyroscope theory show the action of inertial torques only by a term of the angular momentum [10-12]. Such works with mathematical models for gyroscopic effects do not match practical tests [13-15]. This is the reason that every year is published hundreds of works dedicated to gyroscopic effects [16,17].

The physics of gyroscopic effects are complex than represented in know works. Gyroscopic effects of the spinning objects are generated not only by the center mas but also by the distributed rotating masses. It means the geometry of the spinning objects plays a significant role in forming the inertial torques. These torques are represented by the system that forms the fundamental principles of gyroscope theory [18]. The new mathematical models for inertial torques give a significant impact on the theory of dynamics of the rotating objects. This work represents the application of mathematical models for the system of inertial torques generated by the spinning circular cone that can be part of the machines and mechanisms. The analytical approach for computing of the inertial torques generated by the spinning circular cone is similar to the spinning disc [18]. Both objects are different in design and mathematical models for inertial torques also different. Following discussions about formulation the action of the inertial torques acting on the spinning cone will be referred to the computational schemes of the published works that consider the spinning disc [18].

Centrifugal torques acting on a spinning cone

The spinning right circular cone can be the component of numerous mechanisms. This section is considered the inertial forces and torques generated the spinning cone with a constant angular velocity of ω (Figure 1). The circular cone mass elements m are located on the cone surface with the forming line b whose arbitrary radius is r relatively axis oz. The rotating mass elements generate the centrifugal torques located on the line forming cone.

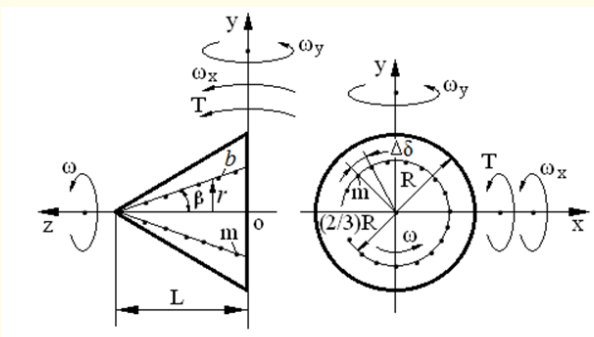

Figure 1

The turn of the arbitrary spinning cone's plane xoy around axis ox leads to change in the plane of the rotating centrifugal force vectors fct which vectors $f_{ct.z}$ are parallel to axis oz of the cone. The initial scheme for the computing of the centrifugal forces $f_{ct.z}$ is the same as in the published paper that considers the spinning disc [18]. The radii y_m of the mass elements location presented on the parallel planes of the circular cone. The integrated product of the centrifugal forces $f_{ct.z}$ that located on the forming line b generates the resistance torque Tct that counteracts to the action of the external torque applied.

The equation for the component of centrifugal force for the arbitrarily chosen plane is as follows:

$$f_{c.z} = f_c \sin\alpha\sin\Delta\gamma = m\ \omega^2 \sin\alpha\sin\Delta\gamma \qquad (1)$$

Where $f_c = m\ \sin\boldsymbol{\omega}^{2}$ is the centrifugal force of the mass element m; $m = (M/2\pi b)\Delta\delta\Delta b$, M is the mass of the cone; b= [(2/3)R]/sinβ is the length of the line that forms the cone surface of the mass element's, R and L is the radius and height of the circular cone respectively, $\Delta\delta$ is the sector's angle of the mass element's location on the plane that parallel to plane xoy; $\Delta b = \Delta r/\sin\beta$ is the line part of the mass element's location, r is the radius of the arbitrary circle plane of the cone; β is the angle of the mass elements location on the cone forming line (Figure 1); $\Delta\gamma$ is the angle of turn for the cone's plane around axis ox ($\sin\Delta\gamma = \Delta\gamma$ for the small values of the angle) [18], other components are a specified above.

The defined parameters are substituted into equation (2) that yield the following:

$$f_{c.z} = \left[\frac{M\ \omega^2 R\sin\beta\sin\alpha}{2\pi(2/3)R\sin\beta}\right]\Delta r\Delta\delta\Delta\gamma = \frac{3M\omega^2}{4\pi R}\Delta\delta\Delta\gamma r\Delta r\sin\alpha \qquad (2)$$

Where all components are as specified above.

The centrifugal forces $f_{ct.z}$ are the distributed load applied across the length of the line forming the cone surface. The centrifugal forces $f_{ct.z}$ is presented by a concentrated load applied to centroid along axis oy. The radius of the location of the centroid is defined by the following proportion: (2/3)R/L = y/(2/3)L, then y_m = (2/3)(2/3) Rsinα = (4/9)Rsinα, which is the distance of the arbitrary cone's mass element's location relative to axis ox.

The resistance torque ΔT_{ct} expresses the action of the centrifugal force $f_{ct.z}$ of the mass element and presented by the following equation:

$$\Delta T_{ct} = f_{ct.z}\, y_m \qquad (3)$$

Where y_m is the arbitrary distance of the location of the cone's mass elements at the arbitrary plane along with axis oz and relative the axis ox, other components are as specified above.

The centroid y_A (point A, figure 2 of [18]) is defined by the following equation:

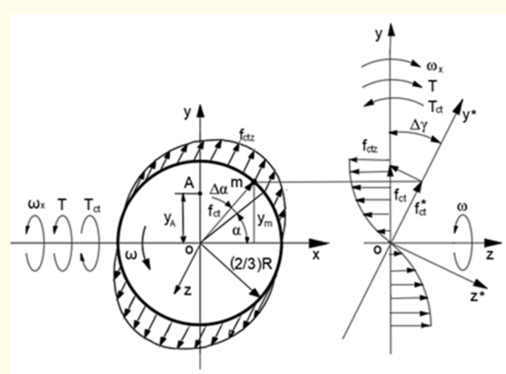

Figure 2

$$y_A = \frac{2R\int_0^{\pi}(1-\cos 2\alpha)d\alpha}{9\int_0^{\pi}\sin\alpha d\alpha} \qquad (4)$$

Where all components are as specified above.

Eqations (4) and (2) are substituted into equation (3) that presented by the integral forms, where $\sin\alpha = \int_0^{\pi/2}\cos\alpha d\alpha$ is replaced by the integral expression, then the following equation emerges:

$$\int_0^{T_{ct}} dT_{ct} = \frac{3M\omega^2}{4\pi R}\times\int_0^{\pi} d\delta\times\int_0^{\pi/2}\cos\alpha d\alpha\times\int_0^{\gamma} d\gamma\times\int_0^{(2/3)R} r dr\times\frac{2R\int_0^{\pi}(1-\cos 2\alpha)d\alpha}{9\int_0^{\pi}\sin\alpha d\alpha} \quad (5)$$

Where the limits of integration for the trigonometric expression of sinus are taken for the quarter of the circle and the result is increased twice.

Solving integral equation (5) yields the following result

$$T_c\Big|_0^{T_c} = \frac{3M\omega^2}{4\pi R}\times\delta\Big|_0^{\pi}\times 2\left(\sin\alpha\Big|_0^{\pi/2}\right)\times(\gamma)\times\left(\frac{r^2}{2}\Big|_0^{(2/3)R}\right)\times\frac{2R\left(\alpha-\frac{1}{2}\sin 2\alpha\right)\Big|_0^{\pi}}{-9\cos\alpha\Big|_0^{\pi}}$$

The change of the upper limit for the trigonometric expression, sinus leads to increasing the result twice. For the expression, sin2α remains the same due to the symmetrical location of the centroid. The solution to the expression giving rise to the following:

$$T_{ct} = \frac{3M\omega^2}{4\pi R}\times\pi\times 2(1-0)\times(\gamma-0)\times\left(\frac{2}{9}R^2-0\right)\times\frac{2R(\pi-0)}{-9(-1-1)} = \frac{MR^2\pi\omega^2}{27}\times\gamma \quad (6)$$

Following steps of solutions are the same as presented in the published work [18], which comments are omitted:

$$\frac{dT_{ct}}{dt} = \frac{MR^2\pi\omega^2}{27}\times\frac{d\gamma}{dt} \quad (7)$$

$$t = \frac{\alpha}{\omega},\ dt = \frac{d\alpha}{\omega},\ \frac{d\gamma}{dt} = \omega_x$$

$$\frac{\omega dT_{ct}}{d\alpha} = \frac{MR^2\pi\omega^2\omega_x}{27} \quad (8)$$

$$dT_{ct} = \frac{MR^2\pi\omega\omega_x}{27}d\alpha \quad (9)$$

$$\int_0^{T_{ct}} dT_{ct} = \int_0^{\pi}\frac{MR^2\pi\omega\omega_x}{27}d\alpha \quad (10)$$

$$T_{ct}\Big|_0^{T_{ct}} = \frac{MR^2\pi\omega\omega_x}{27}\alpha\Big|_0^{\pi} \text{ or } T_{ct} = \frac{MR^2\pi^2\omega\omega_x}{27} \quad (11)$$

The change in centrifugal torques acts on the upper and lower sides of the spinning cone, then the total resistance torque T_{ct} is obtained when the result of equation (11) is increased twice.

$$T_{ct} = \frac{2MR^2\pi^2\omega\omega_x}{27} = \frac{20}{81}\pi^2 J\omega\omega_x \quad (12)$$

Where $J = 3MR^2/10$ is the cone's mass moment of inertia and other components are as specified above.

Common inertial torques acting on a spinning cone

The action of the external torque on the spinning cone leads to a change in the direction of the mass elements' tangential velocity V and produces their acceleration in which directions are parallel to the spinning cone's axis *oz*. The initial scheme for the computing of the change in the tangential velocity V is the same as in the published paper [18]. The accelerated motions of the spinning cone's mass elements generate their inertial forces. The inertial forces f_{in} and their variable radius of location x_m produce the inertial precession torque ΔT_{in} acting around axis *oy* which equation is as follows:

$$\Delta T_{in} = f_{in}x_m = ma_z x_m \quad (13)$$

Where x_m is the distance to the mass element's location along with axis *ox,* other components are as specified above.

The expression for the distance x_m for the mass element's location along axis *ox* is represented by the component of equation (4), but with the change in indices of axes and forces. The expression for the acceleration of the mass element along with axis *oz* a_z is presented by the following equation [18]:

$$\alpha_z = r\omega^2\Delta\gamma\sin\alpha \quad (14)$$

Where all other components are as specified above.

The defined parameters (Equation (2)) and equation (14) are substituted into equation (13) that yields the following equation:

$$\Delta T_{in} = \frac{3M\omega^2}{4\pi R}r\Delta r\Delta\delta\Delta\gamma\sin\alpha\times x_m \quad (15)$$

Where $y_m = x_m$ that represented by the Eq. (4), other parameters are as specified above.

The following solution is the same as for equation (5) of Section 2 and yields the equation for the precession torque acting around axis *oy* in a counter-clockwise direction.

$$T_{in} = \frac{20}{81}\pi^2 J\omega\omega_x \quad (16)$$

The total initial precession torque T_p acting on the circular cone around axis *oy* represents the sum of the precession torques generated by the common inertial forces (Equation (16)) of the mass elements and the change in the angular momentum $T_{am} = J\omega\omega_x$ whose equation is as follows:

$$T_p = T_{in} + T_{am} = \left(\frac{20}{81}\pi^2 + 1\right)J\omega\omega_x \quad (17)$$

Where all components are as specified above.

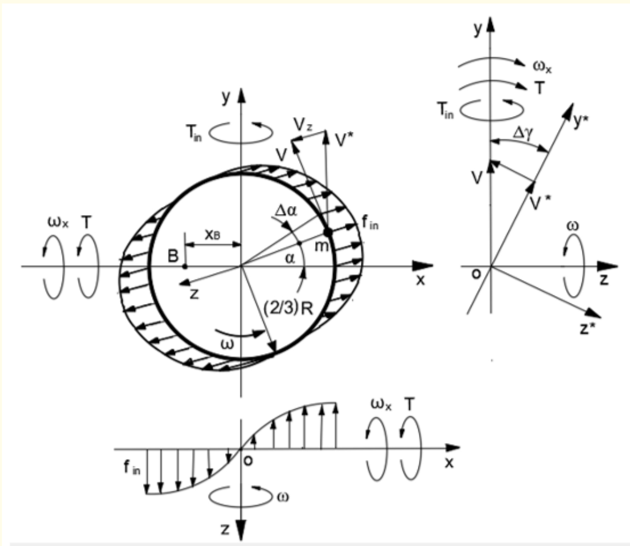

Figure 3: Schematic of acting common inertial forces and torques on the spinning cone.

Coriolis torques acting on a spinning cone

The mass element *m* travels on the arbitrary circle on the cone's plane, which turns on the angle $\Delta\gamma$ around axis *ox.* This turn leads to change in the direction of the tangential velocity *V* of mass elements and produces the acceleration and Coriolis forces of the rotating mass elements. The initial scheme for the computing of the change in the tangential velocity *V* is the same as in the published paper [18]. Similar Coriolis forces are located on parallel planes of the cone. The torque ΔT_{cr} generated by the Coriolis force f_{cr} of the spinning cone is expressed by the following equation:

$$\Delta T_{cr} = f_{cr} y_m = m a_z y_m \quad (18)$$

Where components are represented in equation (3) and (4) of Section 2.

The expression for a_z is represented by the following equation:

$$\alpha_z = \frac{dV_z}{dt} = \frac{d(V\cos\alpha\sin\Delta\gamma)}{dt} = V\cos\alpha\frac{d\gamma}{dt} = r\omega\omega_x\cos\alpha \quad (19)$$

Where $a_z = dV_z/dt$ is the Coriolis acceleration of the mass element along with axis *oz*; $V_z = V\cos\alpha\sin\Delta\gamma$ is the change in the tangential velocity $V = r\omega\cos\alpha$ of the mass element; components are as specified above.

Defined parameters (Equation (2)) are substituted into f_{cr} (Equation (18)) that yield the following expression:

$$f_{cr} = \frac{3M}{4\pi R}\omega\omega_x\Delta\delta r\Delta r\cos\alpha = \frac{3M}{4\pi R}\omega\omega_x r\Delta r\Delta\delta\cos\alpha \quad (20)$$

Then equation (18) is presented by the following expression:

$$\Delta T_{cr} = \frac{3M\omega\omega_x}{4\pi R}\cos\alpha\times r\Delta r\Delta\delta\times y_C \quad (21)$$

The integrated product of Coriolis forces are the concentrated load applied to the centroid (point *C*, figure 4 of [18]) and computed by equation (20), but with its own symbols. Then, the centroid y_C is defined by the following equation:

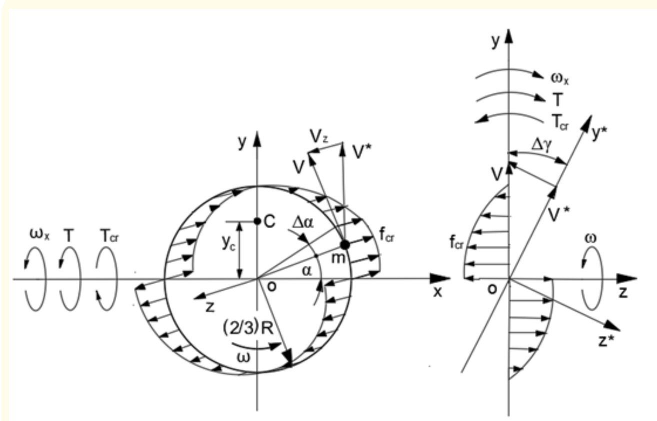

Figure 4: Schematic of the acting Coriolis forces and torques on the spinning cone.

$$y_C = \frac{2R\int_0^{\pi}\sin 2\alpha d\sin\alpha}{9\int_0^{\pi}\cos\alpha\alpha d\alpha} \quad (22)$$

Where all components are as specified above.

Equation (22) is substituted into equation (18), $\cos\alpha = -\int_0^{\pi}\sin\alpha d\alpha$ replaced by the integral expression, then the following equation emerges:

$$\int_0^{T_{cr}} dT_{cr} = \frac{3M\omega\omega_x}{4\pi R}\times\int_0^{\pi} d\delta\times\int_0^{(2/3)R} r dr\times\int_0^{\pi} -\sin\alpha d\alpha\times\frac{2R\int_0^{\pi}\sin 2\alpha d\sin\alpha}{9\int_0^{\pi}\cos\alpha d\alpha} \quad (23)$$

Solving integral equation (23) yields the following result:

$$T_{cr}\Big|_0^{T_{cr}} = \frac{3M\omega\omega_x}{4\pi R}\times\left(\delta\Big|_0^{\pi}\right)\times\left(\frac{r^2}{2}\Big|_0^{(2/3)R}\right)\times\left(-\cos\alpha\Big|_0^{\pi}\right)\times\frac{-R\cos 2\alpha\Big|_0^{\pi/2}}{9\times 2\sin\alpha\Big|_0^{\pi/2}}$$

The change of the upper limit for the trigonometric expression cosine leads to increasing the result twice. For the expression, sin2α remains the same due to the symmetrical location of the centroid. The solution to the expression giving rise to the following:

$$T_{cr} = \frac{3M\omega\omega_x}{4\pi R}\times(\pi-0)\times\frac{2}{9}R^2\times(-)(-1-1)\times\left(\frac{-R(-1-1)}{18(1-0)}\right) = \frac{MR^2\omega\omega_x}{27} \quad (24)$$

Coriolis torques act on the upper and lower sides of the cone's plane, then the total resistance torque T_{cr} is obtained when the result of equation (24) is increased twice:

$$T_{cr} = \frac{2MR^2\omega\omega_x}{27} = \frac{20}{81}J\omega\omega_x \quad (25)$$

Where $J = 3MR^2/10$ is the cone's mass moment of inertia, all parameters are as specified above.

The total resistance torque acting around axis *ox* presents the sum of the resistance torques generated by the centrifugal and Coriolis forces of the cone's mass elements whose equation is as follows:

$$T_r = T_{ct} + T_{cr} = \left(\frac{20}{81}\pi^2 + \frac{20}{81}\right)J\omega\omega_x = \frac{20}{81}(\pi^2+1)J\omega\omega_x \quad (26)$$

Where T_r is the total resistance torque of the spinning circular cone acting around axis *ox,* other components are as specified above.

Working example

The circular cone has a mass of 1.0 kg, the maximal radius of 0.1 m, spinning at 3000 rpm and the angular velocity of processing of 0.05 rpm. Determine the value of the resistance and precession torques acting on the spinning cone.

The solution to this problem uses equations (26) and (17). Substituting the initial data into equations and transformation yield the following result:

$$T_r = T_{ct} + T_{cr} = \frac{20}{81}(\pi^2+1)J\omega\omega_x = \frac{20}{81}(\pi^2+1)\frac{3MR^2}{10}\omega\omega_x =$$
$$\frac{20}{81}(\pi^2+1)\times\frac{3.0\times1.0\times0.1^2}{10}\times\frac{3000\times2\pi}{60}\times\frac{0.05\times2\pi}{60} = 0.013244283 \quad Nm$$

$$T_p = T_{in} + T_{am} = \left(\frac{20}{81}\pi^2+1\right)J\omega\omega_x = \left(\frac{10}{81}\pi^2+1\right)\frac{3MR^2}{10}\omega\omega_x =$$
$$\left(\frac{20}{81}\pi^2+1\right)\times\frac{3.0\times1.0\times0.1^2}{10}\times\frac{3000\times2\pi}{60}\times\frac{0.05\times2\pi}{60} = 0{,}0169606159 \quad Nm$$

Where T_r and T_p are the resistance and precession torques respectively generated by the rotating mass elements of the spinning circular cone.

Results and Discussion

New studies of the gyroscopic effects of the spinning objects have shown the system of inertial torques generated by the rotating masses depends on their forms and geometry. The mathematical models for internal torques acting on the spinning circular cone derived by the methods used for the spinning disc. New mathematical models are significantly different from equations for inertial torques generated by the spinning disc. The new analytical approach enables for describing the physics of the inertial torques generated by the spinning cone, which can be part of mechanisms in engineering. The mathematical models for the inertial torques of the spinning cone can be used by practitioners in industries.

Conclusion

The known publications in the area of gyroscope theory contain the simplified approaches to the gyroscopic effects of the rotating objects that cannot give correct results. The new investigations in the area of gyroscopic effects represented the mathematical models for the inertial torques generated by the rotating masses of the spinning objects. New results clearly describe the physics of gyroscopic effects manifested by different designs of rotating objects. The main contribution and novelty of this work are new mathematical models for the inertial torques acting on the spinning circular cone. The method for deriving the mathematical models for the spinning objects of different designs opens the possibility to solve problems relating to gyroscopic effects**.** The new analytical approach to properties of the spinning cone enables for showing gyroscopic effects and represents a new challenge for future studies of the dynamics of rotating objects.

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