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Research Article

A Study of a Back Order EOQ Model Using Uncertain Demand Rate

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Abstract

In this paper we study an inventory problem under intuitionistic fuzzy environment. Here we have studied the basic Economic order quantity (EOQ) model under intuitionistic dense fuzzy environment. In fuzzy set theory the concept of dense fuzzy set is quite new which is depending upon the number of negotiations/turnovers made by industrial developers. Moreover, we have discussed the preliminary concept on dense fuzzy sets with their corresponding membership functions and defuzzification methods. Then we have used the basic EOQ model into the proposed defuzzification method for optimization. A sensitive analysis, graphical illustration and conclusion are made for justification the new approach.

Keywords: Intuitionistic Fuzzy; Dense Fuzzy; Score Function; Optimization

Introduction

Zadeh [1] is the father of fuzzy set. He first introduced the idea of fuzzy set. Then it has been applied by Bellman and Zadeh [2] in decision making problems. After that, many researchers were being engaged to characterized the actual nature of the fuzzy set [3-8]. The concept of eigen fuzzy number sets was developed nicely by Goetschel and Voxman [9]. Piegat [10] gave a new definition of fuzzy set. Diamond [11,12] gave a new structure of k type fuzzy numbers and finally found star shaped fuzzy sets. Compact fuzzy sets were characterized by Diamond and Kloeden [13,14] in which its parameterization into single valued mappings is possible. Heilpern [15] discussed on fuzzy mappings and fixed point theorem. Chutia., *et al.* [16] developed an alternative method of finding the membership of a fuzzy number, by the same time Mahanta., *et al.* [17] were able to construct the structure of fuzzy arithmetic without the help of α – cuts also. The concept of fuzzy complement functional studied by Roychoudhury and Pedrycz [18]. Cauchy problem under fuzzy control is developed by Bobylev [19,20]. Buckley [21] has generalized and extended the fuzzy sets to practical applications.

In fuzzy set theory, the membership of an element to a set is single value between zero and one. However in reality, it is not always true that the degree of a non-membership of an element in a fuzzy set is equal to 1 minus the membership degree because there may be hesitation degree. Therefore, a generalization of fuzzy set was proposed by [22,23] as intuitionistic fuzzy set(IFS) which incorporate the degree of hesitation called hesitation margin and it is defined as 1 minus sum of membership and non-membership degrees respectively. The notion of defining intuitionistic fuzzy set as generalized fuzzy set is quite interesting and useful in many application area. De., *et al.* [24] gave an intuitionistic fuzzy sets approach in medical Diagnosis. Ejegwa., *et al.* [25] applied intuitionistic fuzzy set in career determination via normalized Euclidean distance method. To derive a score value we use the membership and non-membership functions. De and Sana [26] have developed a backlogging model under IFS using the score value of the objective function. De., *et al.* [27] have studied an EOQ model with backorder considering the interpolating by pass technique over the pareto optimality in intuitionistic fuzzy technique. Beg and Rashid (2014) studied a trapezoidal valued IFS for decision making problems. In intuitionistic fuzzy environment, De [28] has investigated a special type of EOQ model where the natural idle time (general closing time duration per day) has been considered. De and Pal [29,30] made an intelligent decision for a bi-objective inventory problem.

However, in the literature, probably the first time and the latest concept on fuzzy set namely Triangular dense fuzzy set (TDFS) along with the new defuzzification methods and triangular dense fuzzy neutrosophic set (TDFNS) were been developed rigorously by De and Beg [31,32]. After that, several researchers [33-38] uses these types of fuzzy set in EOQ and EPQ models. In their study they choose a Cauchy sequence which might converges to zero. Using this property, they develop the triangular dense fuzzy set where the fuzziness is decreasing with time or learning experiences. They claim that by this assumption a situation will come when no ambiguity will be found. They also incorporated this same concept in neutrosophic sets also.

In our present study, we have introduced a back order EOQ model under Intuitionistic Dense Fuzzy set. In crisp environment the EOQ model is simple but here we mainly give tress on intuitionistic dense fuzzy environment. Here we utilized the solution procedure developed by De and Beg [39]. We have used the score function for the objective function under intuitionistic fuzzy environment. Then we take defuzzified values by means of ranking index value. We get the numerical solution using the LINGO 13.0 software. Finally, graphical illustration, sensitivity analysis are made followed by a conclusion.

Preliminaries

Triangular dense fuzzy set (TDFS)

In this section we introduce different definitions of dense fuzzy set and triangular dense fuzzy set with their graphic representations and examples for subsequent use.

Definition 1

Consider the fuzzy set Ã whose components are sequence of functions generating from the mapping of natural numbers with a crisp number x. Now if all the components converge to the crisp number x as n→∞ then the fuzzy sets under considerations are called dense fuzzy set (DFS).

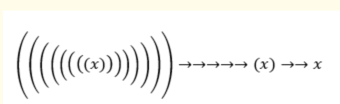

Figure 1: Dense fuzzy set.

Definition 2

Let a fuzzy number Ã = <a_1,a_2,a_3> with a_1=a_2 f_n and a_3=a_2 g_n, where f_n and g_n are the sequence of functions. If f_n and g_n are both converges to 1 as n→∞ then the fuzzy set Ã = <a_1,a_2,a_3> converges to a crisp singleton {a_2}. Then we call the fuzzy set Ã = <a_1,a_2,a_3> as a Triangular dense fuzzy set (TDFS).

Definition 3

Alternative definition of TDFS. Let, B˜ = <f_n,g_n,h_n> be three sequences of the elements of a triangular fuzzy set, where f_n and g_n are the sequence of functions. Now, if f_n→a_2, g_n→a_2 and h_n→a_2 holds for n→∞ then the fuzzy set B˜ converges to a crisp singleton {a_2}. Then we call the fuzzy set B˜ = <f_n,g_n,h_n> as a Triangular dense fuzzy set.

Definition 4

Definition of TDFS based on Cartesian product of two sets. Let Ã be the fuzzy number whose components are the elements of R×N,R being the set of real numbers and N being the set of natural numbers with the membership grade satisfying the functional relation μ:R×N→[0,1]. Now as n→∞ if μ(x,n)→1 for some x∈R and n∈N then we call the set Ã as dense fuzzy set. If Ã is triangular then it is called TDFS. Now, if for some n in N, μ(x,n) attains the highest membership degree 1 then the set itself is called "Normalized Triangular Dense Fuzzy Set" or NTDFS.

Definition 5

Definition of TDFS based on non-membership function. Let A˜ be the fuzzy number whose components are the elements of R×N whose non membership grade satisfying the functional relation γ:R×N→[0,1]. Now as n→∞ if γ(x,n)→0 for some x∈R and n∈N then we call the set Ã as dense fuzzy set. If we consider the fuzzy number Ã of the form Ã = <a_1,a_2,a_3> then we call it "Triangular Dense Fuzzy Set". Now, if for n=0,γ(x,n) attains the highest membership degree 1 then we can express this fuzzy number as "Normalized Triangular Dense Fuzzy Set" or NTDFS.

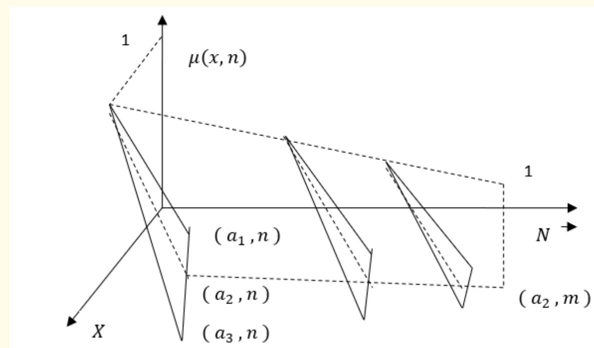

Figure 2: Membership function of NTDFS.

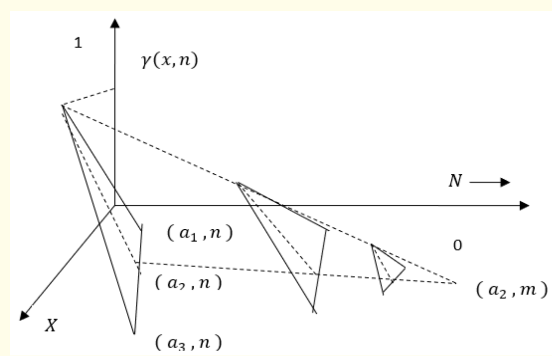

Figure 3: Non-Membership function of NTDFS.

Example 1. As per definitions (1-4) let us assume the TDFS as follows

$$\tilde{A} = \langle a_2\left(1-\frac{\rho}{1+n}\right), a_2, a_2\left(1+\frac{\sigma}{1+n}\right)\rangle, \text{ for } 0< \rho,\sigma < 1$$

The membership function for $0 \le n$ is defined as follows

$$\mu(x,n) = \begin{cases} 0 \ if \ x < a_2\left(1-\frac{\rho}{1+n}\right) \ and \ x > a_2\left(1+\frac{\sigma}{1+n}\right) \\ \left\{\frac{x-a_2\left(1-\frac{\rho}{1+n}\right)}{\frac{\rho a_2}{1+n}}\right\} \ if \ a_2\left(1-\frac{\rho}{1+n}\right) \le x \le a_2 \\ \left\{\frac{a_2\left(1+\frac{\sigma}{1+n}\right)-x}{\frac{\sigma a_2}{1+n}}\right\} \ if \ a_2 \le x \le a_2\left(1+\frac{\sigma}{1+n}\right) \end{cases}$$

Defuzzification method based on α- cuts

The left and the right α- cuts of a triangular dense fuzzy number A˜= μ(x,n) are,

$L^{-1}(\alpha,n) = a_2(1-\frac{\rho}{1+n}+\frac{\rho\alpha}{1+n})$ and $R^{-1}(\alpha,n) = a_2(1+\frac{\sigma}{1+n}-\frac{\sigma\alpha}{1+n})$.

So, $L^{-1}(\alpha,n) + R^{-1}(\alpha,n)$

$$= a_2\left(2-\frac{\rho}{1+n}+\frac{\rho\alpha}{1+n}+\frac{\sigma}{1+n}-\frac{\sigma\alpha}{1+n}\right)$$

$$= a_2\left\{2+\frac{(\rho-\sigma)\alpha}{1+n}+\frac{\sigma-\rho}{1+n}\right\}$$

Now, as per De and Beg (2016) the defuzzification method for the TDFS $(\tilde{A})$ and it is given by

$$I(\tilde{A}) = \frac{1}{2N}\sum_{n=0}^{N}\int_0^1\{L^{-1}(\alpha,n)+R^{-1}(\alpha,n)\}d\alpha$$

Thus, $\int_0^1\{L^{-1}(\alpha,n)+R^{-1}(\alpha,n)\}d\alpha = a_2\left\{2+\frac{\sigma-\rho}{2(1+n)}\right\}$.

Hence $I(\tilde{A}) = \frac{1}{2N}\sum_{n=0}^{N} a_2\left\{2+\frac{\sigma-\rho}{2(1+n)}\right\}$

$$= \frac{a_2}{2N}\left[2N+\frac{\sigma-\rho}{2}\left\{\frac{1}{1+0}+\frac{1}{1+1}+\frac{1}{1+2}+\cdots+\frac{1}{1+N}\right\}\right]$$

Obviously, as $N\to\infty$, $I(\tilde{A})\to a_2$.

Notations

C_1: Holding cost per quantity per unit time.

C_2: Shortage cost per unit quantity per unit time.

C_3: Set up cost per unit time period per cycle.

D_2: Demand in Shortage period ($D_2=D_1\, e^{-t_2}$))

Q_1: Inventory level at time t_1.

Q_2: Shortage during the time t_2.

Assumptions

We have the following assumptions

1. Demand rate is uniform and known.
2. Rate of replenishment is finite.
3. Lead time is zero/negligible.
4. Shortage are allowed and fully backlogged.

Formulation of crisp mathematical model

Let the inventory starts at time t = 0 with order quantity Q_1 and demand rate D_1. After time $t = t_1$ the inventory reaches zero level and the shortage starts and it continues up to time $t = t_1+t_2$. Let Q_2 be the shortage quantity during that time period t2. Also, we assume, that the shortage time demand rate is depending on the duration of shortage time t_2.

Therefore, the mathematical problem associated to the proposed model is shown in Figure 4 and the necessary calculations are given below.

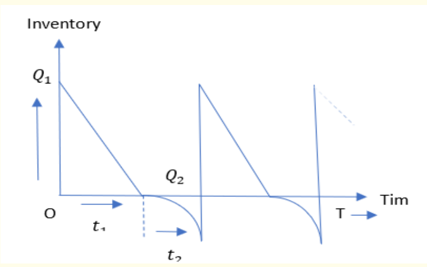

Figure 4: Inventory model with backorder for Intuitionistic dense fuzzy environment.

From Figure 4, using triangle law and area under curvature we get the following relations:

$T = t_1+t_2$ (1)

$Q_1 = D_1 t_1$ (2)

And $Q_2 = \int_0^{t_2} D_2\, dt_2 = \int_0^{t_2} D_1\, e^{-t_2}dt_2 = D_1\int_0^{t_2} e^{-t_2}dt_2 = D_1[1-\, e^{-t_2}]$ (3)

Holding cost $= \frac{1}{2}\, C_1 Q_1 t_1$ (4)

Shortage cost $= \frac{1}{2} C_2 Q_2 t_2 = \frac{1}{2} C_2 D_1(1-e^{-t_2})t_2$ (5)

Set up cost $= C_3$ (6)

$\therefore$ Total cost $= \frac{1}{2}\, C_1 Q_1 t_1 + \frac{1}{2} C_2 Q_2 t_2 + C_3$ (7)

Average total cost $= \frac{1}{T}\left(\frac{1}{2}\, C_1 Q_1 t_1 + \frac{1}{2} C_2 Q_2 t_2 + C_3\right)$

$= D_1\left\{\frac{1}{2}\, C_1\, \frac{t_1^2}{t_1+t_2} + \frac{1}{2}\, C_2\, \frac{(1-e^{-t_2})t_2}{t_1+t_2}\right\} + \frac{C_3}{t_1+t_2}$ (8)

Therefore, our problem is given by

Minimize $F(t_1,t_2) = D_1\psi(t_1,t_2)+\varphi(t_1,t_2)$ (9)

Where $\psi(t_1,t_2) = \frac{1}{2}\, C_1\, \frac{t_1^2}{t_1+t_2} + \frac{1}{2}\, C_2\, \frac{(1-e^{-t_2})t_2}{t_1+t_2}$ (10)

And $\varphi(t_1,t_2) = \frac{C_3}{t_1+t_2}$ (11)

Subject to the condition (1-3).

Fuzzy mathematical model

Since demand rate follows an important role in defining the objective function in an inventory process, so we consider the demand rate assumes flexible values in the propose model and it can be reduce by means of dense fuzzy set. So, the objective function of the crisp model (11) can be written as

$$\tilde{F} = \widetilde{D_1}\psi(t_1,t_2)+\varphi(t_1,t_2) \qquad (15)$$

Where ψ and φ are given by (10) and (11). Now, (15) can also be written as $\widetilde{D_1} = \frac{(\tilde{F}-\varphi)}{\psi}$ (16)

Now using the membership and non-membership function (8) and (9) we have

Now the membership function is $< d_1\left(1-\frac{\rho}{1+n}\right), d_1, d_1\left(1+\frac{\sigma}{1+n}\right) >$

$$\mu(D_1) = \begin{cases} \left\{\dfrac{D_1-d_1(1-\frac{\rho}{1+n})}{\frac{d_1\rho}{1+n}}\right\}, d_1(1-\frac{\rho}{1+n}) \le D_1 \le d_1 \\ \left\{\dfrac{d_1\left(1+\frac{\sigma}{1+n}\right)-D_1}{\frac{d_1\sigma}{1+n}}\right\}, d_1 \le D_1 \le d_1(1+\frac{\sigma}{1+n}) \\ 0, elsewhere \end{cases} \qquad (17)$$

The non-membership function for the demand rate is

$$\gamma(D_1) = \begin{cases} \left\{\dfrac{d_1e^{-n}-D_1}{d_1\rho e^{-n}}\right\},\ d_1(1-\rho)e^{-n} < D_1 < d_1e^{-n} \\ \left\{\dfrac{D_1-d_1e^{-n}}{d_1\sigma e^{-n}}\right\},\ d_1e^{-n} < D_1 < d_1(1+\sigma)e^{-n} \\ 0, elsewhere \end{cases} \qquad (18)$$

So, the score of the demand rate is given by

$S(D_1) = \mu(D_1) - \gamma(D_1)$

$$= \begin{cases} \left\{\dfrac{D_1-d_1(1-\frac{\rho}{1+n})}{\frac{d_1\rho}{1+n}} - \dfrac{d_1e^{-n}-D_1}{d_1\rho e^{-n}}\right\},\ d_1(1-\rho)e^{-n} < D_1 < d_1e^{-n} \\ \left\{\dfrac{d_1\left(1+\frac{\sigma}{1+n}\right)-D_1}{\frac{d_1\sigma}{1+n}} - \dfrac{D_1-d_1e^{-n}}{d_1\sigma e^{-n}}\right\},\ d_1e^{-n} < D_1 < d_1(1+\sigma)e^{-n} \\ 0, elsewhere \end{cases}$$

$$= \begin{cases} \left\{\frac{D_1(1+n+e^n)}{d_1\rho} + \frac{\rho-2-n}{\rho}\right\},\ d_1(1-\rho)e^{-n} < D_1 < d_1 e^{-n} \\ \left\{\frac{2+n+\sigma}{\sigma} - \frac{D_1(1+n+e^n)}{d_1\sigma}\right\},\ d_1 e^{-n} < D_1 < d_1(1+\sigma)e^{-n} \\ 0, elsewhere \end{cases} \quad (19)$$

$$S(D_1) = \begin{cases} k_1 D_1 + k_2,\ d_1(1-\rho)e^{-n} < D_1 < d_1 e^{-n} \\ k_3 - k_4 D_1,\ d_1 e^{-n} < D_1 < d_1(1+\sigma)e^{-n} \\ 0, elsewhere \end{cases} \quad (20)$$

Where $k_1 = \frac{(1+n+e^n)}{d_1\rho}, k_2 = \frac{\rho-2-n}{\rho}, k_3 = \frac{2+n+\sigma}{\sigma}, and\ k_4 = \frac{(1+n+e^n)}{d_1\sigma}$

The left and the right α-cuts of a triangular dense fuzzy number ($\tilde{D_1}$) are $\left[\frac{\alpha-k_2}{k_1}, \frac{k_3-\alpha}{k_4}\right]$.

Therefore, $L^{-1}(\alpha,n) = \frac{\alpha-k_2}{k_1}$ and $R^{-1}(\alpha,n) = \frac{k_3-\alpha}{k_4}$.

$$\text{So, } L^{-1}(\alpha,n) + R^{-1}(\alpha,n) = \frac{\alpha-k_2}{k_1} + \frac{k_3-\alpha}{k_4} \quad (21)$$

Now, using De and Beg (2016) we get

$$I(\widetilde{D_1}) = \frac{1}{2N}\sum_{n=0}^{N}\int_0^1 \{L^{-1}(\alpha,n) + R^{-1}(\alpha,n)\}\, d\alpha$$

$$\text{Thus } \int_0^1 \{L^{-1}(\alpha,n) + R^{-1}(\alpha,n)\}\, d\alpha = \int_0^1 \left(\frac{\alpha-k_2}{k_1} + \frac{k_3-\alpha}{k_4}\right) d\alpha = \frac{1}{2k_1} + \left(\frac{k_3}{k_4} - \frac{k_2}{k_1}\right) - \frac{1}{2k_4}$$

$$I(\widetilde{D_1}) = \frac{1}{2N}\sum_{n=0}^{N}\int_0^1 \{L^{-1}(\alpha,n) + R^{-1}(\alpha,n)\}\, d\alpha = \frac{1}{2N}\sum_{n=0}^{N}\left(\frac{1}{2k_1} + \left(\frac{k_3}{k_4} - \frac{k_2}{k_1}\right) - \frac{1}{2k_4}\right)$$

$$= \frac{1}{2N}\left[\frac{d_1}{2}\left(\frac{8+\sigma-\rho}{2} + \frac{12+\sigma-\rho}{2+e} + \frac{16+\sigma-\rho}{3+e^2} + \cdots + \frac{8+4N+\sigma-\rho}{1+N+e^N}\right)\right] \quad (22)$$

However, to find the membership and non-membership functions of the fuzzy objective functions we take the relation (12) for (16) we get

$$\mu\left(\frac{\tilde{F}-\varphi}{\psi}\right) = \begin{cases} \left\{\frac{\left(\frac{\tilde{F}-\varphi}{\psi}\right) - d_1\left(1-\frac{\rho}{1+n}\right)}{d_1\left(\frac{\rho}{1+n}\right)}\right\}, d_1\left(1-\frac{\rho}{1+n}\right) \le \frac{\tilde{F}-\varphi}{\psi} \le d_1 \\ \left\{\frac{d_1\left(1+\frac{\sigma}{1+n}\right) - \frac{\tilde{F}-\varphi}{\psi}}{\frac{d_1\sigma}{1+n}}\right\}, d_1 \le \frac{\tilde{F}-\varphi}{\psi} \le d_1\left(1+\frac{\sigma}{1+n}\right) \\ 0, elsewhere \end{cases}$$

$$\Rightarrow \mu(\tilde{F}) = \begin{cases} \left\{\frac{1+n}{d_1\rho\psi}\tilde{F} - (\varphi + d_1\psi)\frac{1+n}{d_1\rho\psi} + 1\right\},\ d_1\left(1-\frac{\rho}{1+n}\right)\psi + \varphi \le \tilde{F} \le d_1\psi + \varphi \\ \left\{1 + (\varphi + d_1\psi)\frac{1+n}{d_1\sigma\psi} - \frac{1+n}{d_1\sigma\psi}\tilde{F}\right\},\ d_1\psi + \varphi \le \tilde{F} \le d_1\left(1+\frac{\sigma}{1+n}\right)\psi + \varphi \\ 0, elsewhere \end{cases} \quad (23)$$

Now the left and right α- cuts of the triangular dense fuzzy number $\tilde{F}$ are

$$\left[(\varphi + d_1\psi) + \frac{(\alpha-1)d_1\rho\psi}{1+n}, (\varphi + d_1\psi) + \frac{(1-\alpha)d_1\sigma\psi}{1+n}\right]$$

Therefore

$L^{-1}(\alpha,n) = (\varphi + d_1\psi) + \frac{(\alpha-1)d_1\rho\psi}{1+n}$ and

$R^{-1}(\alpha,n) = (\varphi + d_1\psi) + \frac{(1-\alpha)d_1\sigma\psi}{1+n}$.

So, $L^{-1}(\alpha,n) + R^{-1}(\alpha,n) = 2(\varphi + d_1\psi) + \frac{(\alpha-1)d_1(\rho-\sigma)\psi}{1+n}$

$$I(\tilde{F}) = \frac{1}{2N}\sum_{n=0}^{N}\int_0^1 \{L^{-1}(\alpha,n) + R^{-1}(\alpha,n)\}\, d\alpha$$

$$\text{Thus } \int_0^1 \{L^{-1}(\alpha,n) + R^{-1}(\alpha,n)\}\, d\alpha = \int_0^1 \left\{2(\varphi + d_1\psi) + \frac{(\alpha-1)d_1(\rho-\sigma)\psi}{1+n}\right\} d\alpha$$

$$= 2(\varphi + d_1\psi) - \frac{d_1(\rho-\sigma)\psi}{2(1+n)}$$

$$I(\tilde{F}) = \frac{1}{2N}\sum_{n=0}^{N}\left\{2(\varphi + d_1\psi) - \frac{d_1(\rho-\sigma)\psi}{2(1+n)}\right\} =$$

$$(\varphi + d_1\psi) - \frac{d_1(\rho-\sigma)\psi}{4N}\left\{\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{1+N}\right\} \quad (24)$$

Obviously, as $N \to \infty, I(\tilde{F}) \to \varphi + d_1\psi$

Where, $\varphi(t_1, t_2)$ and $\psi(t_1, t_2)$ can be obtained from (10) and (11).

And the non-membership function of $\tilde{F}$ is given by

$$\gamma(d) = \begin{cases} \left\{\frac{d_1 e^{-n} - d}{d_1\rho e^{-n}}\right\},\ d_1(1-\rho)e^{-n} < d < d_1 e^{-n} \\ \left\{\frac{d - d_1 e^{-n}}{d_1\sigma e^{-n}}\right\},\ d_1 e^{-n} < d < d_1(1+\sigma)e^{-n} \\ 0, elsewhere \end{cases}$$

$$\Rightarrow \gamma(F) = \begin{cases} \left\{\frac{d_1 e^{-n} - \frac{F-\varphi}{\psi}}{d_1\rho e^{-n}}\right\},\ d_1(1-\rho)e^{-n}\psi + \varphi < F < \psi d_1 e^{-n} + \varphi \\ \left\{\frac{\frac{F-\varphi}{\psi} - d_1 e^{-n}}{d_1\sigma e^{-n}}\right\},\ d_1 e^{-n}\psi + \varphi < F < d_1(1+\sigma)e^{-n}\psi + \varphi \\ 0, elsewhere \end{cases} \quad (25)$$

Now the score function of the proposed objective function is obtained as

$S(w) = \mu(w) - \gamma(w)$

$$= \begin{cases} \frac{(1+n)(w-\varphi) - d_1\psi(1+n)\left(1-\frac{\rho}{1+n}\right)}{d_1\rho\psi} - \frac{d_1\psi - (w-\varphi)e^n}{d_1\rho\psi} \\ \frac{d_1\psi(1+n) + d_1\psi\sigma - (1+n)(w-\varphi)}{d_1\sigma\psi} - \frac{(w-\varphi)e^n - d_1\psi}{d_1\sigma\psi} \\ 0, elsewhere \end{cases}$$

$$= \begin{cases} w\left(\frac{1+n+e^n}{d_1\rho\psi}\right) - \varphi\left(\frac{1+n+e^n}{d_1\rho\psi}\right) + \frac{(\rho-1-n)}{\rho} - \frac{1}{\rho} \\ \varphi\left(\frac{1+n+e^n}{d_1\sigma\psi}\right) + \frac{(\sigma+1+n)}{\sigma} + \frac{1}{\sigma} - w\left(\frac{1+n+e^n}{d_1\sigma\psi}\right) \\ 0, elsewhere \end{cases} \quad (26)$$

$$= \begin{cases} \delta_1 w - \delta_2 \\ \delta_3 - \delta_4 w \\ 0, elsewhere \end{cases} \quad (27)$$

Where $\delta_1 = \frac{1+n+e^n}{d_1\rho\psi}, \delta_2 = \phi\left(\frac{1+n+e^n}{d_1\rho\psi}\right) + \frac{(1+n-\rho)}{\rho} + \frac{1}{\rho}, \delta_3 = \phi\left(\frac{1+n+e^n}{d_1\sigma\psi}\right) + \frac{(\sigma+1+n)}{\sigma} + \frac{1}{\sigma}$

and $\delta_4 = w\left(\frac{1+n+e^n}{d_1\sigma\psi}\right)$.

Now the α- cuts of the above score function is obtained as $\left[\frac{\alpha+\delta_2}{\delta_1}, \frac{\delta_3-\alpha}{\delta_4}\right]$ where, the left and right α- cuts are

$L^{-1}(\alpha,n) = \frac{\alpha+\delta_2}{\delta_1}, R^{-1}(\alpha,n) = \frac{\delta_3-\alpha}{\delta_4}$

So, $L^{-1}(\alpha,n) + R^{-1}(\alpha,n) = \frac{\alpha+\delta_2}{\delta_1} + \frac{\delta_3-\alpha}{\delta_4}$

Now, using De and Beg (2016) we get

$$I(\tilde{Z}) = \frac{1}{2N}\sum_{n=0}^{N}\int_0^1 \{L^{-1}(\alpha,n) + R^{-1}(\alpha,n)\}\, d\alpha \quad (28)$$

$$\text{Thus } \int_0^1 \{L^{-1}(\alpha,n) + R^{-1}(\alpha,n)\}\, d\alpha$$

$$= \int_0^1 \left(\frac{\alpha+\delta_2}{\delta_1} + \frac{\delta_3-\alpha}{\delta_4}\right) d\alpha = \frac{1}{2\delta_1} + \left(\frac{\delta_3}{\delta_4} + \frac{\delta_2}{\delta_1}\right) - \frac{1}{2\delta_4}$$

$$= \frac{1}{\delta_1}\left(\frac{1}{2} + \delta_2\right) + \frac{1}{\delta_4}\left(\delta_3 - \frac{1}{2}\right)$$

Now from (28)

$$I(\tilde{Z}) = \frac{1}{2N}\sum_{n=0}^{N}\int_0^1\{L^{-1}(\alpha,n) + R^{-1}(\alpha,n)\}\,d\alpha$$

$$= \frac{1}{2N}\sum_{n=0}^{N}\left[\frac{1}{\delta_1}\left(\frac{1}{2}+\delta_2\right) + \frac{1}{\delta_4}\left(\delta_3 - \frac{1}{2}\right)\right]$$

$$= \frac{1}{2N}\sum_{n=0}^{N}\left[\begin{array}{l}\frac{d_1\rho\psi}{1+n+e^n}\left(\frac{1}{2}+\varphi\left(\frac{1+n+e^n}{d_1\rho\psi}\right)+\frac{(1+n-\rho)}{\rho}+\frac{1}{\rho}\right)\\ +\frac{d_1\sigma\psi}{1+n+e^n}\left(\varphi\left(\frac{1+n+e^n}{d_1\sigma\psi}\right)+\frac{(\sigma+1+n)}{\sigma}+\frac{1}{\sigma}-\frac{1}{2}\right)\end{array}\right]$$

$$= \varphi + d_1\psi + \frac{(\sigma-\rho)d_1\psi}{8} \text{ for } n = 0 \text{ and } N = 1 \qquad (29)$$

Again, the index values of the order quantity and the shortage quantity are given by

$$I(\widetilde{Q_1}) = t_1 I(\widetilde{D_1})$$

$$= t_1\frac{1}{2N}\left[\frac{d_1}{2}\left(\frac{8+\sigma-\rho}{2}+\frac{12+\sigma-\rho}{2+e}+\frac{16+\sigma-\rho}{3+e^2}+\cdots+\frac{8+4N+\sigma-\rho}{1+N+e^N}\right)\right] \qquad (30)$$

And $I(\widetilde{Q_2}) = (1-e^{-t_2})I(\widetilde{D_1})$

$$= (1-e^{-t_2})\frac{1}{2N}\left[\frac{d_1}{2}\left(\frac{8+\sigma-\rho}{2}+\frac{12+\sigma-\rho}{2+e}+\frac{16+\sigma-\rho}{3+e^2}+\cdots+\frac{8+4N+\sigma-\rho}{1+N+e^N}\right)\right] \qquad (31)$$

Thus the equivalent crisp problem of the dense fuzzy model is given by

$$\text{Minimize } I(\tilde{Z}) = \frac{1}{2N}\left[2\varphi N + \frac{d_1\psi}{2}\left(\frac{8+\sigma-\rho}{2}+\frac{12+\sigma-\rho}{2+e}+\frac{16+\sigma-\rho}{3+e^2}+\cdots+\frac{8+4N+\sigma-\rho}{1+N+e^N}\right)\right]$$

Particular cases

Numerical example 1:

Let us consider C_1=2.5, C_2=1.8, C_3=1200, d_1=100, ρ=0.3, σ=0.2 then we get the following results.

Model	Time (t_1^*)	Time (t_2^*)	Time (T^*)	Q_1^*	Q_2^*	Minimum cost (Z^*)
Crisp	3.04	3.04	6.09	304.67	95.24	430.21
Fuzzy	3.06	3.06	6.13	302.86	94.15	427.29
Dense Fuzzy	3.59	3.59	7.18	261.91	70.87	362.53

Table 1: Optimal solution of EOQ model.

Sensitivity analysis

Based on the numerical example consider above for the single production plant model, we now calculate the corresponding outputs for changing inputs parameter one by one. Again the sensitivity analysis is performed by changing of each parameter C_1,C_2,C_3,ρ,σ and d_1 by +30%, +20%, -20% and -30% considering one at a time and keeping the remaining unchanged.

Values of the parameter	% Change	Time (t_1^*)	Time (t_2^*)	Time(T^*)	Q_1^*	Q_2^*	Minimum cost(Z^*)	$\frac{Z^*-Z_*}{Z_*}$ 100%
C_1	+30	3.14	3.14	6.29	229.26	69.73	408.37	12.64
	+20	3.27	3.27	6.55	238.76	70.12	393.75	8.61
	-20	4.02	4.02	8.05	293.37	71.57	327.93	-9.54
	-30	4.30	4.30	8.61	313.97	71.89	308.97	-14.77
C_2	+30	3.58	3.58	7.16	261.12	70.84	372.09	2.63
	+20	3.58	3.58	7.17	261.39	70.85	368.90	1.75
	-20	3.60	3.60	7.20	262.44	70.88	356.15	-1.75
	-30	3.60	3.60	7.20	262.70	70.89	352.96	-2.63
C_3	+30	4.11	4.11	8.22	299.81	71.68	409.23	12.88
	+20	3.94	3.94	7.89	287.75	71.46	394.34	8.77
	-20	3.19	3.19	6.39	233.17	69.90	327.20	-9.74
	-30	2.98	2.98	5.96	217.36	69.18	307.79	-15.09
ρ	+30	3.61	3.61	7.22	260.82	70.28	360.83	-0.46
	+20	3.60	3.60	7.20	261.18	70.48	361.39	-0.31
	-20	3.58	3.58	7.16	262.64	71.25	363.65	0.30
	-30	3.57	3.57	7.15	263.00	71.45	364.22	0.46
σ	+30	3.58	3.58	7.16	262.64	71.25	363.65	0.30
	+20	3.58	3.58	7.17	262.40	71.12	363.28	0.20
	-20	3.60	3.60	7.20	261.43	70.61	361.77	-0.20
	-30	3.60	3.60	7.20	261.18	70.48	361.39	-0.20
d_1	+30	3.13	3.13	6.26	296.95	90.61	417.78	15.24
	+20	3.26	3.26	6.53	285.85	84.12	400.06	10.35
	-20	4.03	4.03	8.06	235.07	57.26	321.49	-11.32
	-30	4.31	4.31	8.63	220.24	50.33	299.27	-17.44

Table 2: Sensitivity analysis with changing parameters from (-30% to +30%).

Discussion on Sensitivity Analysis

- The optimum objective Z* is insensitive for +30% change of holding cost but at -30% change the objective value assumes highly sensitive.
- The optimum objective Z* is moderately sensitive for both +30% and -30% change.
- The optimum objective Z* is insensitive for +30% change of set up cost but at -30% change the objective value assumes highly sensitive.
- The optimum objective Z* is highly sensitive for +30% change of fuzzy flexibility parameter ρ and at -30% change the objective value assumes moderately sensitive.
- The optimum objective Z* is moderately sensitive for +30% change of fuzzy flexibility parameter σ and at -30% change the objective value assumes highly sensitive.
- The optimum objective Z* is insensitive for +30% change of d_1 but at -30% change the objective value assumes highly sensitive.

Graphical Illustrations

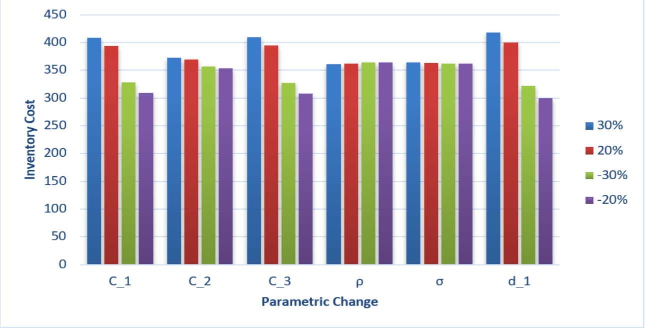

Figure 5: Inventory cost vs parametric change.

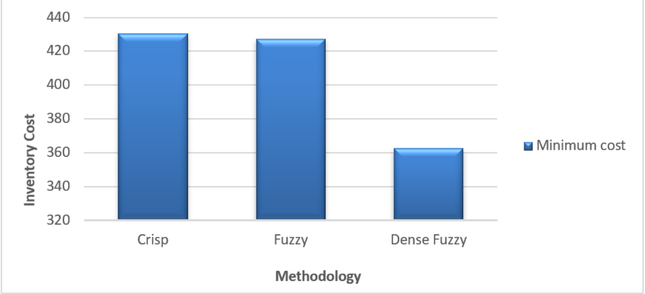

Figure 6: Inventory cost vs Methodology.

Discussion on graphical illustration

From the figure 5 we see that

- The average inventory cost is highly sensitive whenever we made a change of the set up cost (C_3), the holding cost (C_1), and the parameter d_1 from -30% to +30%.
- The average inventory cost is low sensitive whenever we made a change of the parameter σ from -30% to +30%.
- The average inventory cost is low sensitive whenever we made a change of the shortage cost (C_2) and the parameter ρ from -30% to +30%.

From the figure 6 we see that the inventory cost is more less for dense fuzzy model than crisp model and fuzzy model also.

Conclusion

Here we have discussed a simple EOQ model under intuitionistic dense fuzzy environment. In our study we see that the crisp model is inferior to the dense fuzzy model. However, the intuitionistic fuzzy model is quite better than any other model because in this model we get better optimal result. The novelty of this model lies in its new approach of studying the inventory problem which is rare in existing literature.

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