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Research Article

Formulation of an Exact Proof of the Riemann Hypothesis

Fayez Fok Al Adeh*

President of the Syrian Cosmological Society, Syria

***Corresponding Author:** Fayez Fok Al Adeh, President of the Syrian Cosmological Society, Syria.

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Abstract

I have already discovered a simple proof of the Riemann Hypothesis. The hypothesis states that the nontrivial zeros of the Riemann zeta function have real part equal to 0.5. I assume that any such zero is s =a+ bi. I use integral calculus in the first part of the proof. In the second part I employ variational calculus. Through equations (50) to (59) I consider (a) as a fixed exponent, and verify that a = 0.5. From equation (60) onward I view (a) as a parameter [6] (a<0.5) and arrive at a contradiction. At the end of the proof (from equation (73)) and through the assumption that (a) is a parameter, I verify again that a = 0.5.

Subj-class: Functional Analysis, Complex variables, General Mathematics.

Keywords: Definite Integral; Indefinite Integral; Variational Calculus

Introduction

The Riemann zeta function is the function of the complex variable s = a + bi (i = $\sqrt{-1}$), defined in the half plane a >1 by the absolute convergent series

$$\zeta(s) = \sum_{1}^{\infty} \frac{1}{n^s} \quad (1)$$

And in the whole complex plane by analytic continuation [1].

The function $\zeta(s)$ has zeros at the negative even integers -2, -4, ... and one refers to them as the trivial zeros. The Riemann hypothesis states that the nontrivial zeros of $\zeta(s)$ have real part equal to 0.5.

Proof of the hypothesis

We begin with the equation

$$\zeta(s)=0 \quad (2)$$

And with

$$s= a+bi \quad (3)$$

$$\zeta(a+bi)=0 \quad (4)$$

It is known that the nontrivial zeros of $\zeta(s)$ are all complex. Their real parts lie between zero and one.

If 0 < a < 1 then [1]

$$\zeta(s) = s\int_0^\infty \frac{[x]-x}{x^{s+1}}dx \quad (0 < a < 1) \quad (5)$$

[x] is the integer function

Hence

$$\int_0^\infty \frac{[x]-x}{x^{s+1}}dx = 0 \quad (6)$$

Therefore

$$\int_0^\infty ([x]-x)x^{-1-a-bi}dx = 0 \quad (7)$$

$$\int_0^\infty ([x]-x)x^{-1-a}x^{-bi}dx = 0 \quad (8)$$

$$\int_0^\infty x^{-1-a}([x]-x)(\cos(b\log x) - i\sin(b\log x))dx = 0 \quad (9)$$

Separating the real and imaginary parts we get

$$\int_0^\infty x^{-1-a}([x]-x)\cos(b\log x)dx = 0 \quad (10)$$

$$\int_0^\infty x^{-1-a}([x]-x)\sin(b\log x)dx = 0 \quad (11)$$

According to the functional equation [3], if $\zeta(s)$=0 then $\zeta(1-s)$= 0. Hence we get besides equation (11)

$$\int_0^\infty x^{-2+a}([x]-x)\sin(b\log x)dx = 0 \quad (12)$$

In equation (11) replace the dummy variable x by the dummy variable y

$$\int_0^\infty y^{-1-a}([y]-y)\sin(b\log y)dy = 0 \quad (13)$$

We form the product of the integrals (12) and (13). This is justified by the fact that both integrals (12) and (13) are absolutely convergent (this is necessary according to [2]). As to integral (12) we notice that

$$\int_0^\infty x^{-2+a}([x]-x)\sin(b\log x)dx \le \int_0^\infty \left| x^{-2+a}([x]-x)\sin(b\log x)\right| dx$$

$$\le \int_0^\infty x^{-2+a}((x))dx$$

(where ((z)) is the fractional part of z, 0 ((z))<1)

$$= \lim(t\to 0)\int_0^{1-t} x^{-1+a}dx + \lim(t\to 0)\int_{1+t}^\infty x^{-2+a}((x))dx$$

(t is a very small positive number) (since ((x)) =x whenever 0 x<1)

$$= \frac{1}{a} + \lim(t\to 0)\int_{1+t}^\infty x^{-2+a}((x))dx$$

$$< \frac{1}{a} + \lim(t\to 0)\int_{1+t}^\infty x^{-2+a}dx = \frac{1}{a}+\frac{1}{a-1}$$

And as to integral (13) we have [2] $\int_0^\infty y^{-1-a}([y]-y)\sin(b\log y)dy$

$$\le \int_0^\infty \left| y^{-1-a}([y]-y)\sin(b\log y)\right| dy$$

$$\le \int_0^\infty y^{-1-a}((y))dy$$

$$= \lim(t\to 0)\int_0^{1-t} y^{-a}dy + \lim(t\to 0)\int_{1+t}^\infty y^{-1-a}((y))dy$$

(t is a very small positive number)(since ((y))=y whenever 0 ≤ y<1)

$$= \frac{1}{1-a} + \lim(t\to 0)\int_{1+t}^\infty y^{-1-a}((y))dy$$

$$< \frac{1}{1-a} + \int_{1+t}^\infty y^{-1-a}dy = \frac{1}{1-a}+\frac{1}{a}$$

Since the limits of integration do not involve x or y, the product can be expressed as the double integral

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([x]-x)([y]-y)\sin(b\log y)\sin(b\log x)dxdy = 0 \quad (14)$$

Thus

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([x]-x)([y]-y)(\cos(b\log y + b\log x) - \cos(b\log y - b\log x))dxdy = 0 \quad (15)$$

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([x]-x)([y]-y)(\cos(b\log xy) - \cos(b\log \frac{y}{x}))dxdy = 0 \quad (16)$$

That is

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([x]-x)([y]-y)\cos(b\log xy)dxdy =$$

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([x]-x)([y]-y)\cos(b\log \frac{y}{x})dxdy \quad (17)$$

Consider the integral on the right-hand side of equation (17)

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([x]-x)([y]-y)\cos(b\log \frac{y}{x})dxdy \quad (18)$$

In this integral make the substitution $x = \frac{1}{z}$ $dx = \frac{-dz}{z^2}$

The integral becomes

$$\int_0^\infty\int_\infty^0 z^{2-a}y^{-1-a}([\frac{1}{z}]-\frac{1}{z})([y]-y)\cos(b\log zy)\frac{-dz}{z^2}dy \quad (19)$$

That is

$$-\int_0^\infty\int_\infty^0 z^{-a}y^{-1-a}([\frac{1}{z}]-\frac{1}{z})([y]-y)\cos(b\log zy)dzdy \quad (20)$$

This is equivalent to

$$\int_0^\infty\int_0^\infty z^{-a}y^{-1-a}([\frac{1}{z}]-\frac{1}{z})([y]-y)\cos(b\log zy)dzdy \quad (21)$$

If we replace the dummy variable z by the dummy variable x, the integral takes the form

$$\int_0^\infty\int_0^\infty x^{-a}y^{-1-a}([\frac{1}{x}]-\frac{1}{x})([y]-y)\cos(b\log xy)dxdy \quad (22)$$

Rewrite this integral in the equivalent form

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}(x^{2-2a}[\frac{1}{x}]-\frac{x^{2-2a}}{x})([y]-y)\cos(b\log xy)dxdy \quad (23)$$

Thus equation 17 becomes

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([x]-x)([y]-y)\cos(b\log xy)dxdy =$$

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}(x^{2-2a}[\frac{1}{x}]-\frac{x^{2-2a}}{x})([y]-y)\cos(b\log xy)dxdy \quad (24)$$

Write the last equation in the form

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([y]-y)\cos(b\log xy)\{(x^{2-2a}[\frac{1}{x}] - \frac{x^{2-2a}}{x})-([x]-x)\}dx \quad (25)$$

dy=0

Let p <0 be an arbitrary small positive number [5]. We consider the following regions in the x –y plane.

The region of integration I = [0, ∞)× [0, ∞) (26)

The large region I1 =[p, ∞)× [p, ∞) (27)

The narrow strip I2 =[p, ∞)× [0,p] (28)

The narrow strip I3 = [0,p]× [0, ∞) (29)

Note that

I = I1 ∪ I 2 ∪ I 3 (30)

Denote the integrand in the left hand side of equation (25) by

$$F(x,y) = x^{-2+a}\, y^{-1-a}([y]-y)\cos(b\log xy)\{(x^{2-2a}[\frac{1}{x}] - \frac{x^{2-2a}}{x})-([x]-x)\} \quad (31)$$

Let us find the limit of F (x,y) as x → ∞ and y → ∞. This limit is given by

$$\text{Lim } x^{-a}\, y^{-1-a}[-((y))]\cos(b\log xy)[-((\frac{1}{x}))+((x))\, x^{2a-2}] \quad (32)$$

((z)) is the fractional part of the number z, 0 ((z)) < 1

The above limit vanishes, since all the functions [- ((y))], cos (blog xy), - $((\frac{1}{x}))$, and ((x)) remain bounded as x → ∞ and y → ∞

Note that the function F (x,y) is defined and bounded in the region I1. We can prove that the integral

$$\iint_{I1} F(x,y)\,dx\,dy \text{ is bounded as follows} \quad (33)$$

$$\iint_{I1} F(x,y)\,dx\,dy = \iint_{I1} x^{-a}\, y^{-1-a}[-((y))]\cos(b\log xy)[-((\frac{1}{x}))+((x))\, x^{2a-2}]\,dx\,dy \quad (34)$$

$$\leq \left|\iint_{I1} x^{-a}\, y^{-1-a}[-((y))]\cos(b\log xy)[-((\frac{1}{x}))+((x))\, x^{2a-2}]\,dx\,dy\right|$$

$$= \left|\int_p^\infty \left(\int_p^\infty x^{-a}\cos(b\log xy)[-((\frac{1}{x}))+((x))\, x^{2a-2}]\,dx\right) y^{-1-a}[-((y))]\,dy\right|$$

$$\leq \int_p^\infty \left|\left(\int_p^\infty x^{-a}\cos(b\log xy)[-((\frac{1}{x}))+((x))\, x^{2a-2}]\,dx\right)\right| \left|y^{-1-a}[-((y))]\right|\,dy$$

$$\leq \int_p^\infty \left(\int_p^\infty x^{-a}\left|\cos(b\log xy)\right| \left|[-((\frac{1}{x}))+((x))\, x^{2a-2}]\right|\,dx\right) \left|y^{-1-a}[-((y))]\right|\,dy$$

$$< \int_P^\infty x^{-a}[((\frac{1}{x}))+((x))\, x^{2a-2}]\,dx \int_P^\infty y^{-1-a}$$

$$= \frac{1}{ap^a}\int_P^\infty x^{-a}[((\frac{1}{x}))+((x))\, x^{2a-2}]\,dx$$

$$= \frac{1}{ap^a}\{\lim(t\to 0)\int_P^{1-t} x^{-a}[((\frac{1}{x}))+((x))\, x^{2a-2}]\,dx + \lim(t\to 0)\int_{1+t}^\infty x^{-a}[((\frac{1}{x}))+((x))\, x^{2a-2}]\,dx\}$$

where t is a very small arbitrary positive. number. Since the integral

$$\lim(t\to 0)\int_P^{1-t} x^{-a}[((\frac{1}{x}))+((x))\, x^{2a-2}]\,dx$$

is bounded, it remains to show that lim(t → 0)

$$\int_{1+t}^\infty x^{-a}[((\frac{1}{x}))+((x))\, x^{2a-2}]\,dx \text{ is bounded.}$$

Since x >1, then $((\frac{1}{x})) = \frac{1}{x}$ and we have

$$\lim(t\to 0)\int_{1+t}^\infty x^{-a}[((\frac{1}{x}))+((x))\, x^{2a-2}]\,dx$$

$$= \lim(t\to 0)\int_{1+t}^\infty x^{-a}[\frac{1}{x}+((x))\, x^{2a-2}]\,dx$$

$$= \lim(t\to 0)\int_{1+t}^\infty [x^{-a-1}+((x))\, x^{a-2}]\,dx$$

$$< \lim(t\to 0)\int_{1+t}^\infty [x^{-a-1}+x^{a-2}]\,dx$$

$$= \frac{1}{a(1-a)}$$

Hence the boundedness of the integral $\iint_{I1} F(x,y)\,dx\,dy$ is proved.

Consider the region

I4 = I2 ∪ I3 (35)

We know that

$$0 = \iint_I F(x,y)\,dx\,dy = \iint_{I1} F(x,y)\,dx\,dy + \iint_{I4} F(x,y)\,dx\,dy \quad (36)$$

And that

$$\iint_{I1} F(x,y)\,dx\,dy \text{ is bounded} \quad (37)$$

From which we deduce that the integral

$$\iint_{I4} F(x,y)\,dx\,dy \text{ is bounded} \quad (38)$$

Remember that

$$\iint_{I4} F(x,y)\,dx\,dy = \iint_{I2} F(x,y)\,dx\,dy + \iint_{I3} F(x,y)\,dx\,dy \quad (39)$$

Consider the integral

$$\iint_{I2} F(x,y)\,dx\,dy \leq \left|\iint_{I2} F(x,y)\,dx\,dy\right|$$

$$= \left|\int_0^p \left(\int_p^\infty x^{-a}\{((\frac{1}{x}))-((x))\, x^{2a-2}\}\cos(b\log xy)\,dx\right)\frac{1}{y^a}\,dy\right| \quad (40)$$

$$\le \int_0^p \left| \int_p^\infty (x^{-a}\{((\frac{1}{x}))-((x))\, x^{2a-2}\}\cos(b\log xy)\,dx) \right| \frac{1}{y^a}\,dy$$

$$\le \int_0^p (\int_p^\infty \left| x^{-a}\{((\frac{1}{x}))-((x))\, x^{2a-2}\}\right| \left|\cos(b\log xy)\right| dx)\frac{1}{y^a}\,dy$$

$$\le \int_P^\infty \left| x^{-a}\{((\frac{1}{x}))-((x))\, x^{2a-2}\}\right| dx \times \int_0^P \frac{1}{y^a}\,dy$$

(This is because in this region ((y)) = y). It is evident that the integral

$$\int_P^\infty \left| x^{-a}\{((\frac{1}{x}))-((x))\, x^{2a-2}\}\right|$$

dx is bounded, this was proved in the course of proving that the integral $\iint_{I1} F(x,y)\,dx\,dy$ is bounded

Also it is evident that the integral

$$\int_0^P \frac{1}{y^a}\,dy$$

is bounded. Thus we deduce that the integral (40) $\iint_{I2} F(x,y)\,dx\,dy$ is bounded.

Hence, according to equation (39), the integral $\iint_{I3} F(x,y)\,dx\,dy$ is bounded.

Now consider the integral

$$\iint_{I3} F(x,y)\,dx\,dy \qquad (41)$$

We write it in the form

$$\iint_{I3} F(x,y)\,dx\,dy = \int_0^p (\int_0^\infty y^{-1-a}((y))\cos(b\log xy)\,dy)\frac{\{((\frac{1}{x}))-x^{2a-1}\}}{x^a}\,dx \qquad (42)$$

(This is because in this region ((x)) = x)

$$\le \left| \int_0^p (\int_0^\infty y^{-1-a}((y))\cos(b\log xy)\,dy)\frac{\{((\frac{1}{x}))-x^{2a-1}\}}{x^a}\,dx \right|$$

$$\le \int_0^p \left| (\int_0^\infty y^{-1-a}((y))\cos(b\log xy)\,dy) \right| \left| \frac{\{((\frac{1}{x}))-x^{2a-1}\}}{x^a} \right| dx$$

$$\le \int_0^p (\int_0^\infty y^{-1-a}((y))\,dy) \left| \frac{\{((\frac{1}{x}))-x^{2a-1}\}}{x^a} \right| dx$$

Now we consider the integral with respect to y

$$\int_0^\infty y^{-1-a}((y))\,dy \qquad (43)$$

$$= (\lim t \to 0) \int_0^{1-t} y^{-1-a}\times y\,dy + (\lim t\to 0)\int_{1+t}^\infty y^{-1-a}((y))\,dy$$

(where t is a very small arbitrary positive number). (Note that ((y))=y whenever 0 y<1).

Thus we have (lim t → 0)

$$\int_{1+t}^\infty y^{-1-a}((y))\,dy < (\lim t\to 0)\int_{1+t}^\infty y^{-1-a}\,dy = \frac{1}{a}$$

and (lim t → 0) $\int_0^{1-t} y^{-1-a}\times y\,dy = \frac{1}{1-a}$

Hence the integral (43) $\int_0^{1-t} y^{-1-a}((y))\,dy$ is bounded.

Since $\left|\int_0^\infty y^{-1-a}((y))\cos(b\log xy)\,dy\right| \le \int_0^\infty y^{-1-a}((y))\,dy$,

we conclude that the integral

$\left|\int_0^\infty y^{-1-a}((y))\cos(b\log xy)\,dy\right|$ is a bounded function of x.

Let this function be H(x). Thus we have

$$\left|\int_0^\infty y^{-1-a}((y))\cos(b\log xy)\,dy\right| = H(x)\ \ K \text{ (K is a positive number)} \qquad (44)$$

Now equation (44) gives us

$$-K \le \int_0^\infty y^{-1-a}((y))\cos(b\log xy)\,dy \le K \qquad (45)$$

According to equation (42) we have

$$\iint_{I3} F(x,y)\,dx\,dy = \int_0^p (\int_0^\infty y^{-1-a}((y))\cos(b\log xy)\,dy)\frac{\{((\frac{1}{x}))-x^{2a-1}\}}{x^a}\,dx$$

$$\ge \int_0^p (-K)\frac{\{((\frac{1}{x}))-x^{2a-1}\}}{x^a}\,dx = K\int_p^0 \frac{\{((\frac{1}{x}))-x^{2a-1}\}}{x^a}\,dx \qquad (46)$$

Since $\iint_{I3} F(x,y)\,dx\,dy$ is bounded, then $\int_p^0 \frac{\{((\frac{1}{x}))-x^{2a-1}\}}{x^a}$ dx is also bounded. Therefore the integral

$$G = \int_0^p \frac{\{((\frac{1}{x}))-x^{2a-1}\}}{x^a}\,dx \text{ is bounded.} \qquad (47)$$

We denote the integrand of (47) by

$$F = \frac{1}{x^a}\{((\frac{1}{x}))-x^{2a-1}\} \qquad (48)$$

Let δ G [F] be the variation of the integral G due to the variation of the integrand δ F.

Since

G [F] = ∫ F dx (the integral (49) is indefinite) (49)

(here we do not consider a as a parameter, rather we consider it as a given exponent)

We deduce that $\frac{\delta G[F]}{\delta F(x)} = 1$

that is

δ G [F] = δ F (x) (50)

But we have

δ G[F] = $\int dx \frac{\delta G[F]}{\delta F(x)} \delta F(x)$ (the integral (51) is indefinite) (51)

Using equation (50) we deduce that

δ G[F] = ∫ dx δF(the integral (52) is indefinite) (52)

Since G[F] is bounded across the elementary interval [0,p], we must have that

δ G[F] is bounded across this interval (53)

From (52) we conclude that

$$\delta G = \int_0^P dx\, \delta F(x) = \int_0^P dx \frac{dF}{dx} \delta x = [\text{ F } \delta x \text{] (at x = p)- [F } \delta x \text{] (at x = 0)}$$

(54)

Since the value of [F δx] (at x = p)is bounded, we deduce from equation (54) that

lim (x → 0) F δ x must remain bounded. (55)

Thus we must have that

$(\lim x \to 0)$ [δ x $\frac{1}{x^a}$ { (($\frac{1}{x}$))- x^{2a-1} }] is bounded. (56)

First we compute

$(\lim x \to 0) \frac{\delta x}{x^a}$ (57)

Applying L 'Hospital ' rule (this is justified according to [4]) we get

$$(\lim x \to 0) \frac{\delta x}{x^a} = (\lim x \to 0) \frac{1}{a} \times x^{1-a} \times \frac{d(\delta x)}{dx} = 0 \quad (58)$$

We conclude from (56) that the product

$0 \times (\lim x \to 0)$ { (($\frac{1}{x}$))- x^{2a-1} }must remain bounded. (59)

Assume that a =0.5. (remember that we considered a as a given exponent). This value a =0.5 will guarantee that the quantity

{ (($\frac{1}{x}$))- x^{2a-1} } will remain bounded in the limit as (x → 0).

Therefore, in this case (a=0.5) (56) will approach zero as (x → 0) and hence remain bounded.

Now suppose that a< 0.5. In this case we consider a as a parameter [6]. Hence we have

G_a [x]= $\int dx \frac{F(x,a)}{x}$ (the integral (60) is indefinite) (60)

Thus

$$\frac{\delta G_a[x]}{\delta x} = \frac{F(x,a)}{x} \quad (61)$$

But we have that

$\delta G_a[x] = \int dx \frac{\delta G_a[x]}{\delta x} \delta x$ (the integral (62) is indefinite) (62)

Substituting from (61) we get

$\delta G_a[x] = \int dx \frac{F(x,a)}{x} \delta x$ (the integral (63) is indefinite) (63)

We return to equation (49) and write

G = lim (t → 0) $\int_t^p F dx$ (t is a very small positive number 0<t<p)

(64)

= { F x(at p) – lim (t → 0) Fx (at t) } - lim (t → 0) $\int_t^p$ x dF

Let us compute

lim (t → 0) Fx (at t) = lim (t → 0) t^{1-a} (($\frac{1}{t}$)) – t^a = 0 (65)

Thus equation (64) reduces to

G – Fx (at p) = - lim (t → 0) $\int_t^p$ x dF (66)

Note that the left – hand side of equation (66) is bounded. Equation (63) gives us

δ G_a =lim (t → 0) $\int_t^p dx \frac{F}{x} \delta x$ (67)

(t is the same small positive number 0<t<p)

We can easily prove that the two integrals $\int_t^p$ x dF and $\int_t^p dx \frac{F}{x} \delta x$ are absolutely convergent. Since the limits of integration do not involve any variable, we form the product of (66) and (67)

K = lim(t → 0) $\int_t^p \int_t^p$ xdF× dx $\frac{F}{x} \delta x$ = lim(t → 0) $\int_t^p$ FdF $\delta x dx$ $\int_t^p$ × (K is a bounded quantity) (68)

That is

K = lim(t → 0) [$\frac{F^2}{2}$ (at p) - $\frac{F^2}{2}$ (at t)] × [δ x (at p) - δ x (at t)]

(69)

We conclude from this equation that

{ [$\frac{F^2}{2}$ (at p) - lim(t → 0) $\frac{F^2}{2}$ (at t)] ×[δ x (at p)] } (70)

is bounded.

(since lim(x → 0) δ x = 0, which is the same thing as lim(t → 0) δ x = 0)

Since $\frac{F^2}{2}$ (at p) is bounded, we deduce at once that $\frac{F^2}{2}$ must remain bounded in the limit as (t → 0), which is the same thing as saying that F must remain bounded in the limit as (x → 0). Therefore.

$$\lim (x \to 0) \frac{((\frac{1}{x})) - x^{2a-1}}{x^a} \text{ must remain bounded.} \quad (71)$$

But

$$\lim (x \to 0) \frac{((\frac{1}{x})) - x^{2a-1}}{x^a} = \lim (x \to 0) \frac{x^{1-2a}}{x^{1-2a}} \times \frac{((\frac{1}{x})) - x^{2a-1}}{x^a} \quad (72)$$

$$= \lim (x \to 0) \frac{x^{1-2a}((\frac{1}{x})) - 1}{x^{1-a}} = \lim (x \to 0) \frac{-1}{x^{1-a}}$$

It is evident that this last limit is unbounded. This contradicts our conclusion (71) that

$$\lim (x \to 0) \frac{((\frac{1}{x})) - x^{2a-1}}{x^a} \text{ must remain bounded (for a< 0.5)}$$

Therefore the case a<0.5 is rejected. We verify here that, for a = 0.5 (71)remains bounded as (x → 0).

We have that

$$((\frac{1}{x})) - x^{2a-1} < 1 - x^{2a-1} \quad (73)$$

Therefore

$$\lim (a \to 0.5) (x \to 0) \frac{((\frac{1}{x})) - x^{2a-1}}{x^a} < \lim (a \to 0.5) (x \to 0) \frac{1 - x^{2a-1}}{x^a} \quad (74)$$

We consider the limit

$$\lim (a \to 0.5) (x \to 0) \frac{1 - x^{2a-1}}{x^a} \quad (75)$$

We write

$$a = (\lim x \to 0) (0.5 + x) \quad (76)$$

Hence we get

$$\lim (a \to 0.5) (x \to 0) \; x^{2a-1} = \lim (x \to 0) \; x^{2(0.5+x)-1} = \lim (x \to 0) x^{2x} = 1 \quad (77)$$

(Since $\lim(x \to 0) \; x^x = 1$)

Therefore we must apply L 'Hospital ' rule with respect to x in the limiting process (75)

$$\lim (a \to 0.5) (x \to 0) \frac{1 - x^{2a-1}}{x^a} = \lim (a \to 0.5) (x \to 0) \frac{-(2a-1)x^{2a-2}}{a x^{a-1}}$$

$$= \lim (a \to 0.5) (x \to 0) \frac{(\frac{1}{a} - 2)}{x^{1-a}} \quad (78)$$

Now we write again

$$a = (\lim x \to 0) (0.5 + x) \quad (79)$$

Thus the limit (78) becomes

$$\lim (a \to 0.5) (x \to 0) \frac{(\frac{1}{a} - 2)}{x^{1-a}} = \lim (x \to 0) \frac{(0.5+x)^{-1} - 2}{x^{0.5-x}}$$

$$= \lim (x \to 0) \frac{(0.5+x)^{-1} - 2}{x^{0.5} \times x^{-x}} \quad (80)$$

$$= \lim (x \to 0) \frac{(0.5+x)^{-1} - 2}{x^{0.5}} \text{ (Since } \lim (x \to 0) x^{-x} = 1 \text{)}$$

We must apply L'Hospital ' rule [4]

$$\lim (x \to 0) \frac{(0.5+x)^{-1} - 2}{x^{0.5}} = \lim (x \to 0) \frac{-(0.5+x)^{-2}}{0.5 x^{-0.5}}$$

$$= \lim (x \to 0) \frac{-2 \times x^{0.5}}{(0.5+x)^2} = 0$$

Thus we have verified here that, for a = 0.5 (71) approaches zero as (x → 0) and hence remains bounded.

We consider the case a >0.5. This case is also rejected, since according to the functional equation [3], if ($\zeta(s)$=0) (s = a+ bi) has a root with a>0.5, then it must have another root with another value of a < 0.5. But we have already rejected this last case with a<0.5.

Thus we are left with the only possible value of a which is a = 0.5.
Therefore a = 0.5.
This proves the Riemann Hypothesis

Conclusion

The Riemann Hypothesis which states that the nontrivial zeros of the Riemann zeta function have real part equal to 0.5, according to the above, is now proved.

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