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Research Article

# Two-dimensional Auxetic Structures from Rotating Squares

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## Abstract

Mechanical metamaterials constructed from rotating squares connected in a new way by means of rotation axes on the surface of the squares exhibit a unique negative Poisson's ratio (NPR) effect. This effect stems from the highly ordered geometric structure formed by the rigid squares. Under a tensile force, the tested auxetic structures expand horizontally and vertically, with the measure of the relative expansion in these linear dimensions not depending on the number of the elements and the size of the structure, but only on the position of the rotation axes on the surface of the squares.

The paper includes examples of planar and tubular structures made of rigid squares. The analyzed mechanism of the change in the structures' dimensions upon stretching or compression involves both rotational motion and translation of the connected squares.

The structures produced are an improved two-dimensional version of the well-known "rotating squares" model. They can find applications, e.g., in a wide range of adjustment mechanisms.

**Keywords:** Metamaterials; Auxetic Structures; Negative Poisson's Ratio

## Introduction

Auxetic structures are mechanical metamaterials known for their remarkable ability to expand or contract simultaneously in all directions. Physically, this is determined by Poisson's ratio, which is the ratio of the changes in linear dimensions in the horizontal and vertical direction. Auxetics are characterized by a Negative Poisson's ratio (NPR).

$$\text{Poisson's ratio, } \nu = -\frac{\text{lateral strain}}{\text{axial strain}}$$

As a recap, the Poisson's ratio of a material is defined as the dimensionless constant of anisotropic material, expressed as the ratio of the negative relative lateral strain and the relative axial strain of a body subjected to a tensile load.

When conventional materials are stretched, their expansion in the horizontal direction is positive, and expansion in the vertical direction is negative, hence the minus sign in the Poisson's ratio formula. When auxetic materials are stretched, expansion in both directions is positive; therefore, the Poisson's ratio becomes negative.

Auxetics belong to a large family of metamaterials, the idea of which dates back to Veselago (1968) [1], who stated that theoretically, a material could simultaneously exhibit both negative magnetic permeability and negative permittivity.

In general, metamaterials are a new class of materials composed of ordered elements that exhibit unique properties not observed in nature.

Table 1 summarizes the known metamaterials and their properties.

It must be added that the physical parameters of metamaterials can become negative in a certain frequency range of magnetic, acoustic, and gravitational fields.

| Metamaterial | Physical model | Properties/parameters |
|---|---|---|
| Mechanical | Auxetic | Negative Poisson’s ratio |
| Thermal | Thermal diode | Manipulation of heat transport |
| Optical | Resonators, photonic crystals | Negative refractive index |
| Magnetic | Resonators, artificial magnetics | Negative magnetic permeability |
| Acoustic | Resonators, phononic crystals | Negative values for density and volume modulus |

**Table 1:** Metamaterials and their properties [2-4].

The above unique properties of metamaterials are mainly derived from the precise arrangement of their internal structural elements and not just from the inherent properties of their material components. Because a metamaterial is a physical structure that allows for negative values of material constants in response to (as a function of) a given stimulus. Another formulation holds that “metamaterials” are a class of structural composites whose wave functionalities arise as a collective manifestation of its locally resonant component units [2-4].

In this view, a metamaterial is a product of the interaction of a force field (electric, magnetic, electromagnetic, thermal, gravitational,...) with a real but specific material which, due to its structure, acts as a continuous object for this field.

Specific applications of metamaterials include optical materials that act like perfect lenses, perfect acoustic dampers, and mechanical metamaterials with negative thermal expansion and negative stiffness.

In the present work, we devote particular attention to mechanical metamaterials.

The term ‘metamaterial’ was coined by John Pendry [5] in the late 1990s. The properties of mechanical metamaterials are definitely related to their macrostructure rather than the typeof material they are composed of. A peculiar characteristic of metamaterials is that their material constants can take on negative values. Mechanical metamaterials (auxetics) are geometric structures that exhibit a negative Poisson’s ratio.

The main contributors to the development of auxetics include Lakes (1987) [6], Wojciechowski (1989) [7], as well as Evans (1991) [8], and Grima (2000) [9], among others. Although, for the sake of chronological order, there are at least three more precursors in thisfield, namely Gibson (1982) [10], Kolpakov (1985) [11] and Almgren (1985) [12].

Materials with a negative Poisson’s ratio (auxetics) have the very unusual property of expanding when stretched and contracting when compressed. This property gives the material a number of beneficial effects such as increased shear stiffness, increased resistance to cracking when deformed, increased hardness, increased indentation resistance,and increased sound attenuation capacity.

Although metamaterials are artificial materials, auxetics can be divided into natural ones (e.g., cristobalite) and artificially modeled and produced ones [13].

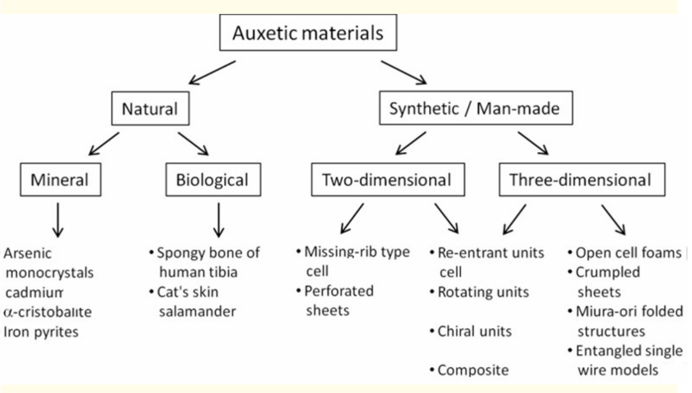


**Figure 1:** Division of auxetic structures according to Mazaev [13].

Most mechanical metamaterials become thinner as they are stretched, but mechanical metamaterials with a negative Poisson's ratio (so-called 'auxetic' materials) become thicker when stretched.

Auxetic materials can be divided into structures of the following types: re-entrant honeycombs, chiral and antichiral, rotating figures, star-shaped, 'missing rib,' 'double arrowhead,' and the kagome lattice. At the micro-molecular scale, auxetic foams, microporous polymers, and liquid crystal polymers can be distinguished [14-16].

## Rotating squares

A particularly interesting set of structures that exhibit a negative Poisson's ratio can be constructed using squares or equilateral triangles connected by hinges [8,9]. These systems exhibit a negative Poisson's ratio if deformed under load, with the squares or triangles remaining rigid but rotating relative to each other thanks to their hinges. The tensile strain generates bidirectional expansion.

The unique properties of these structures mainly stem from the precise arrangement of the geometric elements and also from the inherent properties of their material component, such as their flexibility, which plays a special role in hinges [9].

**Figure 2:** Rotating squares structures as proposed by Grima and Evans [9] in the open position and in the closed position, and the thickening of the vertices forming hinges.

Figure 2b shows the hinges placed on adjacent squares. Usually, the hinges are made by thickening the connecting vertices to form a so-called neck. Under compression, the squares rotate until they reach the closed position. The rotation of hinged squares is not the only mode of deformation of a structure that can uniformly contract or expand since the position of the hinges changes as well.

The rotating square structures shown in figure 2 exhibit particular stress points, i.e., stress concentrations at the connecting vertices of the squares (at the hinges). This inevitably leads to structural damage under cyclical tension and compression. This inconvenience has been eliminated by introducing connections of squares at the overlapping vertices. These connections in the form of rotation axes are an easily attainable and reliable part of the structure.

Typical mechanical metamaterials take into account four elastic constants: Young's modulus E, shear modulus G, bulk modulus K, and Poisson's ratio, which is also relevant for the proposed solution but only to ensure the rigidity of the square elements.

**Figure 3:** Modified structure (3×4) of rotating squares in the closed position and in the open position with marked dimensions of its elements.

The auxetic structure shown in figure 3 consists of a network of square units connected by thin rotation axes located on the surface of the squares. The axes of rotation lie on the diagonals of the square. Upon stretching, the direction of the rotation alternates for adjacent squares.

The factor that determines whether or not a metamaterial constructed from rotating squares is going to have auxetic properties or not depends on whether the squares are sufficiently (perfectly) rigid and whether the hinges allow them to rotate freely. If the squares are deformed, or the hinges are damaged, the structure loses its auxetic properties.

## Geometric relationships in planar systems

The position of the proposed axes of rotation on the surface of the squares can be selected freely but within a limited range. This limitation can be explained based on the analysis of the structure. In this case, too, it is about authenticity in the plane.

A schematic diagram of a mechanical metamaterial with a structure consisting of elementary cells in the form of squares is presented in figure 4.

**Figure 4:** Geometric relationships for the modified rotating square structure – closed position.

Figure 4 shows how the rotating squares are connected, with the axis of rotation (point E) located at a given fixed distance from the edge of each square. If the side length of the square is a, this distance is expressed by a·x, where 0<x<0.25. The parameter x can be usedto characterize a new system of rotating squares. When considering the geometric relationships shown in Figure 4, one can see the similarity of the triangles ABC and CDE.

It follows from the similarity of triangles ABC and CDE that:

$$\frac{a-k}{k} = \frac{a-k-a\cdot x}{a\cdot x} \qquad \text{--------(1)}$$

Considering the expression for the angle θ at the closed position of the structure, oneobtains:

$$tan\frac{\theta}{2} = \frac{a-k}{k} \qquad \text{-------------(2)}$$

$$tan\frac{\theta}{2} = \frac{a-k-a\cdot x}{a\cdot x} \qquad \text{------------(3)}$$

Solving the equation (1) with respect to k gives the relationship between the parameter xand the angle θ for a closed structure [16]:

$$tan\frac{\theta}{2} = \frac{1-\sqrt{1-4\cdot x}}{1+\sqrt{1-4\cdot x}} \qquad \text{-----------(4)}$$

The relationship obtained is shown graphically in figure 5.

**Figure 5:** Variation of the angle θ for the closed position (see Figure 3) as a function of the parameter x [16].

The diagram shows that the maximum position of the axis of rotation corresponds to the value of the parameter x = 0.25, which means that the angle θ reaches its maximum value, and the resulting structure becomes locked – with no possibility of rotation.

A number of useful conclusions can be drawn from the new auxetic structure in the closed position and in the open position shown in figure 3. Through geometric analysis, one can estimate the maximum change in linear dimensions between the closed position and theopen position. Table 2 summarizes the measures of the different segments needed for the calculations.

Depending on the number of elements in the structure (n), the values of X1 and X2 are determined together with their relative change, i.e., the value of expansion when stretched.

The information presented in this diagram is of particular interest. It is the percentage elongation of a structure composed of n squares. The greater the value of the parameter x – the axis of rotation positioned further from the edge of the square – the smaller the expansion of the structure in the open position. The maximum value of expansion upon stretching is 41.4% for x = 0, which is the

**Table 2:** Geometric relationships needed to calculate the dimensions of the structure in closed and open positions (see Figure 3).

**Figure 6:** Effect of two-dimensional expansion of the planar auxetic structure from the closed to the open position.

known auxetic structure of Grima and Evans.Mathematically, the maximum value of the expansion of the rotating squares structure is „Ö2-1".

As the axis of rotation of the squares shifts towards their centers, i.e., as the value of the parameter x increases, the possibility of stretching the structure decreases, and for x = 0.25, the structure does not show such a possibility and becomes locked and thus not susceptible to stretching.

It was found that both for a symmetrical construction (n×n) consisting of the same number of squares horizontally and vertically, and for an asymmetrical construction (n×m), the percentage of expansion depends only on the value of the parameter x and that it is the same in the horizontal and vertical directions. The relative lateral strain is equal to their relative axial strain.

Above all, it should be noted that the ratio of the changes in the size of the structure in the horizontal direction and in the vertical direction between the closed and open positions is the Poisson's ratio which is equal to -1 – regardless of the size of the structure and of the value of the parameter x.

The comparison of the contour outlining the structure, i.e. (X1×X2) for the closed and the open position (figure 3), shows that there is a linear relationship between the ratio of the area (X1×X2) open/(X1×X2)close and the value of the parameter x. The greater the value of x, the closer the ratio is to unity. For x = 0.25, the area (X1×X2) is the same for both the open andthe closed positions. For x = 0, the area (X1×X2) for the open position is twice as large as for the closed position.

**Figure 7:** The ratio of the area of the auxetic structure in the open and closed positions as a function of the parameter x.

The relation presented above is so universal that it does not depend on the count n of the auxetic structure – neither in the even n×n nor in the odd n×m system. The above relationship is due to the regularity of the structure whose geometric elegance is owed to the pioneers of rotating squares, i.e., Grima and Evans [8,9].

One can point out another property of an auxetic structure of that kind. It concerns changes in the position of the axis of rotation of the squares. When the structure is opened, the squares rotate on the axes, but the position of the axes shifts.

**Figure 8:** A diagram of the auxetic structure (4×4, for x = 0.112) in three positions: closed, open with the opening angle θ/2 = 25° and fully open (opening angle θ/2 = 45°) with the marked axes of rotation of the squares.

Let y1 and y2 denote the distances between the axes of rotation in the horizontal and vertical directions, respectively. When the structure is open, the y1/y2 ratio increases from avalue close to zero up to unity. When the structure is opened, the spacing between the axes of rotation in the horizontal direction and in the vertical direction is the same.

From a geometrical perspective, the mechanism presented here is a consequence of the fact that in rhombic units, the altitude of the rhombus decreases along with the increase of the length of its base, thus causing the expansion of the structure (“rhombic expansion mechanism”) leading to a Poisson’s ratio of -1. For practical reasons, the points of attachment between adjacent squares are to be considered as axes of rotation, marked as small circles. It is evident that when the structure is stretched, the square units connected bythe rotation axes not only rotate relative to each other but also translate.

This means that the horizontal diagonals of the rhombi become elongated as θ increases from its initial value to 90 °, thus forcing a decrease in the height of the rhombic areas. All this is caused simply by increasing the distance between points A and B (y1) and decreasing the distance between points C and D (y2). The structure becomes larger as it stretches (uniaxially) thanks to the homogeneity of the rigid squares and, in practice, can prove to be reliable for any number of components.

To highlight the capabilities of the proposed structure, the deformations that occurred when the prototype was stretched have been directly measured. The measurements show that the change in the ratio of the distance between the rotation axes increases to reach a value of 1. This results in an increase in the size of the plane (X1×X2) based on the marked contours of the structure (Figure 8).

As an example, for the parameter value x = 0.112, the change of the y1/y2 ratio as a function of the opening angle is shown in figure 9.

The curve shown in figure 9 results from the movement of rigid squares connected by rotation axes that undergo translation and rotation.

**Figure 9:** The ratio of the spacing of rotational axes as a function of the opening angle θ/2

The presented new auxetic structures can therefore be designed and manufactured at any scale and create a two-dimensional expansion effect achieving NPR.

## Geometric analysis of tubular structures

Tubular structures belong to the group of mechanical metamaterials particularly sought after for engineering applications. For this reason, numerous proposals of that kind can be found in the literature, one group of which consists of medical applications [17-20].

An auxetic planar (2D) structure made of rotating squares connected by axes of rotation on the surface of the squares can be rolled up and joined to form a tubular structure. It can be proven that such a tubular structure also has auxetic properties. Figure 10 shows the initial 2×12 planar structure in the open position and in the closed position.

The determined values of the X1 and X2 dimensions can be used to calculate the size of the expansion as a result of the closed position – open position transition.

By rolling up a planar structure, one can obtain an object in the form of a band. The connected squares are then arranged in a circle, and the more flexible the material of the rigid squares, the better curvature can be attained.

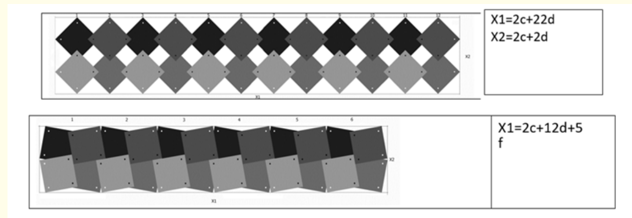


**Figure 10:** Planar 2×12 auxetic structure of rotating squares in the open (top) and the closed (bottom) position.

Figure 11 shows a projection of the tubular structure, including the circle of the structure and the groups of squares (2×2) surrounding it. This regular symmetrical arrangement of the groups of squares in the open position and in the closed position suggests that also in this case the size of the structure may increase upon stretching. The figure illustrates a structure consisting of 24 squares separated by a radius D/2 from the axis of the tubular structure.

When the structure is stretched, each square rotates until right angles are formed between the edges of the squares. At the same time, the position of each square shifts both parallel and perpendicularly to the axis of the tubular structure. In this way, thanks to the moving parts, the tubular structure undergoes expansion.

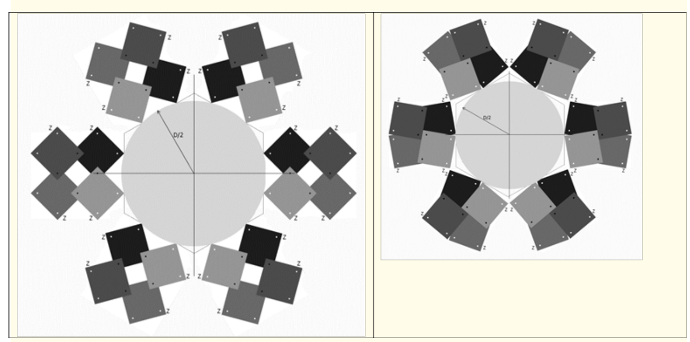


**Figure 11:** Cross-section of a 2×12 tubular auxetic structure with a projection of groups of squares perpendicular to the axis of the structure in the open position and in the closed position.

The arrangement presented in figure 11 contains 6 groups of squares in the closed and in theopen position, with several connections between the squares that have been broken – denoted by the letter Z. The vertices of the squares touch the line of the circle of the tubular structure with a radius D/2. It can be seen that the diameter of the tubular structure depends on the dimensions X1 and X2 of the planar structure. From this relation, it followsthat:

$X1 = D\pi$ ––––––(4)

While the height of the structure is $X2 = H$.

Therefore, the size changes of the tubular structure correspond to the size changes of the planar structure between the open and closed positions. Lateral and axial expansion of the structure is a response to stretching. Conversely, contraction is observed as the open structure is compressed.

The above relationship indicates that the change in the size of the tubular structure will have the same character as the change in the size of the planar structure. In this case as well,these changes can be determined as a function of the parameter x, i.e., the position of the rotation axes on the surface of the squares.

What can be particularly interesting is the possibility of expansion of the tubular structurewhen moving from the closed to the open position. It can be seen that the radius of the structure increases along with the degree of opening. The square elements connected with the rotation axes rotate and shift their position creating an enlarged spatial openworkstructure.

The results of the calculations made for the above structure, i.e. the relative change in thediameter of the tubular structure and the parameter x, correspond to the relationship shown in figure 6.

The obtained relationship between the expansion of the structure along the diameter and the parameter x is close to a straight line and shows that the closer the edges of the squares are to the axes of rotation, the greater expansion of the tubular structure can be obtained. Itis worth noting that the maximum value of the expansion along the diameter of the tubular structure reaches 41.4% for the parameter $x = 0$. Whereas, for $x = 0.25$, the squares overlap so far that it is no longer possible to rotate them, and the expansion of the structure is equal to zero.

Now, given the ratio of the relative size change of the tubular structure along its diameter and along its axis, one can calculate the Poisson's ratio, which, as expected, takes negativevalues. As predicted, these values are equal to -1.

The analyzed model is experimentally tested in the following section.

## Physical models

Through testing constructed geometric models and physical structures, the system has been proven to exhibit full functionality and auxetic properties. Analyzing two-dimensional auxeticstructures constructed from squares, a number of models were made from squares cut from steel sheet.

The structure of the mechanical metamaterial is a metamaterial array consisting of elementary cells in the form of squares, which are made of steel sheet with overlapping vertices. Parts of the square elements near the axis of rotation are sliding surfaces.

The structures presented in figure 12 are characterized by a change in size between the closed and open positions that corresponds exactly to the theoretical values given above.

**Figure 12:** Auxetic structures (3×4) made of thin steel sheet, connections made from pins.

An auxetic structure subjected to a vertical or horizontal force causes the slope of each square to change. This results in the entire structure opening up, which leads to an increase in size in both the horizontal and vertical directions and achieves the NPR effect.

The NPR is also achieved in structures produced from connecting the squares to form a closed circuit.

**Figure 13:** Steel sheet squares forming a band-like structure in the closed and in the open position (Structure 18×3, a = 20 mm, x = 0.1, change in height: 55 mm → 71 mm, change in diameter 109 mm → 125 mm).

The presented design of a band with auxetic properties can be treated as an element that already achieves 3D symmetry. Although the design still lacks more precise connections on the rotation axes, it nevertheless changes dimensions in all directions.

Tubular (pipelike) structures occupy a very prominent place among circular structures as theyare the most anticipated ones in terms of their possible applications. It is a well-known fact that tubular structures are one of the most common mechanical elements used in engineering. By rolling up and connecting the two ends of a planar surface composed of rotating squares, an openwork tubular structure is obtained. By using a thin spring steel sheet for the squares, the resulting structure is in a state of stress, i.e., the deformed (slightly bent) square sheet elements are given a certain amount of potential energy. Such stress state promotes the buckling resistance of the tubular structure.

For the above planar structure, the values of the measured parameters X1 and X2 differ fromthe theoretical values (calculated based on the relationships given in Table 1). These differences are due to shifts associated with the width of the axes of rotation, i.e., the parameter x slightly deviates. Despite this, NPR values equal to -1 are obtained – as has beentheoretically demonstrated.

**Figure 14:** Auxetic structures (16×12) made of thin steel sheet, with connections from pins along with experimental values of structure sizes in the open and closed positions.

Using the structure shown in figure 15 a tube-shaped structure presented in figure wascreated.

**Figure 15:** Tubular structure with rotating squares in the open position (top) and in the closed position (bottom).

The resulting tubular structure in the open position had a height of H(open) = 365 mm and a diameter of D(open) = 86 mm. While in the closed position, the tube sizes were reduced to H(close) = 299 mm and D(close) = 67 mm. The size differences occurring between the planar and the tubular structure are due to the imperfect connections on the rotation axes.

It should be emphasized once more that also the experimental tubular structure made from rotating squares exhibits an NPR effect, in this case, equal to = -1.28, with the rotation- translation mechanism involved as well.

This deviation from the theoretical value results from differences in the connections.

## Conclusion

The present work has studied modified auxetic structures composed of rigid rotating squares. Using earlier work of Grima and Evans, the structures were improved by placing axes of rotation onto the surfaces of the squares making their vertices partially overlap. To define the position of the rotation axes a geometrical parameter x was introduced, denoting the distance of a given axis from the edge of a square with the axis placed on its diagonal.

The improved auxetic structures made from rotating squares connected by rotation axes located on the surface of the squares exhibit a change in dimensions between the closed position and the open position that corresponds to the NPR value.

The introduced parameter x allows for characterizing the structure in terms of geometric relations, including the q angle formed between the edges of the squares in the closed position, as well as for the possible value of the expansion of the structure.

Based on a detailed geometric analysis of structures made of rigid squares, the relationship between the angle θ and the parameter x has been determined. The value of x (in the range $0<x<0.25$) corresponds to the angle θ in the position where the structure's surface is completely covered – the closed position. The angle θ in the closed position increases up to 90° when the structure is stretched to the open position. The relationships given allow for calculating the linear dimensions of the structure in the horizontal direction and in the vertical direction (X1 and X2, respectively).

Theoretical values have been determined for the percentage linear expansion of the planar structure as a function of the parameter x, the magnitude of which does not depend on the direction nor on the number of square elements of the structure. The maximum value of the linear expansion reaches 41.42% (for x = 0). The relative linear expansion is independent of the size of the sides of the squares, nor of the number of the squares, and always leads to an NPR value of -1.

As a result of the expansion of the planar structure, there also has been an increase in the area covered by it (X1×X2). The area calculated from the contour of the structure, i.e. (X1×X2), exhibits expansion when going from the closed to the open position, with the ratio (X1×X2)open/(X1×X2)close being a linear function of the parameter x. As the structure expands, for x = 0, the area doubles, and for x = 0.25, with the squares becoming immobile, the area remains constant.

It has been pointed out that the expansion of the structure associated with its stretching and with the transition between the closed position and the open position, i.e., the auxetic structure mechanism, involves both the rotational movement and the translation of the squares. During the transition between the closed and the open position, the spacing of the rotation axes changes, which confirms this rotation-translation mechanism.

From the analysis of the tubular structure – formed from the planar structure being rolled into a tube, relationships have been obtained for the amount of linear expansion of the tubular structure in the axial direction and in the lateral direction. The relative percentage linear expansion of the tubular structure shows the same tendency as in the planar structure.

On the other hand, the ratio of the size changes along the diameter and along the axis of the tubular structure from the closed to the open position, i.e., the Poisson´s ratio (NPR) takes values -1.

The theoretical considerations and geometric analysis are supplemented by actual physical models of the planar structure and the tubular structure – both based on squares cut from steel sheet and connected by axes in the form of pins. The physical models have confirmed the theoretical predictions presented above. For the planar and tubular structures, an NPR of -1 was obtained, while for the constructed tubular structure, the NPR was lower, with a experimental value of -1.28.

The proposed auxetic structures based on rotating squares can provide advanced functionality in the form of size adjustment to meet specific requirements, e.g., in a wide range of adjustment devices without possible limitations of their production possibilities. These rather complex network structures can be manufactured on a large scale without the use of specialized equipment while ensuring a precise assembly.

Furthermore, the properties of the new structure presented herein are not dependent on the chemical composition of the materials used, although it is a prerequisite that they exhibit a certain degree of stiffness and durability to avoid bending, warping, or stretching of the square units. The material of the axes must also exhibit good mechanical properties, especially shear resistance.

Future designs incorporating auxetic structures include multilayer systems, which will still be two-dimensional structures but may be more useful, for example, for expansion joints. The presented auxetic structures have the potential to raise even more research interest when they become more widespread and are noticed by more innovators.

## Authors' Contribution

J.P. came up with the idea presented here, including the model as well as the geometric relationships, prepared the first draft of the manuscript, and constructed physical models used as proof. M.P. arranged the editing and proofreading work. All of the authors read and agreed tothe published version of the manuscript.

## Bibliography


1 VG Veselago. "The electrodynamics of substances with simultaneously negative values of ε and μ". *Soviet Physics Uspekhi* 47 (1968): 509-514.

2 GS Rajni and A Marwaha. "A Review of Metamaterials and its Applications". *International Journal of Engineering Trends and Technology (IJETT):* 19.6 (2015): 305-310.

3 G Ma and P Sheng. "Acoustic metamaterials: From local resonances to broad horizons". *Science and Advances* 2 (2016): 1-16.

4 G Duan., *et al*. "Boosting magnetic resonance imaging signal-to-noise ratio using magnetic metamaterials". *Communications Physics* 35.2 (2019): 1-7.

5 JP Pendry., *et al*. "Magnetism from Conductors and Enhanced Nonlinear Phenomena". IEEE. Trans. Microwave. Theory Technol. 47 (1999): 2075-2084.

6 RS Lakes. "Foam structures with a negative Poisson's ratio". *Science* 235 (1987): 1038-1040.

7 KW Wojciechowski. "Two-dimensional isotropic system with a negative Poisson ratio". *Physics Letters A* 137 (1989): 60-64.

8 K E Evans. "Auxetic polymers: a new range of materials". *Endeavour* (151991): 170-174.

9 JN Grima and KE Evans. "Auxetic behavior from rotating squares". *Journal of Materials Science Letters* 19 (2000): 1563-1565.

10 LJ Gibson., *et al*. "The mechanics of two- dimensional cellular materials". *Proceedings of the Royal Society of London. Series A* 382 (1982): 25-42.

11 AG Kolpakov. "Determination of the average characteristics of elastic frameworks". *Journal of Applied Mathematics and Mechanics* 49.6 (1985): 739-745.

12 RF Almgren. "An isotropic three-dimensional structure with Poisson' s ratio -1". *Journal of Elasticity* 15 (1985): 427-430.

13 AV Mazaev., *et al*. "Auxetics materials: classification, mechanical properties and applications". IOP Conf. Series: Mater. Sci. Eng. 747 (2020): 012008, 1-8.

14 A Joseph., *et al*. "On the application of additive manufacturing methods for auxetic structures: a review". *Advances in Manufacturing* 9 (2021): 342-368.

15 E Barchiesi., *et al*. "Mechanical metamaterials: a state of the art". *Mathematics and Mechanics of Solids* 24 (2019): 212-234.

16 J Plewa., *et al*. "Experimental studies of auxetic structures from rotating squares". *Materials* (2022).

17 X Ren., *et al*. "A simple auxetic tubular structure with tuneable mechanical properties". *Smart Materials and Structures* 25 (2016): 065012, 1-9.

18 SK Bhullar., *et al*. "Design and Fabrication of Stent with Negative Poisson's Ratio". *International Journal of Mechanical Engineering and Mechatronics* 8.2 (2014): 448-454.

19 F G J Broeren., *et al*. "Spatial pseudo-rigid body model for the analysis of a tubular mechanical Metamaterial". *Mathematics and Mechanics of Solids* 25.2 (2020): 305-316.

20 C Luo., *et al*. "Design manufacturing and applications of auxetic tubular structures: A review". *Thin-Walled Structure* 163 (2021): 7682, 1-12.