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Review Article

Spontaneous Superconductivity

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Abstract

We report on the spontaneous emergence of superconductivity in an exactly solvable SU2 x SU2 fermion model. This happens, without appealing to a pairing interaction, at finite temperatures for special fermion numbers.

Keywords: Superconductivity; Pairing Interaction; Temperature

Introduction

Bohr, Mottelson, and Pines [1], as well as Belyaev [2] were able to successfully adapt to nuclei the superconductivity techniques advanced by Bardeen, Cooper, and Schrieffer [3,4]. Nuclear superconductivity on the basis of nuclear fermion-pairing is common fare in nuclear structure descriptions [5], where one works with finite systems [5].

In this work we unveil peculiar temperature and number of fermions effects displayed by N fermion described by an exactly solvable Lie model, without appeal to a fermion-fermion pairing interaction. Among hundreds of papers on nuclear superconductivity see, in addition to [5], see for example [6-8].

The effects we wish to report on involve a context delineated by the Gibbs canonical ensemble and focused on an exactly solvable model introduced in [9] for which the BCS treatment of superconductivity becomes exact, so that our results are not based on approximate treatments. We start by presenting such model below.

Exactly solvable many-fermion models

The mother model: that of Lipkin., et al. (LM)

This LM model [10,11] was exceedingly helpful in a multitude of investigations regarding the validity and adequancy of different many-body techniques devised so as to inspect and explore the manifold traits of the quantum many body problem.

The LM template is constructed as an SU2 algebra associated to particular operators denominated quasi-spin ones and can produce easily attainable exact solutions to the relevant Schroedinger equation (SE). These eaxct solutions are to be compared with those approximate ones encountered using diverse kinds of theoretical approximations to the problem at hand. In the LM context one speaks an ”angular momentum” language. The relevant Casimir operator is akin to J^2 [10] and has attached to it several multiplets. Lipkin et al. only use the multiplet associated to the system’s ground state. For facing superconductivity, however, one needs the additional multiplets. A practical way to handling them was advanced in reference [9].

As stated above, Cambiaggio and Plastino (CP) [9] proposed a simple LM-extension that allowed for the formulation, in quasi-spin parlance, of a BCS-like construct mimicking the ordinary superconductivity. The construct yields exact solutions. This extension is a generalization of the SU2's LM to an SU2 x SU2 related alternative, involving a variable fermion number.

SU2 x SU2 model for N fermions

This construct [9,12] focuses on s N fermions disseminated in a couple of (N = 2Ω) (2Ω)-fold degenerate single-particle levels. The two levels are separated by an energy gap ϵ. One characterizes the 4Ω multiplicity via the quantum numbers p, μ in this manner: p = 1, . . . , 2Ω and μ = ±1. One deals with the angular momentum SU2 quasi-spin operators [10].

$$J_z = (1/2)\sum_{p,\mu} \mu C^+_{p,\mu} C_{p,\mu}, \tag{1}$$

$$J_+ = \sum_p C^+_{p,+} C_{p,-}, \tag{2}$$

$$J_- = \sum_p C^+_{p,-} C_{p,+}. \tag{3}$$

Cambiaggio and Plastino supplement them with the SU2 "pairing" operators.

$$Q_0 = (1/2)\sum_{p,\mu} C^+_{p,\mu} C_{p,\mu} - \Omega, \tag{4}$$

$$Q_+ = \sum_p C^+_{p,+} C^+_{p,-}, \tag{5}$$

$$Q_- = \sum_p C_{p,-} C_{p,+}. \tag{6}$$

Evidently, usly, Q+ creates, and Q− annihilates , two fermions yielding zero contribution to the values of the operator J_z. We say then that these two fermions "couple" to J_z = 0. Note that any Q-operator will commute with all J-operators, and vice versa (SU2 x SU2). We have a complete orthonormal basis determined by the eigenvalues of J^2, J_z, Q^2, Q_0, i.e., |J, Q, J_z, Q_0). A "pairing" Hamiltonian commuting with the number of fermions operator reads [5] (we take ϵ = 1).

$$H = J_z - (G/2) Q_+ Q_-, \tag{7}$$

With G the pairing-strength. Notably, in this paper we always gtake G = 0. The pairing interaction above has exactly the same appearance that used in nuclear theory [see [5], Eq. (4.140)]. An important quantity for our purpose is the quasi-spin seniority number ν.

$$\nu = 2(\Omega - Q), \tag{8}$$

Which is the number of particles not "paired" to J_z = 0. IIn other words, ν yields the number of "unpaired" particles in a Q-multiplet. As for our multiplets structure, one has [9].

$$J = \nu/2, \tag{9}$$

While

$$J + Q = \Omega. \tag{10}$$

In the case of the Lipkin model we have N = 2Ω, Q_0 = 0, equalities that we also use in this work [9].

At zero temperature, the unperturbed ground state (no interaction) has J = Ω, J_z = -Ω, Q = Q_0 = 0, belonging to the multiplet J = Ω, Q = Q_0 = 0. We represent the exact eigenvalues of H as [9].

$$E(J, Q, J_z, Q_0) = J_z - (G/2)\left[Q(Q+1) - Q_0(Q_0 - 1)\right]. \tag{11}$$

The energy of the unperturbed ground state (ν = N, Q = Q_0) is [9].

$$E_0 = -\Omega. \tag{12}$$

It is important to nothe that the state of quasi-spin seniority zero, for which all fermions are "paired" to J_z = 0, faithfully mimics a nuclear "superconducting" state [9,14], being characterized by ν = 0 and Q = Ω. Its superconducting energy reads.

$$E_s = -(G/2)\left[\Omega(\Omega+1) - Q_0(Q_0 - 1)\right], \tag{13}$$

$$E_s = -(G/2)\left[\Omega(N-\nu) + \frac{\nu}{2}\left(\frac{\nu}{2} - 1\right) - \frac{N}{2}\left(\frac{N}{2} - 1\right)\right]. \tag{14}$$

The pertinent state metamorphoses into the superconducting state of the pairing-interacting model when.

$$G = G_{crit} \geq (4\Omega/N)\frac{1}{2\Omega + 1 - N/2}, \tag{15}$$

Whenever G < G_{crit} the system remains in the unperturbed ground state (UGS). Note that the greater N, the less "labor" is needed. measured in G-units, to force the system to be a superconductor, a nice fact that was not duly emphasized in [9] but becomes critical for our present purposes. In other words, as N grows, G_{crit} diminishes.

SU2 x SU2 model and finite temperatures

At finite temperature T, with β = 1/T, the remnants of the T = 0-phase-transition from unperturbed state to superconductivity are now called cross-overs [13]. For our SU2 x SU2 model, the pertinent Gibbs canonical ensemble treatment was devised and implemented in Ref. [14].

In investigating ground statesjust the J + Q = Ω "band" needs consideration. For finite T instead, all states belonging to different bands are now "accessible" in the pertinent Gibbs' statistical ensemble. One needs then, for writing the partition fucntion Z the degeneracy Y (J, Q) given in Ref. [14]

$$Y(J,Q) = \frac{(2\Omega+2)!(2\Omega)!(2J+1)(2\Omega+1)}{(\Omega+J+Q+2)!(\Omega+J-Q+1)!(\Omega-J+Q+1)!(\Omega-J-Q)!}. \quad (16)$$

We begin with a partial partition function ZM of the type [14].

$$Z_M = \sum_{M=-J}^{M=J} \exp\left[-\beta\left(M - \frac{G}{2}Q(Q+1)\right)\right], \quad (17)$$

And then the true partition function Z becomes.

$$Z = \sum_{J,Q} Y(J,Q) Z_M, \quad (18)$$

Where J and Q run over all the values that verify the restrictions [14,15].

$$0 \le J \le \Omega, \quad (19)$$

$$0 \le Q \le \Omega, \quad (20)$$

$$0 \le J + Q \le \Omega. \quad (21)$$

Now we proceed to evaluate, from Z, our statistical quantifiers. We will need first to modify (21) to the form.

0 ≤ J + Q = s ≤ Ω.

In doing for Z the double sum over J, Q, it is convenient to sum over J+Q = s and over J. Q remains fixed at Q = s – J, where.

0 ≤ s ≤ Ω

And s = 0, 1, 2, 3, . . . , Ω. Also, J = 0, 1, 2,……, s. Accordingly,

$$Z = \sum_{J,Q} Y(J,Q) Z_M = \sum_{s=0}^{\Omega} \sum_{J=0}^{s} Y(J,Q) Z_M. \quad (22)$$

Remind that N = 2Ω. Gibbbs' canonical probability distribution (CPD) reads.

$$P(M,Q,J) = \frac{Y(J,Q)\exp\left[-\beta\left(M - \frac{G}{2}Q(Q+1)\right)\right]}{Z}. \quad (23)$$

Note that for Q = 0 there are no coupled pairs of fermions. This is the case of the ground state multiplet. All the excited multiplets have paired fermions and thus partial superconductivity. These excited multiplets become availabn˜e for occupation as soon as the temperature is different from zero.

We introduce as well the effective superconductivity index X [Cf. Eq. (8)].

$$X = \sum_{J,Q} \sum_{M=-J}^{M=J} P(M,Q)[(N-\nu)/N]. \quad (24)$$

It is equal to unity for a perfect superconductor and vanishes for the unper-turbed system, As a consequence.

$$<\nu> = <1 - X> = \sum_{J,Q} \sum_{M=-J}^{M=J} P(M,Q)\nu, \quad (25)$$

Quantifies at the temperature T the average number of unpaired particles. For full superconductivity we have 1 – X = 0.

Spontaneous pairing

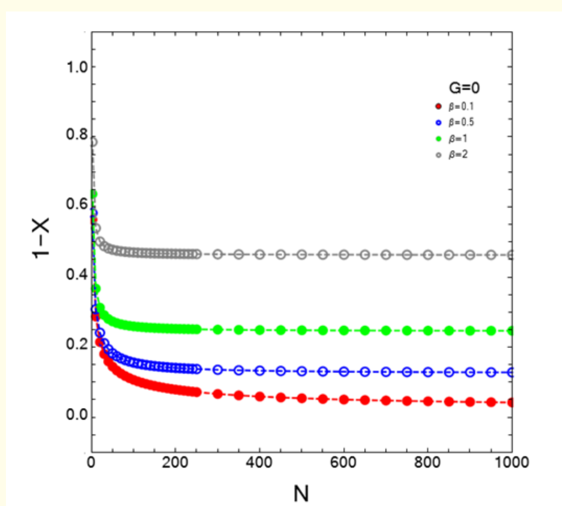

Figure 1: Quantifier 1 - X vs N for several values of β and G = 0. At large N and sufficiently high T we attain superconductivity.

Figure 1 depicts 1 - X vs N for fixed G = 0 (no pairing interaction) and several values of the inverse temperature β. We see that paired fermions emerge at sufficiently high N (partial superconductivity), which seems surprising.

We have seen in the preceding Section that finite T always permit occupation of multiplets with Q = 0, and thus partial superconductivity even in the absence of fermion-fermion interactions. Additionally, we here emphasize the following.

- Note that if both T and N are high enough, partial superconductivity emerges and is clearly appreciated for G = 0.
- The degree of partial superconductivity increases with N.

Conclusions

The theoretical machinery of superconductivity [4] was adapted for describing fermion-pairing in finite systems of N particles more than 60 years ago. In this work we have described N -dependent effects in finite fermions systems, in the absence of pairing interactions. Note that N growth works in the same way as temperature growth. Figure 1 teaches us that temperature-induced partial superconductivity may exist even for a pairing coupling constant G = 0.

Authors' Contributions

The two authors have contributed in equal measure to the preparation of this manuscript.

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