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Review Article

Thermo-statistics of Newtonian Gravity at Short Distances

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Abstract

Classical Newtonian gravitation (CNG) encounters problems at short distances because of the emergence of quantum effects. Experimentally, CNG has been shown to be valid down to 52 micrometers. Here we undertake a classical canonical ensemble treatment of Newtonian gravitation (NG) and discover interesting effects at very short distances.

Keywords: Newtonian Gravity; Statistical Mechanics; Partition Functions

Introduction

In this work we present a thermo-statistical discussion of Newtonian theory of gravitation at short distances [1]. We use to such an effect physical and mathematical theories described in [2,3], which were previously employed in several works (see, for example, [4-7]).

We perform a Gibbs canonical ensemble treatment at fixed temperature T (with kB Boltzmann's constant) so as to obtain the pertinent partition function Z [8-11] for Newtonian gravity.

It is well known that the conventional integration tools needed to obtain Z in this situation lead to infinities [[12] (and references therein)]. This difficulty was overcome by recourse to special mathematical techniques [12] that involve using (a) a generalization of the dimensional regularization approach [12] and (b) the analytical extension of associated integrals made by Gradshtein and Rizik in Table [13]. We must appeal to a rather not conventional mathematical apparatus for adequately analyzing the ensuing classical scenario [4-7].

Now suppose that we deal with two bodies. One has mass M and radius R_0 and the second is a point mass m at a distance r from the first. Both bodies are contained in a spherical box of radius R1. This system is bound by construction. We will consider in this paper the thermal statistics of this physical situation in a Gibbs' canonical ensemble at the temperatures T. The gravitational constant is called G.

The partition function Z

We set $\beta = 1/k_BT$. The partition function now turns out to be

$$Z = \left[\frac{2\pi^{\frac{3}{2}}}{\Gamma\left(\frac{3}{2}\right)}\right]^2 \int\limits_0^\infty dp \int\limits_{R_0}^{R_1} dr (rp)^2 e^{-\beta\left(\frac{p^2}{2m} - \frac{GmM}{r}\right)}. \tag{1}$$

After some tricky mathematics [4-7] and using the integral exponential function Ei [13] one then arrives

$$Z = \frac{\pi^{\frac{3}{2}} V_1}{2} \left\{ e^{\frac{\beta GmM}{R_1}} \left[\left(\frac{\beta GmM}{R_1} \right)^2 + \frac{\beta GmM}{R_1} + 2 \right] - \left(\frac{\beta GmM}{R_1} \right)^3 E_i \left(\frac{\beta GmM}{R_1} \right) \right\}$$
$$- \frac{\pi^{\frac{3}{2}} V_0}{2} \left\{ e^{\frac{\beta GmM}{R_0}} \left[\left(\frac{\beta GmM}{R_0} \right)^2 + \frac{\beta GmM}{R_0} + 2 \right] - \left(\frac{\beta GmM}{R_0} \right)^3 E_i \left(\frac{\beta GmM}{R_0} \right) \right\}, \qquad (2)$$

Where V1 is the volume of a sphere of radius R1 and V0 is the volume of a sphere of radius R_0. We depict Z in figure 1 and see that it is always positive, as it should. From Z we deduce the accompanying specific heat CV, that is displayed in figure 2.

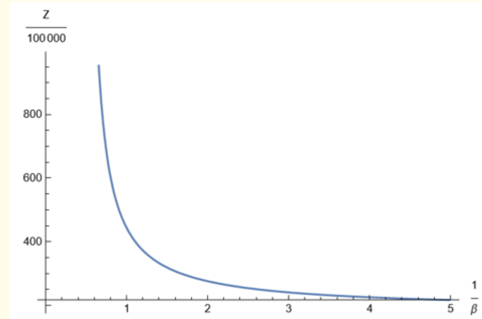

Figure 1: Partition function Z versus the temperature T = 1/β.

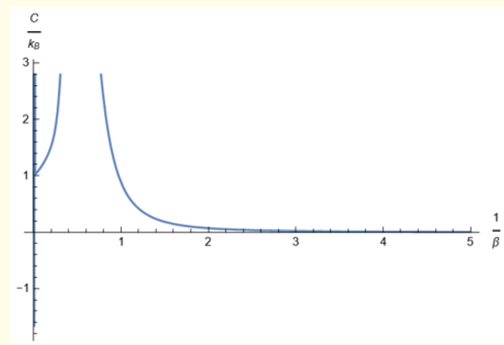

Figure 2: Specific heat CV versus the temperature T = 1/β. The divergence for low T indicates that our classical treatment is no longer value at low temperatures, where quantum effects emerge.

Setting R0 = 0

In order to appreciate, as we wish to do here, the thermal traits of gravitation at short distances r we set $R_0 = 0$. Drastic effects will ensue. Thus, we now deal with two point masses m and M, enclosed in a container of radius R1. The origin-generated sequels will be seen to be dramatic. The pertinent partition function is now

$$Z = -\left[\frac{2\pi^{\frac{3}{2}}}{\Gamma\left(\frac{3}{2}\right)} \right]^2 \int_0^\infty dp \int_0^{R_1} dr (rp)^2 e^{-\beta\left(\frac{p^2}{2m} - \frac{GmM}{r}\right)}. \qquad (3)$$

To this expression we apply the same treatment as we did in our previous Z case above and obtain the following

$$Z = -4\pi^{\frac{5}{2}} \left(\frac{2m}{\beta} \right)^{\frac{3}{2}} \left\{ \frac{R_1^3 e^{\frac{\beta GmM}{R_1}}}{3!} \left[\left(\frac{\beta GmM}{R_1} \right)^2 + \frac{\beta GmM}{R_1} + 2 \right] + \frac{(\beta GmM)^3}{3!} \left[\ln(\beta GmM) - \psi(4) - E_i \left(\frac{\beta GmM}{R_1} \right) \right] \right\} \qquad (4)$$

Where ψ is the poly-gamma function. We plot our new Z in figure 3. One immediately appreciates the fact that this Z becomes negative for most temperatures, which is a theorical disaster. Thus, we conclude that including very short distance (by integrating from 0 in r) in the evaluation of Z wholly contaminates the ensuing thermal physics. These very short distances are unphysical because Newtonian gravity is not valid at them because quantum effects emerge. The accompanying specific heat is displayed in Fig. 4 and diverges at low T because of the emergence of these quantum effects.

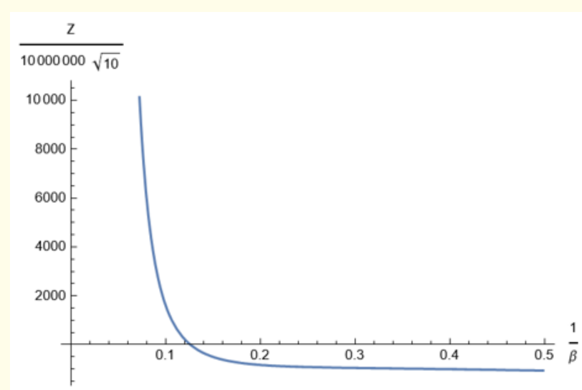

Figure 3: Partition function Z, with the effects of the origin of coordinate r included, versus the temperature T = 1/β. This partition function is unphysical because it becomes negative.

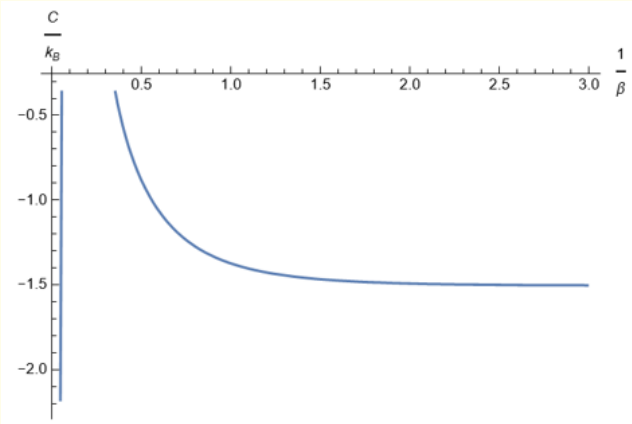

Figure 4: Specific heat CV , with the effects of the origin of coordinate r included, versus the temperature T = 1/β. The divergence signals the presence of quantum effects.

Conclusion

In this work, we have discussed the classical statistical mechanics of Newtonian gravity (NG) at short distances between the interacting masses. This is of interest because we know from experiment that NG is valid down to short distances of about 40 micrometers [1]. What happens with theoretical considerations at smaller distances? We gave the answer here.

Our trick was to integrate the partition function Z both

- With r running from a finite small distance R_0 upwards
- With r running from zero upwards,
- and comparing the pertinent thermal results. These were illuminating indeed.

We confront two different effects in this work. (1) NG is not valid at very short distances because the partition function becomes negative and (2) Quantum effects emerge at low enough temperature. The first type of effects are seen in plotting Z while the second is reflected by the behavior of the specific heat.

Summing up, there seems to be no valid classical thermal description of gravity if the distance between the interacting masses is too short.

Availability

All that might be needed is included in the present manuscript.

Conflict of Interests/Competing Interests

None.

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