

Solution of Direct and Inverse Problems in Investigations of the Internal Gravity Waves Dynamics in Ionosphere Using the Doppler Frequency Shift Method

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Abstract

The solution of direct and inverse problems arising in investigations of the internal gravity waves (IGWs) dynamic via recording of the Doppler frequency shift, is presented. The direct problem is to determine the response of the Doppler shift to IGWs in the region of the radio wave reflection point; the inverse problem is the determination of IGW parameters from experimental data on the Doppler frequency shift. Solutions were obtained in an approximation of the isothermal ionosphere for the heights of the *F*-region. They are presented in a form convenient for their practical use and can have a wide range of applications, including the detection of soliton-like wave structures in the *F*-region of the ionosphere.

Keywords: Internal Gravity Waves; Doppler Frequency Shift; *F*-region

Introduction

In experimental investigations of the ionospheric dynamic processes by use of various sounding methods, one of the main conceptual problems is an adequate interpretation of the temporal variations of the detected signal. In particular, since in most cases the signal reflects fluctuations in the electron concentration, which are associated with the dynamics of ionospheric disturbances (such as the travelling ionospheric disturbances, TIDs), the source of the generation of such TIDs must be determined, and the role of that source can play the internal gravity waves (IGWs) [1,2] and acoustic gravitational waves (AGWs) [3,4] in the neutral component, which, in turn, are excited by impulse type sources of different natures [5-8] (see also [9]). For example, theoretical studies [10-12] first predicted phenomena such as the generation of two-dimensional IGWs solitons in the regions of sharp gradients of the main ionospheric parameters (on the fronts of solar terminator and solar eclipse spot). These received qualitative confirmation in the experiments on ionospheric sounding [13-16]. However, it is

well known that there are no direct methods for the measurement of neutral component dynamic parameters, i.e. IGW characteristics, on ionospheric heights. Thus, the calculation of quantitative dynamic IGW characteristics from the measured data is definitely still relevant at present.

Let us consider here the problem set above on the example of one of the most effective (from the point of view of the dynamic process study method) of the ionosphere-sounding method of recording the Doppler frequency shift (DFS) of the reflected main pulse. Please note that there are two types of problems in IGW studies by the DFS method that coexist and compliment each other:

- Study of the DFS response to IGWs with a priori known parameters (e.g., under a known or modeled mechanism of the IGW excitation), i.e., the direct problem, and
- Interpretation of the recorded IGWs in terms of wave disturbances of the neutral component, i.e., the inverse problem.

In the current research, problems of both types are considered analytically, and their solutions are obtained in a form that is convenient for practical use. Despite of the work in which the problem of DFS reconstruction of the neutral plasma component velocity field was raised for the first time [17], we will not introduce here restrictions on the spatial dimensions of the disturbances and, following the results of our previous work [11], consider the DFS variations for IGWs moving at near-to-horizontal angles.

Direct and inverse problems

In the isotropic case, disturbance of the radio wave phase $\delta\phi = -(\omega/c)\int_L$ leads to a Doppler frequency shift

$$\omega_D = -\frac{\omega}{c} \int_L \frac{dn}{dN} \frac{\partial}{\partial t} \delta N dl \dots\dots\dots(1)$$

where $n = \sqrt{1 - N/N_0}$ is the refraction index and δN are the electron density and its disturbance. Considering region F of the ionosphere, where oxygen atom and ion are dominant in the neutral and ionic component respectively, let us write an equation of continuity for N in the form [6].

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial z} \left[\left(\frac{\partial N}{\partial z} + \frac{N}{2H_i} \right) D_0 e^{z/H_i} - u_z (1 - e^{-vr'}) N \sin I \cos I \right] - \beta N + Q \dots\dots\dots(2)$$

where u_z is vertical component of the neutral particle velocity; v is the delay constant in the disturbance of the ionized component relating to the neutral one; $t' = t - t_0$, t_0 is the time of disturbance start in the neutral component; H_i is the scale height for ions; I is the magnetic inclination; $D_0 \exp(z/H_i) = D_\alpha \sin^2 I$, D_α is the coefficient of ambipolar diffusion; $\beta = \beta_0 \exp(-Pz/H_i)$ and Q are the coefficient of recombination and velocity of ion formation, respectively.

Approximating the concentration profile of charge particles about a reflection point by the exponent, $N = N_0 \exp(z/H_i)$, and integrating the right hand side of the equation (1), taking into account (2) within the limits of $z = H_i$ to $z = 0$, we obtain for the Doppler frequency shift.

$$\omega_D = \left[\beta H_i - \left(3 \frac{D_0}{H_i} + \frac{H_i}{N_0} Q \right) e^{-z/H_i} \right] \Delta k_z + \dots\dots\dots(3)$$

$$+ H_i (1 - e^{-vr'}) \frac{\sin 2I}{2} \left(\frac{1}{2H} + \frac{1}{H_i} \right) \int_L u_z dk_z,$$

where $z = h - h_0$; $\Delta k_z = -2(\omega/c) \cos \theta \sqrt{1 - N(z)/N_0}$, θ is the angle between wavevector and the vertical at level z . The integral in the right hand side of (3) can be calculated by asymptotic Taylor expansion at $H_i \rightarrow 0$:

$$\int_L u_z dk_z \approx \int_{-H_i}^0 \left[u_z(\mathbf{r}_0) + (\mathbf{r} - \mathbf{r}_0) \frac{\partial u_z}{\partial \mathbf{r}_0} \right] dk_z(z) = u_z(0) \Delta k_z \left(1 + \frac{\langle z \rangle}{2H} \right),$$

where $\langle z \rangle \approx -0.258H_i$; $\Delta k_z \approx -1.59(\omega/c) \cos \theta$; ; $u_z(0) = u_z(x, y, t)|_{z=0}$. Then, introducing function $q = Q/\beta N$, at the level of $z = 0$, which corresponds to a reflection point, we obtain from (3).

$$\omega_D = H_i \Delta k_z \left[\beta(1 - q) - 3 \frac{D_0}{H_i^2} + \frac{\sin 2I}{2} \left(\frac{1}{2H} + \frac{1}{H_i} \right) \times \dots\dots\dots(4)$$

$$\times \left(1 - 0.129 \frac{H_i}{H} \right) u_z(0) (1 - e^{-vr'}) \right].$$

Equation (4) is the solution of the direct problem formed above. Reversing equations (3), (4), we easily find u_z as a function of for the inverse problem solution.

Let us write out solutions of the direct and inverse problems in case of $T_e = T_i$:

$$\omega_D = 2\Delta k_z \left[\beta H(1 - q) - 0.75 \frac{D_0}{H} + 0.371 \sin 2I \times u_z (1 - e^{-vr'}) \right], \dots\dots\dots(5)$$

$$u_z = \frac{0.5 \omega_D / \Delta k_z + 0.75 D_0 / H - \beta H(1 - q)}{0.371 \sin 2I [1 - \exp(-vr')]}$$

Let us remark that, at about noon time, when processes of ionization and recombination in ionosphere relatively counterbalance each other, i.e. $q \approx 1$, the equations in (5) become even simpler. During nighttime conditions, ion formation is almost absent, and $q = 0$.

The results of calculations according to formula (4) for conditions in the F-region of the ionosphere correspond to the real conditions when $T_e = T_i$ (solutions (5) are valid), $\Delta k_z = -5.3 \cdot 10^{-2} \text{ M}^{-1}$, $\theta = 0$ (vertical sounding), (region of the magnetic latitude $\phi_m = 45^\circ$), $\beta_0 = 3.1 \cdot 10^{-4} \text{ c}^{-1}$, $D_0 = 3.1 \cdot 10^5 \text{ M}^2 \text{ c}^{-1}$, $q = 0$ (night), and when u_z is a 2D soliton solution of the generalized Kadomtsev–Petviashvili equation (GKP equation) for the upper atmosphere (obtained by Belashov [5], see also [9]) are shown in figure 1.

Figure 1: Profile $u = u_z / \sqrt{gH}$ for the IGW soliton of the KP equation for the upper atmosphere and corresponding DFS variation.

Note that variations ω_D of this type have been repeatedly observed in numerous experiments of Doppler sounding of the ionosphere, which, when the results of this work are taken into account, can be interpreted as the effect caused by moving solitary structures in the F -region – 2D IGW solitons.

Conclusion

The obtained solutions of the direct and inverse problems are clear and quite convenient for practical use in problems related to the identification of IGW disturbances in the records of the indicated DFS signal during ionospheric Doppler sounding of the ionosphere. This representation of the link between DFS and the vertical component of the neutral particles velocity is convenient, because a preliminary reconstruction of the electron density profile over the disturbance velocity field is not needed to solve the direct problem of computation of the Doppler shift. When solving the inverse problem, we can immediately reconstruct the velocity field of the neutral component from the Doppler shift records - directly from the dopplerogram. The presented particular example clearly illustrates the practical significance of the obtained results in studies of IGWs by the DFS method.

The obtained results are currently of particular importance for the detection of soliton-like wave “forerunners” in the F -region,

which are generated in regions of steep gradients of the basic ionospheric parameters during the motion of the solar eclipse spot and solar terminator [12,18,19] and are observed in numerous ionospheric sounding experiments [13-16]. The results can also be useful in for problems related to the impulse impact on the ionosphere of sources such as seismic events [7] and land-surface artificial explosions [8].

Bibliography

1. Bryunelli BE and Namgaladze AA. Fizika ionosfery (Physics of the Ionosphere), Nauka, Moscow (1988).
2. Hocke K and Schlegel K. “A review of atmospheric gravity waves and travelling ionospheric disturbances: 1982-1995”. *Annales Geophysicae* 14.9 (1996): 917-940.
3. Grigor’ev GI. “Acoustic-gravity waves in the earth’s atmosphere (review)”. *Radiophysics and Quantum Electronics* 42.1 (1999): 1-21.
4. Nagorsky PM. “Analysis of the response of an HF radio signal to ionospheric plasma disturbances caused by acoustic shock waves”. *Radiophysics and Quantum Electronics* 42.1 (1999): 31-38.
5. Belashov VYu. “Solitary electron density waves induced by the IGW’s solitons in the ionosphere”. Proc. Int. Symp. on EMC, Nagoya, Japan, Sept. 8-10 IEEE, New York, 1 (1989): 64-67.
6. Belashov VYu. “Dynamics of nonlinear internal gravitational waves at heights of the ionospheric F-region”. *English Translation* 30.4 (1990): 637-641.
7. Pertsev NN and Shalimov SL. “The generation of atmospheric gravity waves in a seismically active region and their effect on the ionosphere”. *Geomagnetism and Aeronomy English to Translation* 36.2 (1996): 223-227.
8. Drobzheva YaV and Krasnov VM. “The acoustic field in the atmosphere and ionosphere caused by a point explosion on the ground”. *Journal of Atmospheric and Solar-Terrestrial Physics* 65.3 (2003): 369-377.
9. Belashov VYu and Vladimirov SV. “Solitary Waves in Dispersive Complex Media”. Theory, Simulation, Applications, Springer, Berlin (2005).
10. Belashov VYu. “The solar terminator front-induced wave disturbances in the ionosphere F layer”. Proc. Int. Symp. on EMC, Nagoya, Beijing, China (1992): 141.

11. Belashov VYu., *et al.* "Dynamics of IGW and traveling ionospheric disturbances in regions with sharp gradients of the ionospheric parameters, Australian Institute of Physics 17th National Congress, Brisbane, Queensland, Australia (2006): WC0111.
12. Belashova ES., *et al.* "Structure and evolution of IGW and TID in regions with sharp gradients of the ionospheric parameters". *Journal of Geophysical Research* 112 (2007): A07302.
13. Belashov VYu and Poddelsky IN. "The complex experimental investigations of the F layer's inhomogeneous structure in the region of solar terminator front". Proc. Int. Symp. on EMC, Nagoya, Beijing, China (1992): 145.
14. Galushko VG., *et al.* "Bistatic HF diagnostics of TIDs over the Antarctic Peninsula". *Journal of Atmospheric and Solar-Terrestrial Physics* 69 (2007): 403-410.
15. Nasyrov IA., *et al.* "The measurement of the ionospheric total content variations caused by a powerful radio emission of "Sura" facility on a network of GNSS-receivers". *Advances in Space Research* 57 (2016): 1015-1020.
16. Nasyrov IA., *et al.* "Study of nonlinear wave structures in the ionosphere, stimulated by the solar terminator and strong radio emission of "Sura" facility, XII Annual Conference "Plasma Physics in the Solar System". (2017): 6-10.
17. Savel'ev VL. "On recovery of the wind field of the neutral component of ionospheric plasma from Doppler frequency shift, Wave Disturbances in the Ionosphere". Nauka, Alma-Ata (1987): 60-66.
18. Belashov VYu. "Analytical solution of direct and inverse problems in the internal gravity waves studies by the Doppler frequency shift method". *Geomagnetism and Aeronomy* 58.5 (2018): 651-653.
19. Belashov VYu and Belashova ES. "Dynamics of IGW and traveling ionospheric disturbances in regions with sharp gradients of the ionospheric parameters". *Advances in Space Research* 56 (2015): 333-340.

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