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Review Article

Mathematical Modelling for Crop Pricing based on Market Value of its Products

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Abstract

The attempt of the research paper is to develop a mathematical model framework to determine the valuation of a crop based on the premise that the key drivers of the crop lie at the consumer's end. The key factors are the crop production costs, crop productivity, service costs and market value of the crop products due to urban tastes and population growth. The framework is designed based on linked differential equations. Farmers are expected to get the valuation of their harvests based on end consumer pay-off, thus providing them reasonable bargaining power to increase their farming profits. It is expected to provide farmers the valuation of the crops before cultivation, which will enable them to choose the most profitable crop. It also provides the farmers better bargaining power and encourages building food processing enterprises to reach the consumers through the most efficient channels. This helps famers to create more successful business groups and organize farming business units. The valuation of the crop considers the risk factors in the value chain and ensures fair crop valuation share based on the end consumer pay-off.

Keywords: Crop Valuation; Differential Equations; Market Dynamics

Introduction

Agriculture is the noblest profession and provides the sustenance for the survival of mankind. Most of the population in low income economies is mostly dependent on agriculture as primary source of income. The farmers with limited resources and land are the primary drivers of the food security of their nation. In the present scenario of climate change, decreasing plant species diversity, increase in adaptive capacity and diversity of agricultural pests, soil fertility depletion, input price rise and volatility of agricultural crop prices it is very difficult for the farmers to market both ends meet.

There is an urgent need to recognize the effort and contributions of the farmers in terms of the valuation of the agricultural products being paid off by the end consumers. It has been noted that the service sectors mostly involved in trading of the harvests enjoys the maximum benefits. Here is an attempt to evaluate the crop harvest based on the valuation of the products paid off by the end consumers. The crop valuation is done based on the market valuation of different crop products.

The model framework considers data from various sources related to agriculture - weather, input costs (pesticides, fertilizer, etc.) based on crop selection, market price dynamics and demand and supply of produce. This considers massive amounts of very high dimensional and unstructured data which are continuously produced and analyzed to support farmers in real time for the best possible market place and price. The system would have an alert system to draw the attention of the government and responsible authorities if a cluster of farmers are at financial risk. This ensures better livelihood for the farming communities.

Method to develop the Model

Let c(t), p(t) and u(t) three dependent variables representing crop valuation, population and urbanization, all depending on time t. The crop valuation includes the crop productivity, costs related to crop production and processing and valuation due its products or derivatives in the market. It has been assumed that the population is practically independent of crop valuation and urbanization. This is considered based on the observation of population growth in under developed and developing countries.

First Euler and later Malthus suggested a model for population growth as,

$$\frac{dp}{dt} = \propto p,\ \propto > 0 \qquad (1)$$

Thus,

$$p(t) = p_0 e^{\propto t}, p_0 = p(0) \qquad (2)$$

Population cannot grow in uninhibited fashion indefinitely, thus replacing $\propto$ of equation (1) by function,

$f(p)$.

$$\frac{dp}{dt} = f(p)\,p \qquad (3)$$

Let's assume, growth rate decreases in proportion (a) to the population, thus

$$f(p)=(\propto -ap),(a>0) \qquad (4)$$

Now from (3),

$$\frac{dp}{dt} = (\propto\ -ap)\,p$$

$$\frac{dp}{dt} = \propto p\left(1-\frac{p}{K}\right), K = \frac{\propto}{a} \qquad (5)$$

at $t = 0,\ p = p_0$.

This is the well-known logistic model for population, as suggested by Verhulst Solving equation (5) we get

$$p\,(t) = \frac{Me^{\propto t}}{1+\frac{M}{K}e^{\propto t}} \qquad (6)$$

where, $M = \frac{p_0}{1-\frac{p_0}{K}}$ (7)

$$\lim_{t\to\infty} p(t) = K \qquad (8)$$

The logistic equation (5) is extensively used with some modification for population theory. The famous Predator-Prey model by Lokta Volterra has been derived from the logistic equation which has been used for various species.

The law of entropy in Informatics suggests that as population grows a census of a nation will miss some number of people which is likely to be caused mostly by the dynamics of the very system of recording the census. So, let us as assume,

$$p(t)= P(t)+ \varepsilon(t) \qquad (9)$$

Where, P(t) is recorded by the census and ε(t) is the error in the record. It is expected that with efforts of statisticians and increase in use of technology in governments the census gets better with time, we may assume ε(t) satisfies the following properties:

i. $\varepsilon(t)>0\ \forall t$

ii. $\varepsilon'(t)<0$,

which implies that ε(t)is decreasing function of t

iii. $\lim_{t\to\infty} \varepsilon(t) = 0$

Then, let us assume that P(t) satisfies the logistic equation,

$$\frac{dP}{dt} = \propto P\left(1-\frac{P}{K}\right), \propto > 0 \qquad (10)$$

And

$$\frac{d\varepsilon}{dt} = -m\varepsilon,\ m > 0 \qquad (11)$$

At $t = 0,\ P = P_0$ and $\varepsilon = \varepsilon_0$ from (9)

$$p_0 = P_0 + \varepsilon_0 \qquad (12)$$

And

$$\frac{dp}{dt} = \propto P\left(1-\frac{P}{K}\right) - m\varepsilon \qquad (13)$$

$$\left.\frac{dp}{dt}\right|_{t=0} = \propto P_0\left(1-\frac{P_0}{K}\right) - m\varepsilon_0 \qquad (14)$$

In order that p(t) is an increasing function at t=0,

$$\propto P_0\left(1-\frac{P_0}{K}\right) > m\varepsilon_0 \qquad (15)$$

Then,

$$\propto > \frac{m\varepsilon_0}{P_0\left(1-\frac{P_0}{K}\right)} \qquad (16)$$

If m=0.01, q=0.01, r=10

$$\propto > 0.0111 \qquad (19)$$

So that population may increase at t=0. If population is small, like some of the northern European countries, may m and q will be small. Then $\propto$ could be much less than 1%, which means rate of growth of population could be lot less than 1%. Hence, we assume that at t=0, p'(t) is positive and p(t) is increasing.

A portion of population lives in the urban areas. Then the rate of change of urbanization is proportional to the rate of change of population.

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$$\frac{du}{dt} = \lambda \frac{dp}{dt} \qquad (20)$$

At t=0, $u=u_0=\lambda p_0$.

The mathematical model for crop valuation has been developed based on four major drivers, such as Market Value of the products of the crop, Service Cost for processing the crops to market and Cost of production to raise the crop. The relation has been composed based on the functional relation between the variables. It is obvious there are the factors will be

c=U+V-W+X+Y (21)

where,

U = increase of c per unit of time due to service cost for processing the units of crop to produce unit products

V = increase of c per unit of time due to cost of production of units of crop

W = decrease of c per unit of time due to productivity of units of crop

X = increase of c per unit of time due to urbanization per unit of time

Y = increase of c per unit of time due to population per unit of time

Thus,

$$\frac{dc}{dt} = \frac{\gamma}{n}c + \delta c - \rho c + \frac{\varphi}{q_1}cu + \frac{\theta}{q_2}cp \quad (22)$$

where, $U = \frac{\gamma}{n}c, V = \delta c, W = \rho c, X = \frac{\varphi}{q_1}cu, Y = \frac{\theta}{q_2}cp$

γ = % of growth of c per unit of μ per unit time; n= crop units processed to produce a product

δ = % of growth of c per unit of μ per unit time

ρ = % of loss of c per unit of μ per unit time

φ= % of growth of c per unit of u per unit time; q_1= unit of products of crop available in urban space

θ= % of growth of c per unit of p per unit time; q_2= unit of products of crop available

μ the units of products processed in unit time.

At t=0, $c=c_0$, solution of equation (22) gives the valuation of the crop based on market dynamics, based on which farmers will empower to bargain the profit share.

Since $u = \lambda p$,

$$\frac{dc}{dt} = \{(\frac{\gamma}{n} + \delta - \rho) + (\frac{\varphi}{q_1}\lambda + \frac{\theta}{q_2})p\}c \quad (23)$$

$$\frac{dc}{dt} = \{(\frac{\gamma}{n} + \delta - \rho) + \eta p\}c \quad (24)$$

$$\text{where, } \eta = (\frac{\varphi}{q_1}\lambda + \frac{\theta}{q_2}) \quad (25)$$

At $t = 0$,

$$\left.\frac{dc}{dt}\right|_{t=0} = \{(\frac{\gamma}{n} + \delta - \rho) + \eta p_0\}c_0 \quad (26)$$

Since, $c_0>0$, $p_0>0$, if η>0 and (γ/n+δ)>ρ

$$\left.\frac{dc}{dt}\right|_{t=0} > 0 \quad (27)$$

Making c an increasing function at t=0

If $\left(\frac{\gamma}{n} + \delta\right) < \rho$ and $\left\{\left(\frac{\gamma}{n} + \delta - \rho\right) + \eta p_0\right\} < 0, \frac{dc}{dt}$ at $t = 0$ will be decreasing function, and crop valuation will start decreasing from the beginning. This implies that, if the crop productivity cost relative to other costs and demand of crop products are low, the crop valuation is destined to be low.

Analytical solution of the model

From (9) we can construct

$$\frac{dp}{dt} = \frac{dP}{dt} + \frac{d\varepsilon}{dt} \quad (28)$$

From (5) and (28)

$$P(t) = \frac{Me^{\alpha t}}{1+\frac{M}{K}e^{\alpha t}} \quad (29)$$

$$\text{where, } M = \frac{P_0}{1-\frac{P_0}{K}} \quad (30)$$

If $\beta = \frac{M}{K}$, then

$$\beta = \frac{\frac{P_0}{K}}{1-\frac{P_0}{K}} \quad (31)$$

This gives,

$$\frac{P_0}{K} = \frac{\beta}{1+\beta} \quad (32)$$

From (10)

$$\frac{d\varepsilon}{dt} = \varepsilon_0 e^{-mt} \quad (33)$$

m is small for a small exponential decay. Thus

$$p(t) = P(t) + \varepsilon(t) = \frac{K\beta e^{\alpha t}}{1+\beta e^{\alpha t}} + \varepsilon_0 e^{-mt} \quad (34)$$

It must be noted that α and m both are constants, one indicates increase of population while the other indicates the decrease of error in census.

Urbanization depends only on population and as such

u(t)= λp(t) (35)

From (24)

$$\int_{c_0}^{c}\frac{dc}{c} = \int_0^t(\frac{\gamma}{n} + \delta - \rho)\, dt + \eta\int_0^t p\, dt \quad (36)$$

$$ln\frac{c}{c_0} = \left(\frac{\gamma}{n} + \delta - \rho\right)t + \eta(I_1 + I_2)$$

where, $I_1 = K\int_0^t \frac{K\beta e^{\alpha t}}{1+\beta e^{\alpha t}}dt = \frac{K}{\alpha}ln\left\{\frac{1+\beta e^{\alpha t}}{1+\beta}\right\}$

And, $I_2 = \varepsilon_0\int_0^t e^{-mt}dt = \frac{\varepsilon_0}{m}(1 - e^{-mt})$

Then from (36)

$\frac{c}{c_0} = e^{F(t)}$ (37)

$F(t) = \left(\frac{\gamma}{n} + \delta - \rho\right) t + \frac{K\eta}{\alpha} ln\left\{\frac{1+\beta e^{\alpha t}}{1+\beta}\right\} + b\,\eta(1 - e^{-mt})$ (38)

where, $b = \frac{\varepsilon_0}{m}$

Analytical solution of the model

From (38), F(0)=0, which satisfies the condition that

$c(0)=c_0 e^{F(0)}=c_0$ (39)

From (38),

$c'(t)=c_0 e^{F(t)} F'(t)$ (40)

$c''(t)=c_0 e^{F(t)} \{F'(t)^2+F''(t)\}$ (41)

Also,

$F'(t) = \left(\frac{\gamma}{n} + \delta - \rho\right) + \frac{K\eta\beta e^{\alpha t}}{1+\beta e^{\alpha t}} + \varepsilon_0 \eta e^{-mt}$ (42)

Since, $b = \frac{\varepsilon_0}{m}$

$F'(0) = \left(\frac{\gamma}{n} + \delta - \rho\right) + \frac{K\eta\beta}{1+\beta} + \varepsilon_0\eta$

$= \left(\frac{\gamma}{n} + \delta - \rho\right) + \eta[\frac{K\beta}{1+\beta} + \varepsilon_0]$

$= \left(\frac{\gamma}{n} + \delta - \rho\right) + \eta p_0$ [from (34)]

$F'(0) = \left(\frac{\gamma}{n} + \delta - \rho\right) + \eta p_0$ (43)

In general, from (34) and (42)

$F'(t) = \left(\frac{\gamma}{n} + \delta - \rho\right) + \eta p(t)$ (44)

Also, $F''(t) = \eta\left\{K\alpha \frac{\beta e^{\alpha t}}{(1+\beta e^{\alpha t})^2} - m\varepsilon_0 e^{-mt}\right\}$ (45)

$= \eta p'(t)$

Case 1: If η>0, then since p(t)>0, F' (t)>0 if (γ/n+δ)>ρ and which is true when the service cost and cost of production are greater than the crop productivity, F(t) is then an increasing function.

η>0, implies that the demand for crop products with urbanization and population growth are in positive trend. Larger values of p(t)will increase F' (t) which mean F(t) will increase faster. However, it will also increase the ratio p/p_0 which will affect (40) adversely.

Also, if ρ is large such that (γ/n+δ)=ρ, since p'(0)>0 [p(t)>0, increasing at, t=0], F'' (t)>0. This means F(0) is a minimum, F(t) is increasing at t=0.

Case 2: If η=0 , then F' (t)=(γ/n+δ-ρ). Then F(t) is an increasing function. Thus, demand of crop products due to population has no effect on crop valuation. Here our model fails.

Case 3: If η<0, then in order F(t) is an increasing function, (γ/n+δ) > ρ+ωp(t), where ω = -η>0. If the condition fails, the valuation of crop is bound to decrease.

Considering the different scenarios in the mathematical model the valuation of the crops will be estimated with statistical estimates of the growth rates for the included parameters in the model.

Big data applications on crop valuation

The model captures the growth rates related to crop production, cost of production, service costs to convert harvest to food products and demands due to population growth and urbanization. It considers variety of data on cropping activities, market dynamics and urban and population growth. The data vary from official agricultural statistics on crop production, sensors measuring weather, plant epidemics, costs of crop products, economics related to food products, market trends and urban and population growths. The seasonal updates on variety of data and measuring the growth estimates for the models; real time data analysis for crop production estimates due to various weather parameter and epidemics and market demands provide the best valuation of a crop. The level of risks borne by each stakeholder qualifies them to get rational share of the pay-off. Thus, the entire ecosystem qualifies for Big Data considering 3Vs (Volume, Variety and Velocity) characteristics of big data.

Review on crop valuation models

Possibly there are very few studies done on crop valuation estimates considering all the related factors from crop production to end consumer consumption. Basically, the studies are done on historic trends of price movements. In a study of Jha and Sinha, the superiority of Artificial Neural Network (ANN) model over linear model methodology has been demonstrated using monthly wholesale price series of soybean and rapeseed-mustard. Another study used cross sectional cum time series for the ten years by keeping in view to estimate input demand and output supply by using normalized Cobb-Douglas profit function for cotton [1-12].

Conclusion

The mathematical model is developed to evaluate crop valuation considering the cost involved in producing the crop-based products, productivity of the crop and the demand of the crop products due to urban tastes and population growth. The model being generic is expected to work any type of season and crop but need to be evaluated with data and statistical estimates. The model estimates are expected to help the farmers to bargain better in the market place. Since the issues in rural livelihood are ever rising, a watchdog system is in demand to minimize the risks involved due to market dynamics, in economics setting.

It has been recorded that cost of food in India increased 7.79 % in June of 2016 over June 2015, Food Inflation in India averaged 8.51 percent from 2012 until 2016. The average daily rural wage for ploughing/tilling in May 2015 was Rs. 272.87, only 3.3% higher in May last year. The rural incomes have eroded as minimum support prices (MSPs) haven't kept pace with rising input costs. Thus, there is an imbalance in rural economy which costs farmers' suicides, rapid growth of urban slums and risks food security of the nation. Thus, an integrated system supported by real time big analytics is needed to have a fair share of profit in agriculture earned due to consumers' pay-off for crop products, based on the incurred risks in the business.

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