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Short Communication

A Short Discussion on Ramanujans İnfinity Sum

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Ramanujan is the one of the most remarkable mathematicians of history, He made very important contributions in mathematics. His magnificient intelligence was first recognised by Thomas Hardy. He derived infinity sum of natural numbers equal to -1/12. His derivation and his logic will be presented and discussed, some contradictions will be given.

It is clearly well known from Gauss the sum of natural numbers from zero to k is given

$$\zeta(s) = \sum_{n-1}^{\infty} \frac{1}{n^s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots$$

$$\zeta(s=-1) = \sum_{n-1}^{\infty} \frac{1}{n^{-1}} = 1 + 2 + 3 + \cdots$$

$$S_A = 1 + 2 + 3 + 4 + \cdots + k + \cdots \infty = \frac{k(k+1)}{2} \text{ as } k \to \infty$$

All secondary school students they know it very well. Despite all Ramanujan derived that this sum equal to -1/12, his logic will be repeated here step by step.

Let $S_A = 1 + 2 + 3 + 4 + \cdots$

And $S_k = k - k + k - k + k - k + \cdots$, and

for $k = 1$; $S_{k=1} = 1 - 1 + 1 - 1 + 1 - 1 + \cdots$

$S_B = 1 - 2 + 3 - 4 + 5 - 6 + \cdots$

$S_B - S_A = 1 - 2 + 3 - 4 + 5 - 6 + 7 \ldots$

$-1 - 2 - 3 - 4 - 5 - 6 - 7 = -4 - 8 - 12 - 16 - \cdots$

$= -4S_1$

All secondary school students they know it very well. Despite all Ramanujan derived that this sum equal to -1/12, his logic will be repeated here step by step.

Then $S_1 = -\frac{S_3}{3}$, it is very trivial.

But $S_{k=1} = 1 - 1 + 1 - 1 + 1 - 1 + \cdots =$

$1 - (1 - 1 + 1 - 1 + 1 - 1 + \cdots) = 1 - S_{k=1}$

and $S_{k=1} = \frac{1}{2}$. At this point there is a debate,

this not only one solution. Also it is possible to imagine the other way

$S_{k=1} = -1 + 1 - 1 + 1 - 1 + 1 - 1 \ldots$

$= -1 - (-1 + 1 - 1 + 1 - 1 + 1 - 1 \ldots)$

$= -1 - S_{k=1}$

And

$S_{k=1} = -\frac{1}{2}$ is equally possible. When

$S_{k=1} = -\frac{1}{2}$ is selected, the infinite sum becomes $S_1 = \frac{1}{12}$.

$S_B - S_{k=1} = 1 - 2 + 3 - 4 + 5 - 6 + \cdots$

$-1 + 1 - 1 + 1 - 1 + 1$

$= -1 + 2 - 3 + 4 - 5 + 6 = -S_B$

$S_B = \frac{S_{k=1}}{2}$; $S_A = -\frac{S_B}{3} = -\frac{1}{3}\frac{S_{k=1}}{2}$

$= -\frac{1}{3}\frac{1}{2}\left(\pm\frac{1}{2}\right) = \pm\frac{1}{12}$

with respect to the selection of $S_{k=1}$

$S_k = k - k + k - k + \cdots, S_k = k - (k - k + k - k + k - k)$

then $S_k = \frac{k}{2}$ and $S_k = -k - (-k - k + k - k + \cdots)$ then $S_k = -\frac{k}{2}$,

$S_A = 1 + 2 + 3 + 4 + \cdots + k + \cdots = \frac{k(k+1)}{2}$ as $k \to \infty, S_1 \to \infty$

$S_B = 1 - 2 + 3 - 4 + 5 - 6 + \cdots$

$S_B - S_k = 1 - 2 + 3 - 4 + 5 - 6 + 7 \ldots$

$\pm(k-1) \mp k \pm (k+1) \mp (k+2) \mp \cdots$

$-k + k - k + k - k + k - k + k$

$S_B - S_k = (1-k) - (2-k) + (3-k) - (4-k) + (5-k) \ldots$

And again $S_A = -\frac{S_B}{3}$, for k is odd number

$S_B - S_k = 1 - 2 + 3 - 4 + 5 - 6 + 7 \ldots$

$-(k-1) + k - (k+1) + (k+2) - (k+3) \ldots$

$-k + k - k + k - k + k - k + k$

$= (1-k) - (2-k) + (3-k) - (4-k) + \cdots$

$-(k-1-k) + (k-k) - (k+1-k) + (k+2-k)$

$= (1-k) - (2-k) + (3-k) - (4-k) + \cdots$

$-(k-1-k) + 0 - 1 + 2 - 3 + 4$

$= (1-k) - (2-k) + (3-k) - (4-k) + \cdots$

$-(k-1-k) - S_B$

$S_B = \frac{S_k}{2} + (1-k) - (2-k) + (3-k) - (4-k) + \cdots$

$-(k-1-k)$

$S_A = -\frac{S_B}{3}$

$= -\frac{1}{3}\left[\frac{S_k}{2} + (1-k) - (2-k) + (3-k) - (4-k) + \cdots - (k-1-k)\right]$

$= -\frac{1}{3}\left[\pm\frac{1}{2}\frac{1}{2} + (1-k) - (2-k) + (3-k) - (4-k) + \cdots - (k-1-k)\right]$

$= \mp\frac{1}{12} - \frac{1}{3}[(1-k) - (2-k) + (3-k) - (4-k) + \cdots - (k-1-k)]$

There does not exist only one unique solution, all derivations depend on the choices of series.

Let us check some cases for k=5

$S_B - S_{k=5} = 1 - 2 + 3 - 4 + 5 - 6 + 7 \ldots$

$-5 + 5 - 5 + 5 - 5 + 5 - 5 + 5$

$= 1 - 2 + 3 - 4 + 0 - 1 + 2 - 3 + 4$

$= 1 - 2 + 3 - 4 - S_B$

$2S_B = +S_{k=5} + 1 - 2 + 3 - 4$

$S_B = +\frac{S_5}{2} + \frac{1 - 2 + 3 - 4}{2}$

$S_A = -\frac{S_B}{3} = -\frac{1}{3}\left[\frac{S_{k=5}}{2} + \frac{1 - 2 + 3 - 4}{2}\right]$

For $k = 9$

$S_B - S_{k=9} = 1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + 9 - 10 + 11 - \cdots$

$-9 + 9 - 9 + 9 - 9 + 9 - 9 + 9$

$S_B - S_{k=9} = 1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + 0 - 1 + 2 - 3 + 4 \ldots$

$= 1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 - S_B$

$S_B = \frac{1}{2}[S_{k=9} + 1 - 2 + 3 - 4 + 5 - 6 + 7 - 8]$

It is very clear that the solution depends on the selection of k and S_k=±1/2, and S_1 may be equally ±1/12 for k=1.

Despite Ramanujan is one of the most important mathematicians in science history, it is not possible that Ramanujan infinite sum is unique absolute solution or derivation.